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It is remarkable that the liquid-curve in fig. 5 should differ so little from the straight line. So it seems that for these mixtures at the chosen temperature the vapour-pressure of the liquid-phase may be represented with near approximation by:

$$P = P_1 (1-x) + P_2 x$$

in which P_1 and P_2 represent the vapour-pressures of the components. If we compare this with the formula given by VAN DER WAALS [1], it appears that the exponents in the latter differ little from zero.

Physics. — „*Considerations concerning the Influence of a Magnetic Field on the Radiation of Light.*" By Prof. H. A. LORENTZ.

§ 1. The assumption that every molecule of a source of light contains a single movable ion, which can be displaced in all directions from its position of equilibrium and is always driven back to that position by the same force, proportional to the displacement, leads to the elementary theory of the phenomenon discovered by Dr. ZEEMAN. Viewed across the lines of force, a single spectral line must, by the action of the field, be tripled, and viewed along the lines of force, be doubled; besides, the components of the triplets and doublets must be polarised in a well known manner.

Whilst the first observations of ZEEMAN were consistent with this theory, and he soon could confirm the theoretical predictions by the observation of distinct triplets and doublets, yet it has become apparent that the case is often less simple. CORNU proved, that f. i. in the case of one of the sodium lines, viewed across the lines of force, the central component of the triplet is doubled, so that in reality a quadruplet is seen. MICHELSON and Th. PRESTON observed in many cases not only a far more complicated structure of the central component, but a similar structure of the outer components of the triplet. According to these observations, the word „triplet" is hardly applicable, though there is always an important difference between the central part of the appearance in the spectrum (the two central lines, f. i. of CORNU's quadruplet) and the outer parts; the first is plane polarised, the plane of polarisation being perpendicular to the lines of force, whereas in the right and left part the plane of polarization is parallel to the lines of force.

§ 2. The facts mentioned evidently make it necessary to replace

[1] v. d. WAALS, Verslag Kon Aknd May 28 '91 p. 416.

the elementary theory by a more complete one. Some time ago, I examined [1]) therefore what phenomena are to be expected, if a molecule, having an arbritary number of degrees of freedom and arbritarily distributed electric charges, is oscillating about a position of equilibrium.

Before returning to this subject, I will consider the conclusions which may be obtained by arguments from symmetry, without entering into the details of the mechanism of radiation.

There can be no doubt, that we may consider the source of light as a system of extremely small particles, oscillating partly with the frequency of the light vibrations; in virtue of their electric charges, these particles must excite in the surrounding aether periodically oscillating dielectric displacements. These constitute the luminous motion radiated by the source.

For briefness' sake this entire system will be indicated by S.

We may now conceive a second system S', which is the image of S, relatively to a fixed plane P. The meaning of this is as follows.

If A is a particle in S, there is in S' a particle A', which is the image of A and of the same physical nature as this particle. Especially, the mass and the electric charge are the same; or, to speak more accurately, in corresponding points of A and A' the same material density and the same density of electric charge will be found. Moreover the particles A' will be *at every moment* the image of the particles A, or, as we may say, the motion of the ions in S' will be the image of the motion in S. If this is the case, the luminous motion in the aether in S' will likewise be the image of the motion in S, in this sense that the vector representing the dielectric displacement in S' is always the image of a corresponding vector in S.

Of course all this will only be possible, if the forces operating on the particles A' are the images of those to which the particles A are subjected. So far as the *mutual* action of the particles is concerned, we may regard this as a consequence of the supposed equality in physical nature. In order that the forces, originated by the external magnetic field, may satisfy the same condition, we will suppose that the vectors, representing the magnetic force in S', may be derived from the corresponding ones in S by first taking their images, and then reversing the directions of these images [2]).

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[1]) Wied. Ann. Bd. 63, p. 278, 1897.

[2]) If the magnetic field is generated by electric currents, we may imagine the required field in S' to be produced by currents, which are the images of the currents in S.

