The so-called cyanogen-bands have completely disappeared; it follows that these bands do not belong to nitrogen, but to a much more easily condensable substance, probably cyanogen.[1])

This is in accordance with STEUBING's observations; the latter found no trace of the cyanogenbands in his experiments, where the presence of any carbon was excluded.[2])

Probably GROTRIAN and RUNGE's nitrogen was not completely free from carbon. This may be due to the fact that they purified their nitrogen by pyrogallicacid-solution; during this operation small quantities of carbon monoxide are usually developed.

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[1]) In some of our experiments we completely immersed the discharge-tube in liquid oxygen, the spectrogram being taken through the walls of the Dewarvessel. During the operation of the Tesla transformer the walls of the Dewarglass show the green fluorescence of cathode-rays. In one of our experiments however some gas was liberated in the space between the walls of the Dewarvessel, so that a red glow appeared, the radiation of which is superposed on that of the discharge-tube. The so-obtained spectrogram is shown in fig. 3. A peculiar phenomenon may be observed. Some of the cyanogen-bands, namely 3855, 3883 and 4168 Å. come out very strongly, whereas the other ones are absent. So it is not impossible. that the cyanogen-bands are due to two different carriers.

[2]) Simular results have been obtained by L. HAMBURGER, who also found no trace of the cyanogenbands in extremely pure nitrogen. Chem. Weekblad (15) 931 1918. (Added in translation).

Physics. — *"The geodesic precession: a consequence of* EINSTEIN'S *theory of gravitation."* By Dr. A. D. FOKKER. (Communicated by Prof. H. A. LORENTZ).

(Communicated at the meeting of October 30, 1920).

It is well known at present what parallel displacement or geodesic translation means in non-euclidean space[1]). And we know also that a compass rigid, moving parallel to itself and completing a closed circuit, in consequence of the curvature of space, will not regain the same orientation which it had before: a certain rotation of curvature will become apparent. Now it occurred to SCHOUTEN that the earth's axis of rotation — provided the earth were a sphere — should remain parallel to itself in the general geodesic sense during the motion of the earth round the sun. Thus, after a year, we must expect the earth's axis to point to a slightly different point of the heavens according to the curvature of space produced by the sun's gravitation. This affords an additional precession which superposes itself on the precessions due to other causes known in astronomy[2]).

The problem however is not so simple as it is put here. Though it can be proved that the axis of rotation will remain parallel to itself in the geodesic sense, yet in reality we have to consider the dragging of the earth's axis along her four-dimensional helicoidal track through time-space and not a circuital displacement in the ecliptic at some definite instant. The problem should be put as one of four-dimensional geometry; it is a problem of mechanics, and not a problem of three-dimensional geometry. If this be done properly, then the result is that we are to expect a precession one and a half times the precession foreseen by SCHOUTEN, viz. 0.019 of a second of arc per annum[3]). This will be shown in the present paper.

The idea at the bottom of the argument is the following. Imagine that in order to describe motions taking place in the neighbourhood of the earth's centre we choose axes such that the time is always

[1]) LEVI CIVITA, Rendic. Cerc. Mat. Palermo, **42**, p. 1, 1917; SCHOUTEN, Direkte Analysis zur n. Relativitätstheorie, Verhandelingen Kon. Akad. v. Wetensch. Amsterdam, XII, no. 6, 1919; WEYL, Raum, Zeit, Materie, Berlin 1920, 3rd ed.; Cf. also an article of the present author in Proceedings Kon. Akad. v. Wetensch. Amsterdam, **21**, p. 505, 1918.

[2]) SCHOUTEN, Proceedings Kon. Akad. v. Wetensch. Amsterdam, **21**, p. 533, 1918; with appendix by DE SITTER.

