## Huygens Institute - Royal Netherlands Academy of Arts and Sciences (KNAW)

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phonomena of retardation, the experimental investigation of the critical phenomena in mixtures has led to reliable results. That phenomena of retardation give rise to important deviations is shown by the fact that Caileeter has obscrved condensation at $25^{\circ}$, whereas my experiments show that the critical point of contact of his mixture after equilibrium of the phases has been obtained, ought to be found at about $23^{\circ}$.

Physics. - „Measurements on the change cf pressure ly substitution of one compone at by the cther in inixtuses of carbonic acid and hydrogen". By Dr. J. Verschaffelt. (Communication No. 47 from the Physical Laboratory at Leiden by Prof. H. K. merlinah Onnes. (Cjntinued.))

## § 1. Change of pressure by substitution.

Tables VI-X contain the results of the determination of isothermals in mixtures with a still larger quantity of hydrogen than those treated of in the former communications. In these mixtures no conden-sation-phenomena appeared in the area observed; and so they cannot reveal to us anything more about the further course of the plaitpointcurve, but in connection with the results communicated before, they show us in what way, at a given temperature and a given volume, the pressure of the mixture depends on its composition.

With the aid of all the determinations communicated we have first calculated the coefficients of pressure, the values of which for different compositions and volumes are given in Table XI. Then we have calculated for one and the same temperature ( $18^{\circ} \mathrm{C}$.) the isothermals of the different mixtures and represented them in a new diagram, the $p-v-x$ diagram. On this diagram we have read the pressures belonging to one and the same volume for different mixtures: table XII contains the values read in this way for some volumes.

As will be explained in $\S 2$ we have chosen the units of volume for the different mixtures so that 1 e.m. ${ }^{8}$ of oach mixture contains the same number of molecules, when the volumes of these mixturcs expressed in the units accepted for this purpose, have the same value. Table XII therefore shows the change in pressure when, starting from one of the two substances in pure condition, we gradually substitute the molecules of this substance for an equal number of molecules of the other substanco.


The character of this „change of pressure by substitution" is illustrated in the annexed figure: in this diagram the abscissae represent the compositions, and the ordinates the pressures for the volume 0,020 and at the temperature $18^{\circ} \mathrm{C}$. From this figure it appears that this change of pressure is not proportional to that of the composition, the pressure is always greater than would follow from a linear relation; if only a small fraction of the number of hydrogen molecules is substituted by an equal number of carbonic acid molecules hardly any change in the pressure takes place.
Table XII shows clearly that neither the coefficients of pressure undergo any material change, when the number of hydrogen molecules substituted by carbonic acid molecules is only a small fraction of the whole number of hydrogen molecules.

## § 2. The unit of volume.

It was desirable to express the volume for each mixture in a special unit of volume, chosen so that one c.m. ${ }^{3}$ of each of the mixtures contains the same number of molecules when its volume, expressed in this unit, has the same value. We can do this by taking for each mixture as unit of volume: the volume which the quantity of substance used would occupy at $0^{\circ} \mathrm{C}$. and 1 atm ., if the mixture behaved like a perfect gas. This volume I will call the theoretically normal volume; it is obtained by multiplying the volume generally called the normal volume viz. the volume which is actually occupied by the substance at $1^{\circ} \mathrm{C}$. and 1 atm ., by a factor expressing the deviation from Arogadno's law.
In connection with Avogadro's law this measure for pure substances has been proposed by Kamerlingh Onnes in 1881. At the same time he pointed out the desirability of expressing henceforth the molecular quantities also in absolute measure. In order to determine the deviation of the normal volume of hydrogen from that which is to be used in applying Avogadro's law, he employed van der Waals's law and the value 0,00050 , deduced by van der

Walls for $a-b$ (Continuiteit p. 91) from Reqnaulifs observations. And so he obtained in KG. M. S. the equation

$$
\left(p+\frac{A}{v^{2}}\right)(v-b)=2262000(1+\alpha t),
$$

which he thought fit for determining the deviation of the gases from Avogadro's law ${ }^{1}$ ).
In the case of hydrogen we shall however use Amagat's observations below 200 atm., which we have also used in determining the pressures. With sufficient approximation they yield for the isothermal of $0^{\circ} \mathrm{C}$.

