The other terms

$$-e^{-2\alpha(t-t_n)}$$

every time disappear very soon after each discontinuity and produce a small bend at the upper part of each jump. See e.g. fig. 4. *The more the galvanometer is damped,* the greater therefore α, the smaller these bends are and *the quicker the galvanometer is able to follow the sudden changes of the flux.*

If we disregard these small bends the deflection of the greatly damped galvanometer is simply given by

$$\theta = \frac{1}{2\alpha} \Sigma \Delta_n N' = \frac{N'}{2\alpha}$$

and the galvanometer deflections are at any moment proportional to the flux going through the induction solenoid[1]). If the H therefore increases proportionally to the time, the deflection of a greatly damped galvanometer accurately describes a hysteresis curve. This extreme case can be better approximated with a moving coil galvanometer than with a string instrument. The curve described by the image of the string galvanometer representing the solution of (1) from which the function N has to be found back, does not lead to such a simple interpretation.

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[1]) This relation is obtained at once from (1) by neglecting the first and third term of the left hand member.

Physics. — *"Note on the paramagnetism of solids."* By Prof. P. EHRENFEST.

(Communicated at the meeting of December 18, 1920).

1. The object of this note is to show that the validity of the CURIE-LANGEVIN law for the susceptibility of *solid* paramagnetic substances may be arrived at on a different theoretical basis from that discussed by WEISS[1]) STERN[2]) and LENZ[3]) (comp. section 3 of this paper), namely on the following assumption: the atoms or molecules of a solid paramagnetic substance contain electrons circulating in definite, practically fixed orbits of lowest quantum-number (which we shall call "rest-orbits"), so that, in the absence of an external magnetic field, there is no perceptible difference in the energy of the two opposite (right and left) directions of circulation. It will be further assumed, that with a magnetic field H at a given temperature T the statistical distribution between right and left which corresponds to H and T[4]) will automatically establish itself. On these assumptions the CURIE-LANGEVIN law will be found to hold c.f. § 4), and, in the case of a crystal symmetry or of a crystalline powder of any crystalline structure, even with the correct numerical factor 3 in the denominator.

2. LANGEVIN'S theory explains the fact, that paramagnetic gases

[1]) P. WEISS, C. R. **156** (1913) 1674.

[2]) O. STERN. Z. f. Phys. **1** (1920) 147.

[3]) W. LENZ. Phys. Zschr. **21** (1920) 613.

[4]) If the motions of the electrons are not submitted to any limitations by means of quantum-conditions and if the law of thermal statistics is applied to all the degrees of freedom, the body is found to be *unmagnetic*; comp. H. A. LORENTZ. Vortr. kinet. Theorie der Mat. u. Elektr. p. 188. (Teubner 1914); H. J. VAN LEEUWEN, Vraagstukken uit de elektronentheorie van het magnetisme p. 54. [Dissertation Leiden 1919]. Accordingly in his theory of paramagnetism LANGEVIN assumes, that the ampère currents, which represent the elementary magnets, are not subjected to thermal statistics in their cyclic co-ordinates. But when, as in our case, quantum conditions are introduced into the statistic scheme, the possibility of paramagnetism returns, even if not a single degree of freedom remains outside the scheme. To this circumstance my attention was drawn by Prof. N. BOHR in a conversation (1919).

and solutions of paramagnetic salts obey CURIE's law; in these cases, taking account of the smallness of

$$a = \frac{\mu H}{rT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

it gives at the limit the CURIE-LANGEVIN relation

$$\chi = \frac{N\mu^2}{3rT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where χ = molair susceptibility; N = AVOGADRO's number $r = \frac{R}{N}$ = BOLTZMANN's constant, μ = magnetic moment of one molecule. Experiment shows that this relation also holds for certain solid paramagnetic substances. In particular KAMERLINGH ONNES has found that gadolinium sulphate follows the law exactly down to the lowest (Helium) temperatures. This is not what one would expect in view of the assumption underlying the derivation of the formula, namely that the orientation of the elementary magnets depends exclusively on: (1) the thermal motions, (2) the directing influence of the external field H. In solid (crystalline) bodies there must be additional forces of a different nature which also play a part in the orientation of the elementary magnets.