[3]) Cf. also a paper by KRAMERS, Proc. Amsterdam, September 1920.

directed along the earth's four-dimensional track and that the origin of space-axes falls along with the earth. Moreover, the original directions of these space-axes at successive instants are to remain parallel to themselves in the general, or natural sense. If our axes of reference are chosen in this way, we may confidently expect the equations of motion to assume a particularly simple form: in fact, as a first approximation, when motions take place very near the origin (i.e. within a domain the two-dimensional cross-sections of which are small compared with the reciprocal of RIEMANN's measure of curvature) then this region may be considered to be homoloidal, that is, free particles are moving in straight lines under no force, and a top spinning round its axis of symmetry will keep its axis of rotation in a fixed direction relative to the axes of reference. As the latter are carried along the axis of time parallel to themselves, so it follows that the same is true for the axis of rotation. [1])

If we proceed to the second approximation, we find that free particles are subject first to forces which we know are the causes of the tides due to the sun's action, and secondly, to forces depending on the velocity of the particle in a manner which in a certain respect resembles CORIOLIS' forces in a centrifugal field. The latter were called by POINCARÉ "forces centrifuges composées". Accordingly the new forces might be designed as *compound tidal forces.*

In order to obtain the second approximation, it is necessary to specify our coördinates in greater detail. In every point-instant of the axis of time we draw all geodesic lines which are perpendicular to the time-track and we desire that these shall define space, three of them being chosen as the axes of space. For convenience sake the latter may be chosen perpendicular to each other.

It will be seen that this space cannot coincide with space as defined by an observer who is at rest with the sun. The two spaces of reference intersect in a surface, which, in each point-instant of the earth's helicoidal track contains the direction in the ecliptic perpendicular to the velocity and the direction perpendicular to the ecliptic. This involves a complication in comparing the relative positions of the two sets of spatial axes of reference. In the case of a planet moving in a circular orbit this difficulty is readily overcome.

[1]) In much the same manner during the moon's motion, as a first approximation, — apart from the sun's perturbing forces, which arise in the second approximation, — the plane of the orbit must keep its position unaltered relative to the falling axes of reference. This results in a motion of the nodes equal to the motion of these axes. De SITTER, proceeding in a totally different manner, arrived at a nodal motion of 1″.91 per century, which is exactly the amount given above for the precession. (Monthly Notices R. A. S. **77**, p. 172, 1916). A comparison with observation could only be made if the nodal motion, resulting from other causes and computed with NEWTON's law of force, were known to one further decimal place than it is at present. (DE SITTER, l.c.).

If then we compare the falling axes, before and after a year's revolution, with axes fixed to the sun and directed to fixed points in the heavens, we find a precession to the amount stated above.

As pointed out by DE SITTER the difficulty in testing the predicted precession by a comparison with observation lies not so much in the limits of accuracy of observation as in the fact that owing to our ignorance of the true values of the earth's principal moments of inertia we do not know with the precision required how much of the observed precession is accounted for by the actions of sun and moon according to NEWTON's law.

We now proceed to the analytical treatment of the problem.

The geodesic falling coordinates.

Consider some point-instant in an arbitrary field of gravitation, where the potentials are denoted by g_{ab}, $(a, b = 0, 1, 2, 3)$, x_0 being the time and $x^{(1)}$, $x^{(2)}$, $x^{(3)}$ space-coordinates. In the usual way we write the symbols of CHRISTOFFEL:

$$\left\{ {ab \atop n} \right\} = \Sigma\, g^{mn} \left[{ab \atop m} \right] = \Sigma\, g^{mn} \cdot \tfrac{1}{2} \left[\frac{\partial g_{am}}{\partial x^b} + \frac{\partial g_{bm}}{\partial x^a} - \frac{\partial g_{ab}}{\partial x^m} \right].$$

where g^{cm} are the algebraical complements of the g_{cm}.

A vector V^a is displaced parallel to itself over an interval dx^m, if its components decrease during the displacement according to the formula

$$d V^a = - \Sigma \left\{ {bm \atop a} \right\} V^b\, dx^m .$$

In the point-instant considered: $x^a{}_0$ $(a = 0, 1, 2, 3)$, choose a vector of unit length having time-character $A^a{}_0$:

$$\Sigma\, g_{ab}\, A^a{}_0\, A^b{}_0 = 1,$$

and three other vectors of unit length, all perpendicular to the former and to one another: $A^a{}_1$, $A^a{}_2$, $A^a{}_3$, such that

$$\Sigma\, g_{ab}\, A^a{}_\mu\, A^b{}_\mu = -1, \quad \text{and} \quad \Sigma\, g_{ab}\, A^a{}_i\, A^b{}_j = 0 \quad \text{if} \quad i \neq j.$$

As in our argument the component of time and the components of space will be treated in a different way, we shall establish the rule that whenever a suffix is indicated by a Greek character, it will not be liable to take the value 0.