$$
\left.p(v-0,000690)=0,99931^{2}\right),
$$

when the normal volume is chosen as unit. Hence in order to obsain the theoretically normal volume we must multiply the volume at $0^{\circ}$ and 1 atm . by 0,99931 .
From the weight of 1 liter of hydrogen: 0,08955 gramme at $0^{\circ}$ and $760 \mathrm{~m} . \mathrm{m}$. mercury, $0^{\circ} \mathrm{C}$. and $45^{\circ}$ latitude and the molecular weight of $\mathrm{CO}_{2}: 43,89$, we deduce the weight of a liter carbonic acid to be 1,9652 gramme under the same circumstances ; Reanaudip found for this 1,9771 , which means a deviation of 0,0060 from the theoretical value with respect to hydrogen. The density of carbonic acid is therefore 0,0053 higher than the theoretically normal density, so that in order to find the theoretically normal volume of pure carbonic acid we must multiply the volume at $0^{\circ}$ and 1 atm . by 1,0053 .
An observation made by Braun ${ }^{3}$ ) gives us some information on the deviations of Avogadro's law shown by mixtures of carbonic acid and hydrogen. He has found that when we mix two equal volumes of hydrogen and carbonic acid under a pressure of about $71 \mathrm{c} . \mathrm{m}$. mercury, the pressure rises $0,097 \mathrm{c} . \mathrm{m}$. thus 0,0014 . And so we may accept that when taking two equal volumes of hydrogen and carbonic acid we obtain by mixing a

[^0]double volume under a pressure of 1.0014 atm . Let $n$ be the number of molecules which one volume of a perfect gas would contain at $0^{\circ}$ and 1 atm., so that $0,99931 n$ is the number of molecules of hydrogen and $1,0053 n$ the number of molecules of carbonic acid, then it is easy to see that one volume of the mixture at $0^{\circ}$ and 1 atm . must contain $1,0009 \mathrm{nmol}$. And so in order to find the theoretically normal volume of a mixture consisting of an equal number of molecules of hydrogen and carbonic acid we must multiply the volume at $0^{\circ}$ and 1 atm . by 1,0009 .
Let, as in V. D. Waals's paper, $x$ be the proportion of the number of hydrogen-molecules to the whole number of molecules, then we can represent the deviation from Avogadro's law by the numbers: 1,0053 for $x=0,1,0009$ for $x=0,5$, and 0,99931 for $x=1$. The deriation for intermediate mixtures can be found with sufficient approximation by applying a parabolic formula of the form $y=a+b x+c x^{2}$; and then we find that the deviation may be represented by
$$
y=0,99931+0,0060(1-x)^{2} .
$$

According to this formula the influence of small admixtures of carbonic acid with the hydrogen is very small, a fact to which the attention will be drawn later on.
For the reduction to $0^{\circ} \mathrm{C}$. we had to use the coefficients of expansion of the mixtures. As a first approximation I might have calculated the coefficients of expansion with the aid of a linear formula from the coefficients of expansion of the pure substances: 0,00366 for $\mathrm{H}_{2}, 0,00371$ for $\mathrm{CO}_{2}$. But led by the previous result regarding the deviations from Avogadro's law I thought it probable that the dependence of the coefficient of expansion on the composition $x$ would show the same characteristics; and so I was obliged to put it thus:

$$
a_{p}=0,00366+0,00005(1-x)^{2}
$$

In determining the theoretically normal volume of hydrogen it was mentioned that by starting from different experimental data we arrive at different deviations from Avogadro's law.
This is even more so in the case of carbonic acid. If by the side of the number 0,99950 for hydrogen formerly accepted by Kamerlingir Onnes we also borrow from v. D. Walls's Continuiteit (p. 76) the number 1,00646 for carbonic acid (deduced from Reanauly's isothermal lines) and if we substitute this for Regnault's determination
of the density of carbonic acid we should find for the mixture $1 / 2$ hydrogen $-1 / 2$ carbonic 1,00158 , whence :

