Mathematics. — "*A Null System* (1, 2, 3)." By Prof. JAN DE VRIES.

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1. We consider as given a congruence $[\varrho^3]$ of twisted cubics with the base points C_1, C_2, C_3, C_4, C_5 [1]) and the crossing straight lines a and b.

Through a point N there passes one curve ϱ^3; let r be the tangent at N and t the transversal of a and b through N. We conjugate $\nu \equiv rt$ to N as a *null plane*.

The curves ϱ^3 touching a plane ν have their points of contact in a conic ϱ^2. The transversal t lying in ν, cuts ϱ^2 in the *null points* N_1 and N_2 of ν.

If ν revolves round the straight line l, t describes a scroll $(t)^2$ and ϱ^2 a cubic surface through l. The locus of N is accordingly a twisted curve λ^5, which has evidently l, hence also a and b, as trisecants.

We have therefore a *null system* with the characteristic numbers $\alpha = 1$, $\beta = 2$, $\gamma = 3$.

2. The points C_k are *singular*; for C_k carries one straight line t but ∞^2 straight lines r. The null planes of C_k form a pencil of planes round t as axis.

Also the points A of a and B of b are *singular*. For each of them carries ∞^1 straight lines t which are combined to a plane pencil. The null planes of each of these points form a pencil of which the axis lies in the tangent r. These axes form two cubic scrolls $(r)^3$.

Other *singular points* S can only arise through coincidence of the straight lines t and r. Now the tangents of the curves ϱ^3 form a complex of the 6^{th} order and this complex has a scroll $(n)^{12}$ in common with the bilinear congruence $[t]$. On each straight line n there lies a point S to which any plane through n corresponds as null plane.

As l is intersected by 12 straight lines n, the corresponding curve λ^5 contains 12 points S.

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[1]) The principal properties of this congruence are to be found for instance in R. STURM: *Die Lehre von den geometrischen Verwandtschaften*, Part IV, p. 470.

3. The null points of the planes passing through the point P, lie on a surface $(P)^4$. For P is the null point of one definite plane of the sheaf and on a straight line l through P there lie the null points of three planes through l.

The intersection of the surface $(P)^4$ and $(Q)^4$ consists of the curve λ^5 corresponding to PQ, the straight lines a and b, and a curve σ^9 which is the locus of the *singular points* S and which passes evidently through the 5 base points C_k.

Three surfaces $(O)^4$, $(P)^4$ and $(Q)^4$ have in the first place the curve σ^9 in common. The points which they have further in common, are apparently the points of intersection of $(O)^4$ with the curve λ^5 corresponding to PQ. To them there belong the 12 points S on λ^5 and the 2×3 points A and B on λ^5; the remaining two are the null points of the plane OPQ.

4. Any plane α through a is *singular*; it contains a plane pencil (t) and each ray t cuts the conic ϱ^2 (§ 1) in two null points. Analogously the planes β through b are *singular*.

Also the ten planes σ each containing three base points C, are *singular*. For in σ_{123} there lies a pencil of conics of which each individual is combined with the straight line $C_4\,C_5$ to a curve ϱ^3; they cut the straight line t in σ_{123} in an involution of null points.

The surface $(P)^4$ contains the conics α^2 and β^2 lying in the planes Pa and Pb, and the intersection p of these planes. The straight line p is singular in this respect that it is a null ray for *each* of its points. The *singular null rays* p form the bilinear congruence with the director lines a and b.

Also the ten straight lines $C_k\,C_l$ are *singular*; for through each point on such a straight line r_{kl} there passes one straight line t, while r_{kl} may be considered as a tangent.