We change variables by introducing the coordinates z^i according to the following formulae:

$$x^a - x^a{}_0 = \Sigma A^a{}_i z^i - \tfrac{1}{2} \Sigma \genfrac\{\}{0pt}{}{bm}{a} A^b{}_i A^m{}_j z^i z^j -$$

$$- \tfrac{1}{6} \Sigma \left[\frac{\partial}{\partial x^n} \genfrac\{\}{0pt}{}{bm}{a} - \genfrac\{\}{0pt}{}{ms}{a} \genfrac\{\}{0pt}{}{bn}{s} - \genfrac\{\}{0pt}{}{bs}{a} \genfrac\{\}{0pt}{}{mn}{s} \right] A^b{}_i A^m{}_j A^n{}_k z^i z^j z^k -$$

$$- \tfrac{1}{3} \Sigma Q^a{}_{b,mn} A^b{}_\mu (A^m{}_\nu A^n{}_0 - A^n{}_\nu A^m{}_0) z^\mu z^\nu z^0 -$$

$$- \tfrac{1}{6} \Sigma Q^a{}_{b,mn} A^b{}_0 (A^m{}_\nu A^n{}_0 - A^n{}_\nu A^m{}_0) z^\nu z^0 z^0 \quad . \; . \; . \; . \; . \; . \; . \; .$$

By $Q^a{}_{b,mn}$ we have denoted the same form within brackets which is found in the foregoing line. Note the symmetry possessed by $Q^a{}_{b,mn}$ in the suffixes b and m. If we put

$$R^a{}_{b,mn} = Q^a{}_{b,mn} - Q^a{}_{b,nm} ,$$

then $R^a{}_{b,mn}$ is the same as a four-index symbol of RIEMANN:

$$R^a{}_{b,mn} = \{ba, mn\},$$

and for its covariant components we have the identities which will be used in the following:

$$R_{ab,mn} = - R_{ba,mn} = - R_{ab,nm} = R_{mn,ab} ,$$

and

$$R_{ab,mn} + R_{bm,an} + R_{ma,bn} = 0.$$

We proceed to show that the above transformation actually affords the geodesic falling coordinates alluded to in the introduction.

The axis of z^0 coincides with a particle's track. Put every $z^\mu = 0$, and we get

$$x^a - x^a{}_0 = A^a{}_0 z^0 - \tfrac{1}{2} \Sigma \genfrac\{\}{0pt}{}{bm}{a} A^b{}_0 A^m{}_0 z^0 z^0 - \tfrac{1}{6} \Sigma Q^a{}_{b,mn} A^b{}_0 A^m{}_0 A^n{}_0 z^0 z^0 z^0 \, . \, .$$

As a second approximation, this is the equation for the geodesic line starting from the point-instant $x^a{}_0$ with initial direction parameters $A^a{}_0$ and where z^0 is the interval along the arc. Thus our time-axis is along a particle's track. Denote the second member of this equation by ξ^a.

The *axes of space are everywhere geodesics,* as far as the approximation goes, and perpendicular among themselves and to the axes of time. For put $z^0 = \tau$ and let the other coordinates vanish with the exception of one z^μ; on rearranging terms we get

$$x^a - x^a{}_0 - \xi^a = A^a{}_\mu z^\mu -$$

$$- \Sigma \genfrac\{\}{0pt}{}{bm}{a} A^b{}_\mu A^m{}_0 z^\mu \tau - \tfrac{1}{2} \Sigma Q^a{}_{b,mn} A^b{}_\mu A^m{}_0 A^n{}_0 \tau \tau z^\mu -$$

$$- \tfrac{1}{2} \Sigma \genfrac\{\}{0pt}{}{bm}{a} A^b{}_\mu A^m{}_\mu z^\mu z^\mu - \tfrac{1}{2} \Sigma Q^a{}_{b,mn} A^b{}_\mu A^m{}_\mu A^n{}_0 \tau z^\mu z^\mu -$$

$$- \tfrac{1}{6} \Sigma Q^a{}_{b,mn} A^b{}_\mu A^m{}_\mu A^n{}_\mu z^\mu z^\mu z^\mu . \; . \; . \; . \; . \; . \; . \; . \; .$$

