visual representation + a representation of a respiration-movement, performed in a special way. Self-observation calls this complex the apperceived or the attentively observed visual representation.

Though psychology cannot be treated now experimentally, yet the theory of physiological parallel processes justifies the opinion that attention and representations of movements originated by special respiration movements, are closely connected. Closer investigations on monkey's brains will be necessary. The other movements expressing attention will have to be submitted to a systematical investigation in the same way, to throw further light on the problem treated in this paper.

Physics. — „*Simple deduction of the characteristic equation for substances with extended and composite molecules*". By Prof. J. D. VAN DER WAALS.

If the quantity of substance inclosed in a certain volume is considered as consisting of material points, which may also be done with extended molecules, composed of atoms, the equation

$$\Sigma\, {}^1/_2\, m\, V^2 = {}^1/_4 \frac{d^2 \Sigma\, mr^2}{dt^2} - {}^1/_2\, \Sigma\, (Xx + Yy + Zz) \ldots . \quad (a)$$

holds, provided that the quantities occurring in this equation be applied to all material points.

If groups of these material points are united to separate systems, as is the case with molecules, which cannot be considered as one single point, the equation mentioned above, becomes:

$$\Sigma\, {}^1/_2\, m\, V_z^2 + \Sigma\Sigma\, {}^1/_2\, \mu\, V_r^2 = {}^1/_4 \frac{d^2 \Sigma\, m\, r_z^2}{dt^2} + {}^1/_4 \frac{d^2\, \Sigma\Sigma\, \mu\, r_r^2}{dt^2} -$$

$$- {}^1/_2\, \Sigma\, (Xx_z + Yy_z + Zz_z) - {}^1/_2\, \Sigma\Sigma\, (Xx_r + Yy_r + Zz_r) \ . \ . \quad (b)$$

in which the index z relates to the centres of gravity of the systems, and the value r indicates the value of a quantity relatively to the centre of inertia.

For the stationary condition of the centres of gravity as well as of the systems themselves, this equation is simplified to:

$$\Sigma\, {}^1/_2\, m\, V_z^2 + \Sigma\Sigma\, {}^1/_2\, \mu\, V_r^2 = - {}^1/_2\, \Sigma\, (Xx_z + Yy_z + Zz_z) -$$

$$- {}^1/_2\, \Sigma\Sigma\, (Xx_r + Yy_r + Zz_r) \ . \ . \ . \ . \ . \ . \quad (c)$$

The condition which is required for considering a group of points as a system is, that these points keep always together, whatever may happen and that the quantity $\Sigma\Sigma\,\mu\,r_r^2$ keeps constant.

For the term $-{}^1/_2\,\Sigma\,(Xx_z + Yy_z + Zz_z)$ we may write ${}^3/_2\,(N + N_1)\,v$, so that the latter equation may be written:

$$\Sigma\,{}^1/_2\,m\,V_z^2 + \Sigma\Sigma\,{}^1/_2\,\mu\,V_r^2 = {}^3/_2\,(N + N_1)\,v -$$

$$- {}^1/_2\,\Sigma\Sigma\,(Xx_r + Yy_r + Zz_r) \quad . \quad . \quad . \quad . \quad (d)$$

In these equations collisions taking place between material points, cannot furnish a value, as in every point where a collision takes place, there are two forces of opposed direction, which, working at the same point, destroy each other. The forces in the term ${}^1/_2\,\Sigma\Sigma\,(Xx_r + Yy_r + Zz_r)$ are simply the attractive forces between the points of the system and possibly also the attractive forces which are exercised on a system by the surrounding ones.

It is true that in transforming $-{}^1/_2\,\Sigma\,(Xx_z + Yy_z + Zz_z)$ to ${}^3/_2\,(N + N_1)\,v$ it has been assumed for these latter forces, that for a system, which does not lie near the surface ΣX is equal to 0, but from this does not follow that $\Sigma X x_r$ is equal to 0.

If to the moving systems themselves the virial equation is applied, we get the equation

$$\Sigma\,\Sigma\,{}^1/_2\,\mu\,V_r^2 = -\,{}^1/_2\,\Sigma\,\Sigma\,(X'x_r + Y'y_r + Z'z_r) \quad . \quad . \quad (e)$$

provided that in X', Y' and Z' all forces, also those which exist on the surfaces as pressures, are taken into account. These systems move in a space, in which the pressure is $N + N_1$ per unity of surface, and if we were justified in considering the pressure as really exercised on the surface of every system, the value furnished in the second member of the equation would be equal to ${}^3/_2\,(N + N_1)\,b_1$, if we represent the volume of all the systems together by b_1.

