KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN TE AMSTERDAM.

PROCEEDINGS OF THE MEETING

of Saturday February 25th 1899.

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The following papers were read:

Mathematics. — „*On the orthoptical circles belonging to linear systems of conics.*" By Prof. J. DE VRIES.

1. The locus of the points through which we can draw orthogonal tangents to the conic

$$\frac{x^2}{A} + \frac{y^2}{B} = C,$$

is the circle represented by

$$x^2 + y^2 = (A + B)\,C.$$

This orthoptical circle (MONGE's circle) is real for the ellipses and the hyperbolae situated in the acute angles of their asymptotes, and imaginary if the hyperbola lies in the obtuse asymptote-angles; it degenerates into a point for the rectangular hyperbola ($A+B=0$) and for the system of two right lines ($C=0$). With the parabola it is represented by the directrix.

The orthoptical circle being concentric with the conic to which it belongs, the centres of the orthoptical circles ω of a pencil of conics lie on the conic γ, containing the centres of the pencil.

If now, following Mr. FIEDLER, we represent each circle by the vertices of the two right cones of which it is the base-circle, the system (ω) is represented by a skew curve of the 4th order Ω_4. Indeed, each plane perpendicular to the plane V of the orthoptical circles intersects the conic in two points and bears two pairs of points representing orthoptical circles.

The skew curve Ω_4 being intersected in four points by a plane parallel to V, there are in (ω) four circles with given radius.

So the system (ω) contains four point-circles; it is evident that we find these in the double points of the three degenerated conics and in the centre of the orthogonal hyperbola belonging to the pencil.

2. The system (ω) contains two orthoptical right lines, viz. the directrices of the two parabolae in the pencil. Each of these lines is represented by two planes inclined to the plane V at angles of 45°, i.e. parallel to four generatrices of each rectangular cone Π_2 having its vertex in a point P of V and placed symmetrically with respect to this plane.

The cone Π_2 bearing the images of all the circles through P, and the above named four points at infinity representing two right lines not passing through P, the remaining four points of intersection of Π_2 with Ω_4 will represent two circles (ω) intersecting in P. Therefore:

The orthoptical circles of a pencil of conics form a system with index two.

This can also be shown as follows. The tangents through P to the conics of the pencil are arranged in a (2,2)-correspondence, each ray through P being touched by two curves. This system has two pairs of rays in common with the involution of the lines, intersecting in P at right angles; so through P pass two circles ω.

Two planes intersecting the plane V in the right line l at angles of 45° have four pairs of points in common with the curve Ω_4. This shows that l is touched by four orthoptical circles. Their points of contact are the coincidences of the (2,2)-correspondence, determined on l by the system (ω).

3. If we represent the form

$$x^2 + y^2 + ax + by + c$$

by C, the system (ω) is represented by the equation

$$C_1 + 2\lambda C_2 + \lambda^2 C_3 = 0.$$

The power of the point (x, y) with regard to the circle indicated by a definite value of λ is then represented by

$$\frac{C_1 + 2\lambda C_2 + \lambda^2 C_3}{1 + 2\lambda + \lambda^2}.$$

This expression becomes independent of λ if we assume the radical centre of C_1, C_2, C_3 for (x, y). So all the circles ω have a common radical centre or, in other words, all orthoptical circles cut a fixed circle Δ at right angles.

As Δ must bear the point-circles of (ω) we may conclude to the following theorem:

The circle through the diagonal points of a complete quadrangle contains the centre of the orthogonal hyperbola determined by the vertices of the quadrangle. Through its centre pass the directrices of the parabolae which can be drawn through those vertices.

Considering the obtained results as a property of the rectangular hyperbola it can be expressed in the following terms:

The diagonal points of each complete quadrangle inscribed in a rectangular hyperbola can be connected by a circle with the centre of that curve.

4. All circles ω being orthogonal to the fixed circle Δ their images lie on the orthogonal hyperboloid of revolution with one sheet cutting V in Δ at right angles.

So the image Ω_4 is the section of this hyperboloid with the cylinder, which is orthogonally cut by V in the conic γ.

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When the base points of the circle form an orthocentric group [1]) all the conics are rectangular hyperbolae; so the system (ω) consists entirely of point-circles.

The orthogonal circle Δ, bearing them, passes through the diagonal points of the complete orthogonal quadrangle and coincides with the circle of FEUERBACH (nine-points circle), containing as is known the centres of all the orthogonal hyperbolae through the vertices and the orthocentre of a triangle.

5. According to the method of FIEDLER the orthoptic circles of a net of conics will be represented by a surface, intersecting the plane V of the net in the locus of the orthoptical point-circles.

Any point P determines a pencil belonging to the net, one of the base points of which is P; so through P passes an orthogonal hyperbola. From this follows, that the orthogonal hyperbolae form a pencil situated in the net. As was said above the orthoptical point circles of this pencil lie on the circle Δ, passing through the diagonal points and the middle points of the six sides of the quadrangle of the base points.

The remaining point-circles ω are centres of pairs of lines and form the cubic curve, called the HESSIAN of the net.