This is, to the second approximation, the equation for the geodesic starting from the point-instant $x^a{}_0 + \xi^a$ with initial direction parameters

$$A^a{}_\mu - \Sigma \genfrac\{\}{0pt}{}{bm}{a} A^b{}_\mu A^m{}_0 \tau - \tfrac{1}{2} \Sigma Q^a{}_{b,mn} A^b{}_\mu A^m{}_0 A^n{}_0 \tau \tau,$$

and where z^μ is the interval measured along the arc. We notice that these parameters are the components of *the unit vector $A^a{}_\mu$, translated geodesically* from the origin of time *along the timetrack,* with an accuracy up to the second approximation. As a geodesic translation does not alter the mutual angles of the translated vectors, it follows that the axes of space and time remain perpendicular.

In the same way it may be shown that every spatial radius, that is a line $z_0 = \tau$, $z^{(1)} = \lambda_1 s$, $z^{(2)} = \lambda_2 s$, $z^{(3)} = \lambda_3 s$, with $\lambda_1{}^2 + \lambda_2{}^2 + \lambda_3{}^2 = 1$, is a geodesic, s being the interval along the arc from the origin.

The potentials g'_{ij} in geodesical falling coordinates.

We shall calculate the new values g'_{ij} by means of the transformation formula

$$g'_{ij} = \Sigma \, p_{ai} \, p_{bj} \, g_{ab} ,$$

where

$$p_{ai} = \partial x^a / \partial z^i .$$

In calculating the p_{ai} the symmetry of $Q^a{}_{b,mn}$ in the suffixes b and m is of great use. It enables us to arrange terms in a practical way. We get

$$p_{ao} = A^a{}_0 - \Sigma \genfrac\{\}{0pt}{}{bm}{a} A^b{}_0 A^m{}_i z^i - \tfrac{1}{2} \Sigma Q^a{}_{b,mn} A^b{}_0 A^m{}_i A^n{}_j z^i z^j -$$

$$- \tfrac{1}{2} \Sigma Q^a{}_{b,mn} A^b{}_i (A^m{}_j A^n{}_0 - A^n{}_j A^m{}_0) z^i z^j,$$

and for any $\mu \neq 0$, we get

$$p_{a\mu} = A^a{}_\mu - \Sigma \genfrac\{\}{0pt}{}{bm}{a} A^b{}_\mu A^m{}_i z^i - \tfrac{1}{2} \Sigma Q^a{}_{b,mn} A^b{}_\mu A^m{}_i A^n{}_j z^i z^j -$$

$$- \tfrac{1}{2} \Sigma Q^a{}_{b,mn} A^b{}_\mu (A^m{}_\tau A^n{}_0 - A^n{}_\tau A^m{}_0) z^\tau z_0 -$$

$$- \tfrac{1}{6} \Sigma Q^a{}_{b,mn} A^b{}_\sigma (A^m{}_\tau A^n{}_\mu - A^n{}_\tau A^m{}_\mu) z^\sigma z^\tau .$$

In the second lines of both formulae we shall replace $Q^a{}_{b,mn}$ by $\frac{1}{2} R^a{}_{b,mn}$. This is permitted because the bracket forms are skew-symmetrical in the suffices m and n.

In the first lines we find exactly the components of the vectors $A^a{}_i$ shifted geodesically from the origin to the point-instant denoted by z^j. Thus, as far as these parts of p_{ai} are concerned, the transformation formula $\Sigma p_{ai} \, p_{bj} \, g_{ab}$ gives 1, -1 or 0 for $i = j = 0$, $i = j = \mu$, or $i \neq j$ respectively. We get

$$g'_{00} = 1 + 0 - \tfrac{1}{2} \Sigma R_{ab,mn} A^a{}_0 A^b{}_i (A^m{}_j A^n{}_0 - A^n{}_j A^m{}_0) z^i z^j.$$