It will also be assumed that the properties of the image of the source of light, as far at least as we are concerned with them in really observable phenomena, are the same as those of the source itself, so that the latter may be substituted for the image. Finally, we suppose that the entire luminous motion in the aether is developed by means of FOURIER's theorem into simple harmonic motions; when the total luminous motions in S and S' are each other's images, the same will evidently be true of those parts of the luminous motions, which have a determinate period T — or rather periods between two definite limits T and $T + d\,T$.

§ 3. Let Q be a straight line, drawn from any point in the source of light parallel to the lines of force, and let L denote the luminous motion *with a definite period* T, existing at a distant point of Q. By taking the image of the whole system, relatively to a plane parallel to the line Q, it is easily seen that the image L' is exactly the luminous motion, that would exist in the point considered, if, the source of light remaining unchanged, the direction of the field were reversed. Hence L' may very well differ from L, but, in all observable properties, L' must remain unchanged, if the reflecting plane be turned around the line Q as axis.

Whence it follows, that, if all vibrations of L are resolved parallel to a line R, perpendicular to Q, the intensity produced by the components must be independent of the direction of R. Indeed, R_1 and R_2 being two lines perpendicular to Q, and I_{r1} and I_{r2} the intensities corresponding to them in the manner indicated, we may give to the reflecting plane two positions P_1 and P_2 in such a way that the image of R_1, relatively to P_1, coincides with that of R_2, relatively to P_2. Indicating by R' the direction of these coinciding images and by $I'_{r'}$ the intensity corresponding to this direction of vibration in L'— this quantity remaining the same, as was remarked above, for every position of the reflecting plane — we may write $I_{r1} = I'_{r'}$ and $I_{r2} = I'_{r'}$; hence $I_{r1} = I_{r2}$.

In this way we come to the conclusion that the light propagated along the lines of force, and having a definite period T, or, in other words, occupying a definite place in the spectrum, cannot be polarised plane or elliptically, neither completely, nor partially. It can only be unpolarised, or circularly polarised; in the latter case the polarization can be partial as well as complete.

The light would be unpolarised, if an influence of the magnetic field did not exist at all. As far as we know, the components of the doublets seen along the lines of force are completely circularly pola-

rised. From the above considerations it however appears that the radiation might also be partially circularly polarised. We see at the same time that, if in a given place of the spectrum the polarization is righthanded, it must become lefthanded at the same place by reversing the magnetization.

§ 4. Arguments of the same kind may be used when the observations take place across the lines of force. Now, we place the reflecting plane perpendicular to these lines. The magnetic field remains unchanged; consequently, the luminous motion must have the same properties as its image. Hence the light, observed in a given place of the spectrum, cannot be circularly nor elliptically polarised — neither completely nor partially. It must be either unpolarised light, or plane polarised — wholly or in part — the plane of polarization being parallel or perpendicular to the lines of force.

It needs scarcely be mentioned, that all observations are in agreement with this conclusion.

§ 5. A closer examination of the mechanism of radiation gives us a relation between the light radiated along and the one radiated across the lines of force. At least one conclusion concerning this point lies at hand.

Let M be a single molecule ot the source of light, and let three rectangular axes OX, OY, OZ be drawn, the first along the line of force. Let e be the electric charge in a point of the molecule, having the coordinates x, y, z; then we may call $\Sigma\, e\, x$, $\Sigma\, e\, y$, $\Sigma\, e\, z$ — calculated for the entire molecule — the components of the electric moment of the particle.

These quantities are continually changing, and will perhaps be extremely complicated functions of the time. By means of FOURIER's theorem, we may however separate the parts that have a determinate period T. We will confine ourselves to these parts and denote them by $\mathfrak{M}x$, $\mathfrak{M}y$, $\mathfrak{M}z$.