As this pressure, however, is transferred on every system by the collisions with the other systems, in calculating this value, we must consider that pressure as exercised at a distance twice as great, so on the surface of a volume, whose lineal dimension is twice that of the sytem; at least for spherical systems. Of the value obtained in this way, the half is to be taken, because a pressure exercised by the first system on the second is at the same time a pressure, which is exercised by the second on the first. The equation (e) becomes then if we put $b = 4\,b_1$

$$\Sigma\,\Sigma\,{}^1/_2\,\mu\,V_r^2 = {}^3/_2\,(N+N_1)\,b - {}^1/_2\,\Sigma\,\Sigma\,(Xx_r + Yy_r + Zz_r)\ .\ (f)$$

If (f) is subtracted from (d) the well-know equation is obtained

$${}^3/_2\,(N+N_1)\,(v-b) = {}^1/_2\,\Sigma\,m\,V_z^2.$$

The equation (f) may be considered to contain the condition for the stationary state of the molecules themselves. In the form given it is, however only applicable, if the molecule is supposed to be composed of material points, which do not form again separate systems. If the latter is the case, the equilibrium of every separate system will give rise to a new equation, which, however, will not change the equation ${}^3/_2\,(N+N_1)\,(v-b) = {}^1/_2\,\Sigma\,m\,V_z^2$.

For a mixture consisting of $n_1 + n_2$ molecules, we find the virial-value of the surface-pressure of all the molecules together through the observation, that the amount of the pressure on the unity of surface for the two kinds of molecules is proportionate to the numbers which are found in unity of volume and therefore also proportionate to n_1 and n_2. For collisions with a molecule of the first kind, a surface-pressure amounting to $\frac{n_1}{n_1+n_2}(N+N_1)$ must be assumed. and for collisions with molecules of the second kind a surface-pressure of $\frac{n_2}{n_1+n_2}(N+N_1)$.

We find for the quantity with which ${}^3/_2\,(N+N_1)$ is to be multiplied in order to indicate the value of the virial of the pressure, which is exercised on the surfaces of the moving systems, the same value as Mr. LORENTZ (Wied. Ann. 1881, Bd. XII, Heft 1) has found, viz.:

$$b = \frac{{}^2/_3\,\pi\,(\sigma_1^3\,n_1^2 + \sigma_2^3\,n_2^2 + 2\,\sigma^3\,n_1\,n_2)}{n_1+n_2}$$

It is easy to deduce, by the preceding way of obtaining the characteristic equation that the value of b is equal to 4 times the volume of the molecules only in case of infinite rarefaction, and that it must be smaller in case of less great rarefaction; it is not even difficult in that case, to give a first approximation of the way, in which b depends on the volume of the substance. By the calculation of the equation (f) we find the value of the virial of the pressure on the moving systems to amount to half the value of the virial of a pressure $N+N_1$, exercised on as many surfaces as there are systems,

but the systems are supposed to be limited by a spherical surface, described with a radius, which has twice the length of the radius of the systems themselves. Let us call these larger spheres: distance-spheres.

All these distance-spheres are supposed to lie quite outside one another and to have no points in common. As the volume of all these spheres together is 8 times as great as the volume of the molecules, the case that all these spheres lie outside one another is by no means possible, if the volume is smaller than $2\,b$.

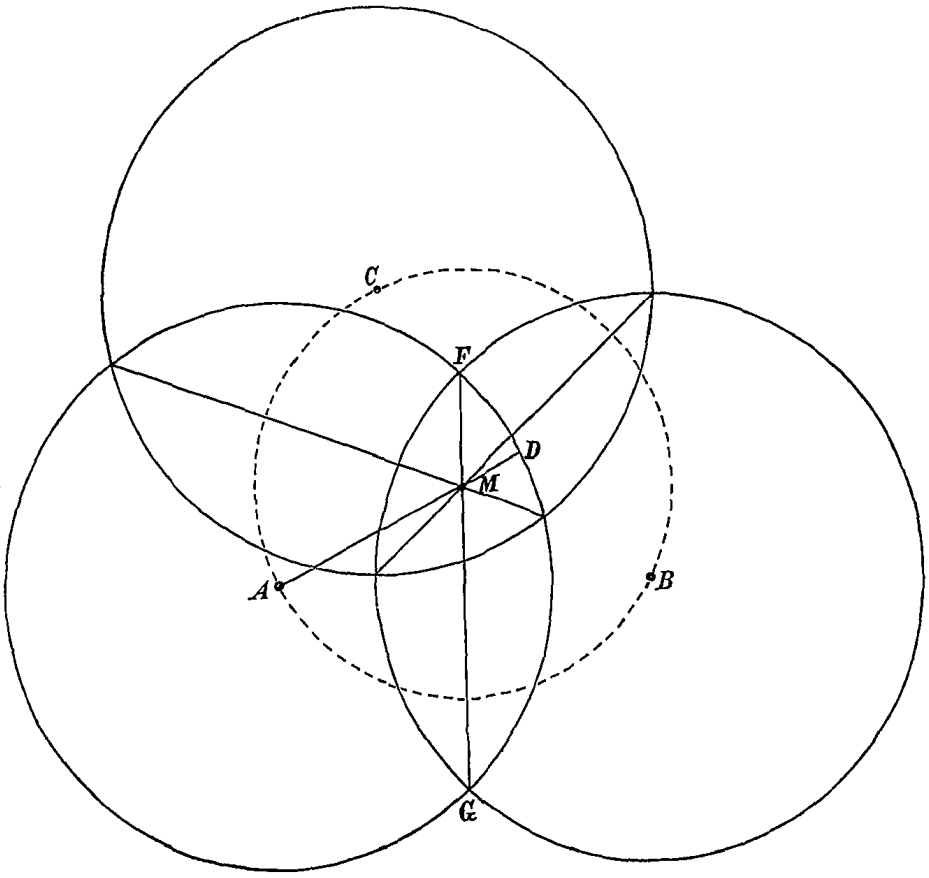
But even if the volume is so large, that the distance-spheres would lie quite outside one another, if the molecules are supposed to be spread in the space at regular distances, a great number of distance-spheres are sure to cover one another in consequence of the fact they are spread quite irregularly. Now the question is, in how far the computation of the value of the virial of the pressure $N + N_1$ is to be modified in consequence of this fact. If we have some molecules, lying in such a way that the distance-spheres intersect, we have not two entire spherical surfaces on which pressure is exercised but a surface consisting of two parts of spherical surfaces. The pressure within the space enclosed by them, is the same, as if it consisted of two separate parts, but the value of the virial of the pressure for the two molecules together amounts to twice $^3/_2(N + N_1)(B - S)$, if B is the volume of a distance-sphere and S the volume of the segment which is cut off from a distance-sphere by the plane of their intersection. In other words, we must take into account only that part of the distance-sphere that reaches up to the plane of the intersection, instead of the whole distance-sphere.