Obviously in the last term the value 0 for j contributes nothing to the sum. Because of the skew-symmetry of $R_{ab,mn}$ in a and b, the value 0 can be disregarded also for i, and the skew-symmetry im m and n allows us to write:

$$g'_{00} = 1 + \Sigma R_{ab,mn} A^a{}_0 A^b{}_\sigma A^m{}_0 A^n{}_\tau z^\sigma z^\tau.$$

Proceeding to $g'_{0\mu}$, we get

$$g'_{0\mu} = 0 + 0 - \tfrac{1}{4} \Sigma R_{ab,mn} A^a{}_\mu A^b{}_i (A^m{}_j A^n{}_0 - A^n{}_j A^m{}_0) z^i z^j -$$
$$- \tfrac{1}{4} \Sigma R_{ab,mn} A^a{}_0 A^b{}_\mu (A^m{}_\tau A^n{}_0 - A^n{}_\tau A^m{}_0) z^\tau z^0 -$$
$$- \tfrac{1}{12} \Sigma R_{ab,mn} A^a{}_0 A^b{}_\sigma (A^m{}_\tau A^n{}_\mu - A^n{}_\tau A^m{}_\mu) z^\sigma z^\tau.$$

Taking $i = 0$ in the first sum, this part cancels out against the second sum (skew-symmetry of $R_{ab,mn}$ in a, b). The remaining part is taken together with the third sum, and we get

$$g'^{0\mu} = \tfrac{2}{3} \Sigma R_{ab,mn} A^a{}_\mu A^b{}_\sigma A^m{}_0 A^n{}_\tau z^\sigma z^\tau.$$

Finally for $g'_{\mu\nu}$ we find:

$$g'_{\mu\nu} = - \varepsilon_{\mu\nu} + 0 - \tfrac{1}{12} \Sigma R_{ab,mn} [A^a{}_\nu A^b{}_\sigma (A^m{}_\tau A^n{}_\mu - A^n{}_\tau A^m{}_\mu) +$$
$$+ A^a{}_\mu A^b{}_\sigma (A^m{}_\tau A^n{}_\nu - A^n{}_\tau A^m{}_\nu)] z^\sigma z^\tau -$$
$$- \tfrac{1}{4} \Sigma R_{ab,mn} (A^a{}_\nu A^b{}_\mu + A^a{}_\mu A^b{}_\nu)(A^m{}_\tau A^n{}_0 - A^n{}_\tau A^m{}_0) z^\tau z^0,$$

where $\varepsilon_{\mu\nu} = 1$ for $\mu = \nu$ and $\varepsilon_{\mu\nu} = 0$ for $\mu \neq \nu$. Having regard to the skew-symmetries of $R_{ab,mn}$ we reduce this expression to

$$g'_{\mu\nu} = - \varepsilon_{\mu\nu} + \tfrac{1}{3} \Sigma R_{ab,mn} A^a{}_\mu A^b{}_\sigma A^m{}_\nu A^n{}_\tau z^\sigma z^\tau.$$

If we remember the transformation formula for $R_{ab,mn}$:

$$R'_{ij,rs} = \Sigma p_{ai} p_{bj} p_{mr} p_{ns} R_{ab,mn},$$

we at once see that without lowering the degree of approximation, we may abridge the forms for g'_{ij} into:

$$g'_{00} = 1 + \Sigma R'_{0\sigma,0\tau} z^\sigma z^\tau,$$
$$g'_{\mu 0} = \tfrac{2}{3} \Sigma R'_{\mu\sigma,0\tau} z^\sigma z^\tau,$$
$$g'_{\mu\nu} = - \varepsilon_{\mu\nu} + \tfrac{1}{3} \Sigma R'_{\mu\sigma,\nu\tau} z^\sigma z^\tau.$$

It must be noticed that these gravitation potentials *depend no more on the time* z^0. The field in our geodesic falling coordinates is stationary as far as our approximation goes.

The $R'_{ij,rs}$ are closely associated with RIEMANN's measure of curvature. If only particles are considered moving so *near the centre* that the squares of the distances multiplied by the measure of curvature may be neglected altogether, then the g'_{ij} may be considered to be *constant* and to have the homoloidal values 1, —1, —1, —1.