$$
y=0.99950+0,00136(1-x)+0,00560(1-x)^{2}
$$

From the point of view of a consistent application of Van Der Waals's law a calculation of the theoretically normal volume of the carbonic acid from the normal volume and the compressibility would be preferable to the method of calculation followed by me. But then the proportion of the weights of one volume of carbonic acid and hydrogen in the state of perfect gases and the molecular weights would be discordant, and it are the latter which we want as a basis for our choice of units of volume (with a view to v. D. Wanls's theory of mixtures). The influence of the differences yielded by the calculations from the different data is not such as to render doubtful the character of the change of pressure when substitution takes place, but still in my determinations the discrepancies differ little from the errors of observation. From this we see once more the necessity of accurate observations with perfectly pure substances in order to make progress in the knowledge of the laws which govern the gasenus state.

| TABLE VI. | $x=0,3528$. |  |  |
| :---: | :---: | :---: | :--- |
| Volume | $p_{16}{ }^{\circ} \cdot 00$ | $p_{31} 0^{\circ} \cdot 60$ |  |
| 0,0 |  |  |  |
| 2810 | 34.44 | 36.85 |  |
| 2585 | 37.17 | 39.80 |  |
| 2442 | 39.15 | 41.96 |  |
| 2299 | 41.38 | 44.38 |  |
| 2159 | 43.77 | 46.95 |  |
| 2014 | 46 | 54 | 49.99 |
| 1869 | 49.70 | 53.45 |  |
| 1724 | 53.35 | 57.40 |  |
| 1587 | 57.35 | 61.80 |  |
| 1480 | 60.90 | 65.75 |  |
| 1340 | 66.35 | 71.75 |  |
| 1207 | 72.40 | 78.45 |  |
| 1067 | 80.30 | 87.25 |  |
| 09255 | 89.90 | 98.30 |  |
| 07040 | 101.60 | 12.0 |  |
| 06510 | 115.3 |  |  |
|  |  |  |  |

TABLE VII. $x=0,4993$.

| $\begin{gathered} \text { Volume } \\ 0,0 \end{gathered}$ | $T_{18{ }^{0} \cdot 00}$ | $p_{38}{ }^{0} \cdot n 0$ |
| :---: | :---: | :---: |
| 3168 | 32.11 | 35.84 |
| 2975 | 34.09 | 35.95 |
| 2825 | 35.78 | 37.78 |
| 2667 | 37.85 | 39.97 |
| 2512 | 40.08 | 42.36 |
| 2355 | 42.60 | 45.07 |
| 2206 | 45.34 | 47.97 |
| 2045 | 48.71 | 51.55 |
| 1889 | 52.35 | 55.50 |
| 1736 | 56.80 | 60.20 |
| 1624. | 60.45 | 64.05 |
| 1468 | 66.35 | 70.40 |
| 1323 | 73.05 | 77.65 |
| 1172 | 81.85 | 87.10 |
| 1017 | 03.15 | 99.50 |
| 08694 | 107.3 | 114.0 |

TABLE VIII.
$x=0,6445$.

| $\overline{\text { Volume }}$ | $p_{16}{ }^{\circ} \mathrm{C} 80$ | $p_{24}{ }^{\circ} \cdot 20$ | $p_{31}{ }^{\circ} \cdot 00$ |
| :---: | :---: | :---: | :---: |
| 2079 | 34.85 | 35.82 | 36.85 |
| 2746 | 37.76 | 38.79 | 39.95 |
| 2600 | 39.90 | 41.00 | 42.13 |
| 2446 | 42.35 | 43.53 | 44.80 |
| 2294 | 45.10 | 46.34 | 47.88 |
| 2143 | 48.21 | 49.60 | 51.00 |
| 1989 | 51.90 | 53.35 | 54.95 |
| 2836 | 56.05 | 57.60 | 59.35 |
| 1686 | 60.90 | 62.65 | 64.50 |
| 1576 | 65.20 | 67.05 | 69.05 |
| 1433 | 71.45 | 73.60 | 75.80 |
| 1284. | 79.50 | 82.00 | 84.55 |
| 1140 | 89.25 | 92.05 | 94.85 |
| 09935 | 102.2 | 105.5 | 108.9 |
| 08495 | 119.2 |  |  |