We come, accordingly, to the same result which I had obtained in another way before (Verslag Kon. Ak. van Wetenschappen Amsterdam, 31 October 1896).

A second approximation is also mentioned there, and though the determination of the value of that correction leads to such long calculations, that as yet I have not brought them to an end, yet I will make some remarks on the way in which this value might be obtained.

If A, B and C are taken for the instantaneous position of the three centres of the distance-spheres and M for the centre of the circumscribed circle, the mean value of the volume limited by the surface of the distance-sphere A and the two planes FM and AMD, will represent the second correction.

If we put $AM = \alpha$ and $\angle AMG = C$, and the radius of the distance-sphere $= R$, the value of the volume FMD will be



$$I = {}^2/_3\, R^3\, Bg\, tg \left[tg\, C \frac{\sqrt{(R^2 - \alpha^2)}}{R} \right] -$$

$$- \alpha\, sin\, C \left(R^2 - \frac{\alpha^2\, sin^2\, C}{3} \right) Bg\, tg \frac{\sqrt{(R^2 - \alpha^2)}}{\alpha\, cos\, C} +$$

$$+ \frac{\alpha\, sin\, C\ \alpha\, cos\, C \sqrt{(R^2 - \alpha^2)}}{3}.$$

If C moves on the circle ABC, the centre of which is M, α and C remain the same and consequently I keeps its value also.

Let provisionally, the distance of A and B remain invariable and let C move arbitrarily, then M moves along the line FG. If we call the heigt of M above AB equal to h, I may be considered as function of h by observing that $\alpha^2 = h^2 + \frac{r^2}{4}$ (r the distance AB) and $sin\, C = \frac{r}{2\,\alpha}$. If the whole figure is turned round line AB, and if

the whole space, in which C may be found is divided into volume-elements ΔV, we have to determine

$$\int \frac{N}{v} I \Delta V.$$

As I is known as function of h, ΔV must also be given as dependent on h.

If we represent the angle which CM forms with FG by φ, the annular volume-element, in wich C lies, is to be represented by

$$2\pi \, d\varphi \, dh \, (h + \alpha \cos \varphi)^2 .$$

If we take φ between 0 and the value which it has, when C lies on the distance-sphere of A, twice the value of this integral is to be taken.

As the value of φ is quite determined by h, when C lies on the sphere of A, the integration must be done with respect to h, and the limits are to be determined, between which h is to be taken.

The highest value of h is of course $\sqrt{\left(R^2 - \frac{r^2}{4}\right)}$; the lowest value may be found from

$$h + \sqrt{\left(h^2 + \frac{r^2}{4}\right)} = \sqrt{\left(R^2 - \frac{r^2}{4}\right)}$$

or

$$h = \frac{R^2 - \frac{r^2}{2}}{2\sqrt{\left(R^2 - \frac{r^2}{4}\right)}}$$

The lowest value, however, cannot descend below $-\sqrt{\left(R^2 - \frac{r^2}{4}\right)}$ which would be the case if $r > R\sqrt{3}$. This is the cause that the integration, must be done in two tempo's, and that we have to calculate

$$\int_R^{R\sqrt{3}} dr \int dh + \int_{R\sqrt{3}}^{2R} dr \int_{-\sqrt{\left(R^2 - \frac{r^2}{4}\right)}}^{\sqrt{\left(R^2 - \frac{r^2}{4}\right)}} dh \, .$$