Three pairs of lines of the net belong to the pencil of orthogonal hyperbolae and consist of orthogonal right lines. The double points must lie on the HESSIAN as well as on the circle Δ; so these curves must touch each other in three points.

Both loci of point-circles forming together a curve of the 5^{th} order, the image of the system of orthoptical circles is a surface of the 5^{th} order S_5.

Each right line of V is touched in each of its points by *one* conic of the net; each right line determines the direction of the axis of a parabola belonging to the net. From this follows, that the point at infinity of each right line cutting V at angles of 45° is to be regarded as the image of an orthoptical right line.

So the points of contact of the asymptotes of the rectangular hyperbola representing all the circles through two points belong to the ten points common to S_5 and that hyperbola. The remaining eight points of intersection representing four circles it is evident from this that *we can draw four orthoptical circles of the system through any two points.*

[1]) That is to say: four points, each of which is the orthocentre of the triangle having the others for vertices.

6. If U_x and U_y represent the differential quotients of the quadratic function U with regard to x and y, the centres of the conics of the net

$$U + \lambda V + \mu W = 0$$

are indicated by the relations

$$U_x + \lambda V_x + \mu W_x = 0,$$
$$U_y + \lambda V_y + \mu W_y = 0.$$

So in general any point (x, y) is the centre of *one* conic. On the other hand any of the four points determined by

$$U_x : U_y = V_x : V_y = W_x : W_y$$

is the common centre of an infinite number of conics. For each of these points the above mentioned linear equations are dependent of each other, so that λ and μ are connected by a linear relation; so in this way a pencil of conics is characterized.

Consequently the surface S_5 contains four right lines perpendicular to V.

7. Let us now consider the orthoptical circles belonging to the conics with four common tangents. Any right line through the point P determining a conic of the tangential pencil, the tangents through P form an involution. This contains only one pair of rectangular rays; so *P lies on one orthoptical circle.*

The wellknown property according to which the centres of the tangential pencil lie in a right line, also proves that the orthoptical circles of a tangential pencil form a pencil.

According to the circles of that pencil passing through two fixed points or intersecting a fixed circle orthogonal or touching each other in a fixed point, the system (ω) will be represented by a rectangular hyperbola with its real axis perpendicular to V or situated in V, or by two right lines cutting V at angles of 45°.

So the system (ω) contains no more than two point-circles or, in other words, the tangential pencil can contain only two rectangular hyperbolae.

A conic of the tangential pencil degenerating into two points, the line joining these points is the diameter of the corresponding ω. In connection with the statement above the wellknown property results from this:

The circles described on the diagonals of a complete quadrilateral as diameters belong to a pencil.

8. Finally we consider a tangential net.

Two right lines through P which intersect at right angles are touched by *one* conic of the net. If the right angle included by these lines turns about the vertex P the pairs of tangents drawn through Q to the variable conic form an involution. For every ray s, drawn through Q, determines a tangential pencil belonging to the net, having the pairs of an involution (r, r') in common with the point P. The conic touched by the orthogonal pair (r, r') has a second tangent s' through Q, forming with s a pair of the involution indicated above. The orthogonal pair (s, s') determining a conic for which r and r' are at right angles to each other, *we can draw but one orthoptical circle through the points P and Q.*

According to the circles (ω) possessing an orthogonal circle or intersecting a fixed circle in two diametrically opposite points or passing through a fixed point, the obtained net of circles is represented by an orthogonal hyperboloid of revolution with one or two sheets or by an orthogonal right cone.

Chemistry. — "*On solubility and meltingpoint as criteria for distinguishing racemic combinations, pseudoracemic mixed crystals and inactive conglomerates.*" By Prof. BAKHUIS ROOZEBOOM.

Though several times attention has been drawn to the phenomena of solubility and melting in order to distinguish between the types mentioned above, no certainty has been attained as yet.

1. *Solubility.* We only get a clear insight in the phenomena of solubility by drawing attention to the number of solubility curves obtainable at a given temperature.

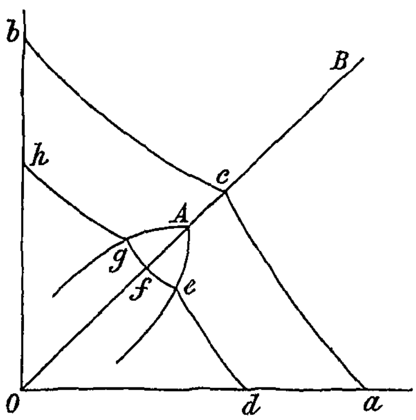

Fig. 1.

If Oa is the proportion of the saturated solution of the dextrosubstance D, and Ob the same for the laevosubstance L, these two are equal at the same temperature.

By adding L to the D-solution and vice-versa we now get, if no racemic combination appears at the temperature used, nothing else than two solubilitycurves, starting from the points a and b and meeting in c. Their precise direction depends on the manifold actions that can take place in the solution. From the perfect equivalence of L and D it results however necessarily that c must always lie on the line OB, which halves the angle of