TABLEIX.
$x=0,7963$.

| $\begin{array}{c}\text { Volame } \\ 0,0\end{array}$ | $p_{18}{ }^{\circ} \cdot 30$ |
| :---: | :---: |
| 3203 | 32.61 |
| 2990 | 35.81 |
| 2723 | 39.47 |
| 2474 | 43.49 |
| 2253 | 47.78 |
| 2032 | 53.00 |
| 1840 | 58.60 |
| 1630 | 66.20 |
| 1484 | 72.95 |
| 1335 | 81.35 |
| 1212 | 89.65 |
| 1090 | 100.1 |
| 09650 | 213.2 |
|  |  |
|  |  |

TABLEX.
$x=0,8972$.

| Volume <br> 0,0 | $p_{17}{ }^{\circ} \cdot 10$ |
| :---: | :---: |
| 3234 | 33.46 |
| 2989 | 36.29 |
| 2840 | 38.15 |
| 2692 | 40.31 |
| 2532 | 42.94 |
| 2384 | 45.72 |
| 2217 | 49.22 |
| 2076 | 52.65 |
| 1904 | 57.50 |
| 1764 | 62.25 |
| 1635 | 67.40 |
| 1485 | 74.55 |
| 1336 | 83.15 |
| 1173 | 95.20 |
| 1028 | 109.5 |

TABLEXI. Coefficients of teusion, ( $15^{\circ}-30^{\circ} \mathrm{C}$.)

|  | Compositions $x=$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.0060 | 0.0491 | $10.0095^{1}$ | ${ }_{1}^{1} 0.1990 \mid$ | 0,3528 | 0.4993 | 0.6445 | 0.7963 | 0.8972 | 1.0000 |
| 30 | 0.162 | 0.159 | 0.156 | '0.150 | 0.142 | 10.134 | 0.130 | 0.127 | 0.126 | 0.126 |
| 28 | 175 | 172 | 168 | 163 | 155 | 146 | 141 | 138 | 136 | 135 |
| 26 | 190 | 186 | 183 | 177 | 168 | 159 | 154 | 150 | 148 | 146 |
| 24 | 209 | 204. | 199 | 193 | 183 | 174 | 168 | 163 | 161 | 159 |
| 22 | 229 | 223 | 218 | 211 | 201 | 191 | 185 | 379 | 176 | 173 |
| 20 | 257 | 249 | 242 | 234 | 223 | 212 | 204 | ]97 | 194 | 192 |
| 18 | 295 | 285 | 276 | 265 | 250 | 237 | 227 | 221 | 216 | 214 |
| 16 | 346 | 333 | 321 | 306 | 286 | 270 | 256 | 249 | 244 | 242 |
| 14. | 412 | 395 | 381 | 360 | 331 | 312 | 296 | 286 | 281 | 278 |
| 12 | 503 | 481 | 461 | 431 | 395 | 373 | 353 | 339 | 331 | 327 |
| 10 | 620 | 590 | 566 | 433 | 495 | 464 | 436 | 415 | 405 | 397 |

TABLE XII. Pressures at $18^{\circ}$.