Equations of motion for free particles in geodesical falling coordinates.

We put forward the simplifying assumption that only particles

will be considered moving slowly relative to the falling axes and that the square of their velocities will be negligible compared with the square of the velocity of light, which, in our coordinates, is nearly unity.

The equations of motion are

$$\frac{d^2 z^\alpha}{ds^2} = - \Sigma \left\{ \begin{matrix} i\,j \\ \alpha \end{matrix} \right\} \frac{dz^i}{ds} \frac{dz^j}{ds}.$$

With the above assumption we may put $dz^0/ds = 1$, and we need only consider combinations where i or j or both of them are 0. In the CHRISTOFFEL symbols the differential coefficients of g'_{ij} are not known beyond the first powers of the coordinates; therefore the reciprocals g'^{ij} may be taken to be 1, —1, —1, —1, and 0. This makes

$$\left\{ \begin{matrix} i\,j \\ \alpha \end{matrix} \right\} = - \left[\begin{matrix} i\,j \\ \alpha \end{matrix} \right]$$

Calculating we find:

$$\left[\begin{matrix} 0\,0 \\ \alpha \end{matrix} \right] = - \Sigma R'_{0\alpha,0\tau} z^\tau,$$

and

$$\left[\begin{matrix} 0\,\beta \\ \alpha \end{matrix} \right] = \tfrac{1}{3} \Sigma (R'_{\alpha\beta,0\tau} + R'_{\alpha\tau,0\beta} - R'_{\beta\alpha,0\tau} - R'_{\beta\tau,0\alpha}) z^\tau,$$
$$= - \Sigma R'_{\beta\alpha,0\tau} z^\tau - \tfrac{1}{3} \Sigma (R'_{\beta\alpha,\tau 0} + R'_{\alpha\tau,\beta 0} + R'_{\tau\beta,\alpha 0}) z^\tau.$$

The bracket vanishes by symmetry of the $R'_{\beta\alpha,\tau 0}$, thus

$$\left[\begin{matrix} 0\,\beta \\ \alpha \end{matrix} \right] = - \Sigma R'_{\beta\alpha,0\tau} z^\tau.$$

Finally the *equations of motion for free particles* become:

$$\frac{d^2 z^\alpha}{dz_0{}^2} = - \Sigma R'_{0\alpha,0\tau} z^\tau - \Sigma R'_{\beta\alpha,0\tau} z^\tau \frac{dz^\beta}{dz_0}.$$

Here we can put

$$\Sigma R'_{23,0\tau} z^\tau = 2\omega_1,$$
$$\Sigma R'_{31,0\tau} z^\tau = 2\omega_2,$$
$$\Sigma R'_{12,0\tau} z^\tau = 2\omega_3,$$

This brings the last term into the form

$$- 2 [\omega \,.\, \mathrm{v}].$$

Interpreting the equation of motion we note that the first term in the right hand member accounts for the forces causing the tidal effects. The second member has the form of a CORIOLISIAN force, but the peculiarity is that the rotation vector ω figuring in it, is a linear function of the coordinates and thus on opposite sides of the planet has the opposite direction. It is conveniently called the compound tidal force. It might come into play when we consider the motions of a satellite.

Resuming, we can say that as a first approximation the equations of motion for free particles in the geodesic falling system are just the same as those in classical dynamics under no forces. When we have mutual forces between the particles, their effects on the motions will be quite the same as predicted by classical dynamics. In particular, a spinning top will keep the direction of its axis of rotation unaltered relative to the axes of reference, i.e. our geodesic falling coordinates. Hence when referred to the original coordinates, the spinning top will for its axis of rotation show whatever precession the geodesical falling axes might exhibit.

The same must be said for the plane of the orbit of a particle, moving under a central force.

If the tidal forces are considered, their effect in changing the direction of the axis of rotation relative to the falling coordinates would be zero if the earth were of spherical shape. If not, the precession caused by them is to be taken in reference to the falling axes, and the precession of the latter will be superposed on the precession due to the tidal forces.