3. It follows from the papers by WEISS, STERN, and LENZ, that in the case of forces of that kind which depend on a crystalline structure the same relation is obtained, if only it is assumed that:

1. The potential energy Φ of the forces which try to keep the axis of an elementary magnet parallel to a definite direction R is centrally symmetrical, i. e. it is equal for any two opposite orientations of the elementary magnets.

2. When the temperature or the field changes, the statistical distribution of the orientations, which according to BOLTZMANN corresponds to the new values of T and H, and to Φ, actually establishes itself; this involves, that there is no retardation in the necessary reversal of the elementary magnets (false equilibrium).

4. Let us now consider a solid body containing electrons whose "rest-orbits" satisfy the conditions mentioned in § 1. For a particular electron let the magnetic moment of its orbit be μ and its projection on the direction of the field $\pm \mu \cos \vartheta_0$, according to whether it circulates to the right or left [1]). At the temperatures considered we

[1]) Strictly speaking μ itself depends on the orientation of the orbit relatively to the field H, since the velocity of the electron is affected by it. In a magnetic field it is not simply the *mechanical* momentum of the electron that is to be

can leave out of account the possibility of the electron jumping to an orbit of higher quantum-number. Now the times during which the electron moves to the right and to the left are in the ratio [1])

$$e^{a\cos\vartheta_0} : e^{-a\cos\vartheta_0} \qquad \left(a = \frac{\mu H}{rT}\right) . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the time-average of the projection of its magnetic moment on the direction of H is therefore given by

$$\frac{\mu \cos\vartheta_0 \, e^{a\cos\vartheta_0} - \mu\cos\vartheta_0 \, e^{-a\cos\vartheta_0}}{e^{a\cos\vartheta_0} + e^{-a\cos\vartheta_0}} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Since a is small we may put

$$e^{\pm a\cos\vartheta_0} \backsim 1 \pm a\cos\vartheta_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

hence (4) becomes

$$\cos^2\vartheta_0 \frac{\mu^2 H}{rT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and the susceptibility χ for N such electrons will be

$$\chi = \frac{N\mu^2}{rT} \overline{\cos^2\vartheta_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

the mean being taken over the possible orientations of the "rest-orbits" of the electrons. For a crystal of cubic symmetry or a powder of arbitrary crystalline structure we have obviously

$$\overline{\cos^2\vartheta_0} = \frac{1}{3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

whereby equation (2) is arrived at.

5. *Additional remarks.*

1. According to the above theory the Röntgen-reflection of a crystal would not be changed by magnetisation. By a very sensitive

taken equal to $\frac{h}{2\pi}$ or a multiple of it, but the *electro-kinetic* momentum of the electron (reduced to mechanical units) has to be added. However, even with a very high value of H, this term is small compared to the other. We may therefore neglect this *dia*magnetic action, depending on induction, just as LANGEVIN did in his fundamental theory.

[1]) In the power of e we must put the quantity which remains constant during a "collision". For an electron, which in a constant field H changes from a right-hand to a left-hand motion, this quantity is *not* the sum of the mechanical and electro-kinetic energies, but a kind of "ROUTH-*Function*" has to be taken. (Comp. Dissertation by H. J. VAN LEEUWEN. Leiden 1919, p.p. 11, 18, 52—54, an extract of which is soon to appear in the Journal de Physique). A simple calcution on this basis, with the approximation referred to in the previous footnote, gives the ratio (3).

null-method COMPTON and ROGNLEY [1]) have actually established the absence of any such effect for the (ferro-magnetic) crystals of magnetite.