If now the dimensions of the molecule are very small compared with the wave-length, then, the observer being supposed at a distance of a great many wave-lengths, it may be deduced from theory, that in all points of OY, light is produced merely by the variations of $\mathfrak{M}x$ and $\mathfrak{M}z$, $\mathfrak{M}x$ producing vibrations along, and $\mathfrak{M}z$ across the lines of force. Similarly for points of OX and OZ.

Suppose, that, when viewing across the lines of force, f. i. from a point of OY, in a given place of the spectrum light is seen

which is entirely plane polarised, the plane of polarization being perpendicular to the lines of force.

Then, at the place in question, there will not be any luminous motion produced by $\mathfrak{M}z$, and, because the molecules are vibrating independently of each other and hence light emitted by one can never be totally destroyed by that originating from another, $\mathfrak{M}z$ must vanish in all molecules. Of course the same argument applies to $\mathfrak{M}y$; hence it follows, that no light can be observed from any point of OX, that is to say in the direction of the lines of force.

BECQUEREL and DESLANDRES have found [1]) that one of the iron lines, when viewed across the lines of force, becomes a triplet, the central and side components of which, as compared with those of the ordinary triplets, have interchanged their states of polarization [2]). The foregoing reasoning entitles us to predict, that only the middle component of this triplet will be visible, when the phenomenon is observed in the direction of the lines of force.

§ 6. In the paper cited above, I have established the equations of motion for infinitely small vibrations of a molecule, having n degrees of freedom, and placed in a magnetic field. I called $p_1, p_2, \ldots p_n$ the general coordinates, chosen in such a manner, that they are 0 in the position of equilibrium, and that they are principal coordinates as long as there is no external magnetic force. I obtained for the equations of motion

$$a_1 \ddot{p}_1 + b_1 \dot{p}_1 - (c_{1.2} \dot{p}_2 + c_{1.3} \dot{p}_3 + \ldots + c_{1.n} \dot{p}_n) = 0, \left.\begin{matrix} \\ \\ \end{matrix}\right\} \ldots (1)$$

$$a_2 \ddot{p}_2 + b_2 \dot{p}_2 - (c_{2.1} \dot{p}_1 + c_{2.3} \dot{p}_3 + \ldots + c_{2.n} \dot{p}_n) = 0, \left.\begin{matrix} \\ \end{matrix}\right.$$

etc.,

where a and b are constants, independent of the magnetic force. The influence of the field is expressed by means of the terms containing the quantities c, which are all proportional to the intensity of the field.

They further satisfy the relations

$$c_{r.s} = - c_{s.r} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

To determine the possible periods of vibration, we put, according to a known method, in (1):

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[1]) Comptes rendus, 4 avril 1898.

[2]) The same phenomenon has been observed in the case of some iron-lines by AMES, EARHART and REESE, Johns Hopkins Univ. Circular. Vol. 17, No. 135.

$$p_1 = \mu_1\, e^{lt}\,, \quad p_2 = \mu_2\, e^{lt}\,, \quad \cdot \quad \cdot \quad \cdot \quad p_n = \mu_n\, e^{lt}\,,$$

and eliminate $\mu_1, \mu_2, \ldots \mu_n$. If

$$\frac{b_1}{a_1} = k_1{}^2, \quad \frac{b_2}{a_2} = k_2{}^2, \quad \cdot \quad \cdot \quad \cdot \quad \frac{b_n}{a_n} = k_n{}^2.$$

and

$$\frac{c_{r.s}}{a_r} = -\, e_{n.s},$$

the result may be put into the form

$$
\begin{vmatrix}
l^2 + k_1{}^2, & e_{1.2}\, l, & e_{1.3}\, l, & \cdots & e_{1.n}\, l \\
e_{2.1}\, l, & l^2 + k_2{}^2, & e_{2.3}\, l, & \cdots & e_{2.n}\, l \\
\cdot & \cdot & \cdot & & \cdot \\
\cdot & \cdot & \cdot & & \cdot \\
\cdot & \cdot & \cdot & & \cdot \\
e_{n.1}\, l, & e_{n.2}\, l, & e_{n.3}\, l, & \cdots & l^2 + k_n{}^2
\end{vmatrix}
= 0 \quad . \quad (3)
$$

In consequence of the relation (2), the development of the determinant will contain only even powers of l^2. Hence an equation is obtained, of the nth degree in l^2. From the circumstances of the case it follows, that the roots of this equation are all real and negative; hence n pairs of imaginary values of l are obtained. If $+\, i\, k'_r$ and $-\, i\, k'_r$ are two of those values, there will be a mode of vibration with the frequency (number of vibrations in the time $2\, \pi$) k'_r.