The common tidal forces are but part of the second approximation. The remaining part is a compound tidal force at right angles and proportional to the velocity, proportional to the distance from the centre and, like the CORIOLISian forces, may be determined as a (three-dimensional) vectorial product of the velocity into a vector which, by means of certain components of the RIEMANNian bivector-tensor of curvature, is a linear function of the radius vector from the centre. For the present we shall leave these forces aside, and turn to the question of how much the amount of the precesssion of the falling axes may be.

The precession of the geodesic falling axes in the case of a planet moving in a circular orbit.

As we pointed out already, a complication in finding the precession of the falling axes arises from the fact that the space of the falling axes makes some angle with space as defined by an observer who has his coordinates fixed to the sun. These spaces intersect in a plane perpendicular to the velocity. By confining ourselves to circular orbits, matters present themselves much less complicated.

In each point-instant of the helicoidal track of the planet we draw *four local axes*: one coinciding with the direction of the track; a second in the direction away from the sun along a radius vector; a third perpendicular to the ecliptic; and the last one with a time

component and a component tangent to the circular orbit; in such a manner that these four directions will be all perpendicular to each other. Now, if the planet with the geodesical falling axes comes across some particular set of local axes, the axes of time, both the falling and the local, will coincide, and therefore the spaces of the falling and of the local axes too will be the same. Thus the position of the falling axes relative to the local ones can be stated and the positions before and after a revolution compared.

The gravitational field of the sun is given by the form of the infinitesimal interval:

$$ds^2 = (1 - \alpha/r)\, dt^2 - \frac{dr^2}{1 - \alpha/r} - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\varphi^2.$$

In this field a circular motion is possible in the plane $\theta = \frac{1}{2}\pi$, with "radius" R and with angular velocity

$$d\varphi/dt = \omega = \sqrt{\alpha/2R^3}.$$

Now, everywhere along the track define four vectors $A^a{}_0, A^a{}_1, A^a{}_2, A^a{}_3$, as follows

	(0)	(1)	(2)	(3)
$A^a{}_0$:	$\sqrt{\frac{2R}{2R-3\alpha}}$,	0,	0,	$\frac{1}{R}\sqrt{\frac{\alpha}{2R-3\alpha}}$,
$A^a{}_1$:	0,	$\sqrt{1-\alpha/R}$,	0,	0,
$A^a{}_2$:	0,	0,	$1/R$,	0,
$A^a{}_3$:	$\sqrt{\frac{\alpha R}{(R-\alpha)(2R-3\alpha)}}$,	0,	0,	$\frac{1}{R}\sqrt{\frac{2(R-\alpha)}{2R-3\alpha}}$.

It will be seen that these vectors are all of unit length and perpendicular to one another. They define the local axes.

A set of these vectors in one particular point-instant can be taken as the starting vectors of the geodesic falling coordinates. To find the directions of the falling axes after a lapse of interval ds (components $A^a{}_0 ds$) we need the values of CHRISTOFFEL's symbols. These are, in coördinates t, r, θ, φ:

$$\left\{ {01 \atop 0} \right\} = \frac{\alpha}{2R(R-\alpha)},$$

$$\left\{ {00 \atop 1} \right\} = \frac{\alpha(R-\alpha)}{2R^3}, \quad \left\{ {11 \atop 1} \right\} = \frac{-\alpha}{2R(R-\alpha)}, \quad \left\{ {22 \atop 1} \right\} = -(R-\alpha), \quad \left\{ {33 \atop 1} \right\} = -(R-\alpha)\sin^2\theta,$$

$$\left\{ {12 \atop 2} \right\} = \frac{1}{R}, \quad \left\{ {13 \atop 3} \right\} = \frac{1}{R},$$

$$\left\{ {33 \atop 2} \right\} = -\sin\theta\cos\theta, \qquad \left\{ {23 \atop 3} \right\} = \frac{\cos\theta}{\sin\theta}.$$ The remaining symbols vanish.