2. In the theory of LANGEVIN-WEISS the rotational movement of the elementary magnets gives its own contribution to the kinetic energy and therefore to the specific heat, whereas in our theory the corresponding term does not occur. At first sight this appears strange, since the form of equation (2) seems to point to equipartition. But a similar result will always be obtained, in cases, where the lowest quantum-motions which are possible, possess a very small difference of energy $\Delta\varepsilon$ with respect to each other (in our case $2\,\mu H cos\,\vartheta$ with a field H). In those cases there is always a range of temperatures T, where T is small enough for the higher quantum-orbits to be disregarded, and at the same time large enough with respect to $\Delta\varepsilon$.

3. Although the transition between the right- and left-hand motions requires an amount of energy small as compared to νT, still it may require the coincidence of favourable circumstances to bring about the corresponding reversal of the motion (moment of momentum). Since we are dealing with a quantic process, it is probably difficult to treat this question quantitatively. In general we may expect, that the corresponding retardations in the establishment of the magnetisation would show themselves most easily at very low temperatures and rapidly alternating fields [2]). (For light-vibrations χ is always $= 0$). They would for instance give rise to a kind of hysteresis and a corresponding development of heat, when gadolinium-sulphate is periodically magnetized in opposite directions.

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[1]) A. H. COMPTON and O. ROGNLEY. Is the atom the ultimate magnetic particle? Phys. Review. **16** (1920) 464.

[2]) The possibility of this retardation was pointed out by LENZ in his address at Nauheim (l.c. p. 615) from the point of view of the sudden reversals of the magnetic atoms. Previously to this in the beginning of July 1920 the question was discussed by Prof. KAMERLINGH ONNES and me, both from the point of view of WEISS' theory and of the assumptions ol this paper, together with the possibility of testing it experimentally.

Palaeontology. — *"The identity of the genera Poloniella and Kloedenella."* By Miss J. E. VAN VEEN. (Communicated by Prof. J. W. MOLL.)

(Communicated at the meeting of December 18, 1920).

In the year 1896 a treatise appeared by Prof. Dr. GÜRICH about the Palaeozoïcum of the Polish middle mountain range. In this treatise the author instituted the new genus *Poloniella* (4, p. 388) for a few carapaces and valves of formerly unknown Ostracoda, originating from the middle devonian Ostracoda marl of Dombrowa near Kielce. These remains he united into one species viz. *Poloniella devonica.*

Some twelve years afterwards the two American palaeonotologists Dr. ULRICH and Dr. BASSLER supplied a contribution to the knowledge of the *Beyrichiidae.* On this occasion the new genus *Kloedenella* (8, p. 317) was founded also, under which group they intended to bring together eight species at the least.

In 1914 Prof. Dr. BONNEMA (2, p. 1087; 3, p. 1105) was able to amplify the characteristics of the genus *Kloedenella* as given by ULRICH and BASSLER as a result of his investigation into the nature of the Ostracod, which Dr. AUREL KRAUSE formerly described under the name *Beyrichia hieroglyphica.*

In comparing what the above mentioned authors have said about the genera *Poloniella* and *Kloedenella,* it is obvious that the latter are identical. It should however be observed that what BONNEMA takes to be the anterior part of the carapace — and rightly in my opinion — is considered the posterior part by the others. As a natural result the valve, which is the left one, according to BONNEMA, is called the right one by the others.

Thus BONNEMA found as the most characteristic feature of the genus *Kloedenella* that the right valve before the straight part of the hinge line has a notch in which a process of the left valve fits. (fig. 3).

In the genus *Poloniella* a similar connection of the valves seems to be present. GÜRICH does not mention this fact emphatically, but as he writes: "Ganz am vorderen Ende jedoch tritt der linke Saum wieder zurück und auf der hinteren Kantenhälfte springt der rechte Saum sogar stark über", I should conclude from this that it occurs here also.

Besides BONNEMA had found that in the genus *Kloedenella* the right valve overlaps the left one at the hinge line, whereas the opposite is the case with the free edges. In accordance herewith, GÜRICH writes ".... greift am Schlossrande die linke Klappe in einer gradlinigen Leiste vorspringend über den entsprechenden Rand