| $\begin{gathered} \text { Volume } \\ 0,0 \end{gathered}$ | Compositions $x=$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.0000 | 0.0494 | 0.0995 | $0.1090$ | 0.3528 | 0.4993 | 06445 | 0.7963 | 0.8972 | . 0000 |
|  |  |  |  |  |  |  |  |  |  |  |
| 30 | 28.901) | 29.68 | 30.37 | 31.33 | 32.70 | 33.81 | 34.77 | 35.80 | 36.19 | 36.31 |
| 29 | 29.73 | 30.43 | 31.33 | 32.25 | 33.75 | 34.92 | 3595 | 37.09 | 37.46 | 37.59 |
| 28 | 30.58 | 31.35 | 32.25 | 33.24 | 34.85 | 36.12 | 37.19 | 38.43 | 38.83 | 38.97 |
| 27 | 31.45 | 32.32 | 33.23 | 34.34 | 36.04 | 37.39 | 38.56 | 39.81 | 40.29 | 40.45 |
| 26 | 32.40 | 33.33 | 3428 | 35.53 | 37.32 | 38.80 | 40.05 | 41.33 | 41.87 | 4204 |
| 25 | 33.36 | 34.40 | 35.37 | 36.77 | 38.64 | 40.23 | 4164 | 42.98 | 43.62 | 43.77 |
| 24 | 34.38 | 35.53 | 36.54 | 38.04 | 40.12 | 41.80 | 43.33 | 44.80 | 45.48 | 45.65 |
| 23 | 35.47 | 3673 | 3778 | 39.43 | 41.72 | 43.57 | 45.17 | 46.73 | 47.48 | 47.70 |
| 22 | 36.55 | 38.00 | 39.11 | 40.91 | 43.40 | 45.46 | 47.18 | 48.90 | 49.67 | 49,94 |
| 21 | 37.75 | 39.40 | 40.55 | 42.50 | 45.26 | 47.50 | 4939 | 51.20 | 5215 | 52.40 |
| 20 | 139.08 | 40.85 | 42.10 | 44, 26 | 47.30 | 49.65 | 51.80 | 5375 | 54.80 | 55.10 |
| 19 | 40.40 | 42.40 | 13.76 | 46.15 | 49.49 | 52.10 | 54.45 | 56.65 | 57.80 | 58.10 |
| 18 | 41.90 | 44.02 | 45.56 | 48.22 | 51.90 | 54.05 | 57.45 | 59.85 | 61.15 | 61.50 |
| 17 | 43.43 | 45.70 | 4747 | 50.40 | 54.60 | 58.00 | 60.75 | 63.45 | 64.90 | 65.25 |
| 16 | 45.00 | 47.52 | 49.55 | 52.80 | 57.50 | 61.30 | 64.55 | 67.45 | 69.10 | 6950 |
| 15 | 40.64 | 49.45 | 51.85 | 55.45 | 6080 | 65.00 | 68.75 | 72.10 | 73.85 | 74.35 |
| 14 | 48.35 | 51.65 | 54.30 | 58.40 | 64.60 | 69.30 | 73.35 | 77.50 | 79.25 | 79.95 |
| 13 | 50.05 | 54.00 | 56.00 | 61.75 | 68.80 | 74.30 | 78.80 | 83.55 | 85.65 | 86.45 |
| 12 | 51.85 | 56.40 | 59.85 | 65.60 | 73.60 | 80.00 | 85.30 | 90.85 | 93.10 | 94.10 |
| 11 | 53.65 | 59.00 | 63.05 | 69.75 | 7910 | 86.75 | 93.00 | 99.25 | 102.0 | 103.1 |
| 10 | 55.50 | 61.65 | 66.55 | 74.25 | 85.40 | 94.40 | 102.1 | 109.6 | 112.9 | 114.3 |

Chemistry. - "On the velocity of electrical reaction". By Dr. Erns'r Cohen, (Communicated by Prof. H. W. Bakhois Roozeboom).

1. If two elements, arranged as follows.

Electrode, rever- $\mid$ Saturated solution of a $\mid$ Electrode, reversible with respect to the anion. salt $S$ in presence of the sible with respect stable solid phase of the to the cathion. salt.
and

[^1]
[^0]:    1) The problem in general has been treated by v. w. Wats in his communications "On the accurate determination of the molecular weight of gases from their density." Proc. Nov. '98 and "Variation of volume and of pressure in mixing IL." Proc. Dec. '98
    ${ }^{2}$ ) See Kamrrlingh Onnes. Verl. Kon. Akad. v. Wet. 1881. Algemeene theorie der vloeistoffen, p. 5-7. Abstract: Arch. Néerl. t. XXX.
    ${ }^{3}$ ) Wied. Ann., 34, p. 948,1888 . Kuenen has pointed out a similnr change of pressure in mixing compressed gases by stirring.
[^1]:    ${ }^{2}$ ) This column has been deduced from isothermal determinations of pure carbonic acid, not yet published.