Now, if we calculate the geodesic increment along ds of the vector components:

$$dA^{a}_{i} = -\Sigma \begin{Bmatrix} bm \\ a \end{Bmatrix} A^{b}_{i} A^{m}_{0} ds,$$

we find

$$dA^{a}_{0} = 0,$$
$$dA^{a}_{2} = 0,$$

but

$$dA^{a}_{1} = -\sqrt{a/2R^3}\,.\,A^{a}_{3}\,ds, \text{ or } = -\omega\, A^{a}_{3}\,ds,$$

and

$$dA^{a}_{3} = +\sqrt{a/2R^3}\,.\,A^{a}_{1}\,ds, \text{ or } = +\omega\, A^{a}_{1}\,ds.$$

From this we infer that the falling axes of $Z^{(1)}$, $Z^{(3)}$, after the lapse of interval ds, as compared with the local axes reached after the interval, show a retrograde rotation of amount ωds in the plane of these axes. Meantime the planet's anomaly has increased by ωdt. Thus, the two angular velocities are the same if the one is measured in ds and the other in dt. The ratio is

$$ds = \sqrt{(1-3a/2R)}\,.\,dt.$$

In the circular planetary motion this will continue uniformly, and it follows that when the planet has completed a revolution, the falling axes will not yet have completed theirs if compared with the local axes passed by during their motion. At the instant the falling axes will have completed a revolution, the radius vector will make an angle of

$$2\pi \sqrt{\frac{2R}{2R-3a}},$$

with the radius from which they started. Relative to this new radius everything will be in exactly the same position as it was in the beginning of the revolution,

Neglecting higher powers of a/R we conclude that there is a precession which, per annum, amounts to the excess of the angle between the two radii over 2π, i.e.

$$2\pi \left[\sqrt{\frac{2R}{2R-3a}} - 1\right] = 2\pi\,.\,\tfrac{3}{4}\frac{a}{R}$$

per annum.

For the earth, it is 0.019 of a second of arc per annum.

Zoology. — *"Die Verwandtschaft der Merostomata mit den Arachnida und den anderen Abteilungen der Arthropoda"*. Von J. Versluys und R. Demoll. (Communicated by Prof. Weber).

(Communicated at the meetings of Sept. 25, and October 30, 1920).

I.

Noch immer gehen die Ansichten über den phylogenetischen Zusammenhang der grossen Abteilungen der Arthropoden, der *Onychophora*, *Myriapoda*, *Hexapoda*, *Arachnida* und *Crustacea* erheblich auseinander. Und es ist vor Allem die verschiedene Beurteilung der Verwandtschaft der Merostomen mit den Arachniden, welche zu so sehr verschiedenen Auffassungen in diesen Fragen führt.

Im Mittelpunkte der Erörterung steht der einzige lebende Vertreter der Merostomen, die Gattung *Limulus*. Diese Form lebt im Meere und atmet durch Kiemen, welche anscheinend von Gliedmassen getragen werden. Dementsprechend wurde das Tier zuerst den Crustaceen zugerechnet. Weitere Untersuchung schien diese Auffassung zu bestätigen; namentlich machte die Entdeckung grossen Eindruck, dass die junge Larve von *Limulus* im Körperaufbau den Trilobiten, diesen alten, ausgestorbenen Vertretern der Crustaceen, ähnlich ist. Man sprach geradezu von einem Trilobiten-stadium in der Entwicklung von *Limulus*.

Andrerseits hatte schon 1829 Straus Dürkheim mit grossem Nachdruck auf eine Blutsverwandtschaft von *Limulus* mit den Arachniden hingewiesen. Ihn folgten einige andere Forscher, bis 1881 und den darauffolgenden Jahren Ray Lankester das *Limulus*-problem einer eingehenden Prüfung unterzog. Er wies dabei eine tatsächlich überrasschende Uebereinstimmung im Baue von *Limulus* mit den Arachniden nach, ganz besonders mit den Scorpioniden. Lankester zweifelte aber andrerseits nicht an der Verwandtschaft von *Limulus* mit den Trilobiten und anderen Crustaceen. Da *Limulus* im Vergleich zu den Crustaceen eine viel mehr spezialisierte Form ist, musste er annehmen, dass *Limulus* von den Trilobiten oder damit verwandten Crustaceen abstammt. Die Arachniden mussten dann wieder von *Limulus* oder dessen weniger spezialisierten vorfahren, den Gigantostraken, abstammen, wobei die Stammformen der Arachniden vom Meeresleben zum Landleben übergegangen wären.