Evidently, without the field, the frequencies would become

$$k_1, \quad k_2, \quad \cdot \quad \cdot \quad \cdot \quad k_n\,,$$

and it is clear that, if there is a magnetic field, each of these frequencies is modified into a value k'_r, differing very slightly from k_r.

In the cited paper I had restricted the development of (3) to terms containing the products of two factors e. Denoting by Π the product

$$(l^2 + k_1{}^2)\; (l^2 + k_2{}^2) \quad , \quad \cdot \quad \cdot \quad (l^2 + k_n{}^2)$$

and by $\Pi_{r.s}$ the value, got from this by omitting the factors $l^2 + k_r{}^2$ and $l^2 + k_s{}^2$, we obtain

$$\Pi - \Sigma\, l^2\, e_{r.s}\, e_{s.r}\, \Pi_{r.s} = 0, \quad \ldots \ldots \ldots \quad (4)$$

where the sum is to be extended to all combinations of unequal indices r and s.

I inferred from this equation, that a triplet can only be observed, if three of the values k are equal, or, in other words, if the system has three equivalent degrees of freedom. This will also be clear when it is considered, that by a continuous decrease of the magnetic field, the three components of the triplet may be made to coincide, so that the simple spectral line, as seen out of the field, may be considered as consisting of three coinciding lines. Applying the same argument to CORNU's quadruplet, it seems natural to suppose that the lines, which are apt to undergo this modification, consist already, under ordinary circumstances, of four coinciding lines, or otherwise, that now we have four equivalent degrees of freedom, or four equal values k.

Yet, the origin of a quadruplet cannot be explained by equation (4). Indeed, if k_1, k_2, k_3, k_4 are the frequencies having the same value k, there are in each term of (4) at least two factors $l^2 + k^2$. Hence the equation must still have two equal roots $- k^2$, and besides only two roots, differing very little from $- k^2$.

§ 7. It was however brought to the notice of the author by Mr. A. PANNEKOEK that in this case equation (4) is incomplete, because some of the terms neglected are of the same order of magnitude as those retained, and that, by returning to equation (3), an explanation of the quadruplet may be arrived at.

If $k_1^2 = k_2^2 = k_3^2 = k_4^2 = k^2$, certainly four roots of equation (3), if not $= - k^2$, will differ only very little from this value.

If l^2 is one of these values (we need not occupy ourselves with the other values of l^2), then the four quantities $l^2 + k_1^2$, $l^2 + k_2^2$, $l^2 + k_3^2$, $l^2 + k_4^2$ will be small. On the other hand, the quantities

$$l^2 + k_5^2, \quad l^2 + k_6^2, \quad . \; . \; . \quad l^2 + k_n^2 \quad . \; . \; . \; . \quad (5)$$

will have values, which by no means become small. Since all the quantities $e\,l$ are likewise small, the elements (5) of the determinant will exceed by far all other elements, and we shall obtain a sufficient approximation, when we take in the development of the determinant only those terms, which contain all the quantities (5). Evidently, the equation serving for the determination of the values of l^2, which differ only slightly from $- k^2$, will therefore be

$$\begin{vmatrix} l^2 + k^2, & e_{1.2}\,l, & e_{1.3}\,l, & e_{1.4}\,l \\ e_{2.1}\,l, & l^2 + k^2, & e_{2.3}\,l, & e_{2.4}\,l \\ e_{3.1}\,l, & e_{3.2}\,l, & l^2 + k^2 & e_{3.4}\,l \\ e_{4.1}\,l, & e_{4.2}\,l, & e_{4.3}\,l, & l^2 + k^2 \end{vmatrix} = 0.$$

If we develop this determinant, all terms, which have an odd number of factors $e_{r.s}\,l$, are excluded by (2). Hence the equation may be written

$$(l^2 + k^2)^4 + A\,(l^2 + k^2)^2 + B = 0, \quad \cdots \cdots \quad (6)$$

where A contains terms with 2 factors $e_{r.s}\,l$, and B terms with 4 factors of this kind. In all these factors l^2 may be replaced by $- k^2$. Consequently, A and B can be found, and A is now proportional to the square and B proportional to the fourth power of the intensity of the field.

From (6) we get two values of $(l^2 + k^2)^2$, which are both real and positive, because, as was already remarked, real values must be found for l^2. Hence the solution of (6) may be represented by

$$(l^2 + k^2)^2 = \alpha^2, \quad \cdots \cdots \cdots \quad (7)$$

and

$$(l^2 + k^2)^2 = \beta^2, \quad \cdots \cdots \cdots \quad (8)$$

where α and β are known, say positive, quantities. By reason of what has been remarked about A and B, the values of α and β will be proportional to the intensity of the field.

Finally from (7) and (8) the following *four* values of l^2 are obtained:

$$l^2 = - k^2 + \alpha, \quad - k^2 - \alpha, \quad - k^2 + \beta, \quad - k^2 - \beta,$$

so that in fact there must be seen a quadruplet in the spectrum. In order that the four lines of this quadruplet may be perfectly sharp, it is however necessary, that in a given magnetic field the quantities α and β are independent of the direction into which the molecule is . turned, or, what comes to the same thing, that, for a given position of the molecule, α and β are independent of the direction of the magnetic force.

Mr. PANNEKOEK has also remarked, that a similar reasoning applies when an arbitrary number, e. g. p, frequencies k are equal. In this case we come to the conclusion that, for a given position of the

molecule in the field, the simple spectral line must be separated into a p-fold line, in such a way, that the position of the different components is symmetrical to the right and left of the original line. From this it follows, that if p is odd, one component remains at the place of the original line.

It seems however very difficult to conceive a system, having really, as is necessary for quadruplets, four equivalent degrees of freedom, especially if in addition to this, it is required that the values of α and β must be independent of the direction of the magnetic force, relatively to the molecule. I have not been able to find out a system, really fulfilling these conditions. It is true, it might be argued that the very existence of a quadruplet *proves* the equality of four frequencies, when there is no magnetic field, and hence that the above theory of the quadruplet must be true, even though the mechanism has not yet been found out. However I have some scruples to be satisfied with this view of the case, for I think, it is not yet quite certain, that the vibrations which produce light are really to be described by equations of the form (1).

Physics. — *On an Asymmetry in the Change of the spectral Lines of Iron, radiating in a Magnetic Field.* By Dr. P. ZEEMAN.

1. It is known that in the elementary treatment of the influence of magnetic forces on spectral lines according to LORENTZ's theory it is sufficient, if only one spectral line is considered, to suppose that in every luminous atom is contained one single moveable ion moving under an attraction proportional to the distance from its position of equilibrium. All motions of such an ion can be resolved into linear vibrations parallel to the lines of force and two circular vibrations, righthanded and lefthanded perpendicular to the lines of force. The period of the first mentioned vibration remains unchanged, those of the last are modified, one being accelerated and the other retarded. The doublets seen along the axis of the field, the triplets seen across it are in this manner simply explained and also the observed polarisation-phenomena. Besides we must expect according to the theory that the outer components of the triplet are of equal intensity and likewise the two circularly polarized components of the doublet. Eye observations as well as the negatives taken by myself and others have always confirmed till now this most simple symmetrical distribution of intensities. The question arises cannot the external magnetic forces, sufficient to direct the molecular