the os pubis, leaves the pelvis at the place, where the vasa femoralia is located (see Fig. 4), while it covers the posterior part of

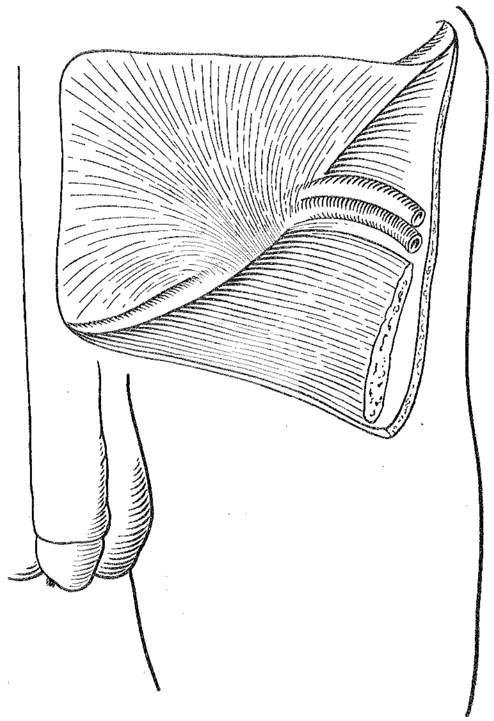

Fig. 4.
Front-view of the f. transversalis after detaching and folding back all layers of the anterior abdominal wall lying before it.

this bone, i.e. the interior of the pelvic wall, medially behind the corpus ossis pubis, we may naturally expect that in one spot or other, situated between these two places, the fascia transversa must cross the os pubis. Now, this crossing-point could easily be found in our anatomical preparation, because at the same place a transparent spot became at once conspicuous. Moreover this spot was less resistant to the pressure of the palpating finger, so that it is not out of the bounds of probabilities to state that there, that is exactly on the inside of THOMSON's ligament, the fascia yields to the peritoneum and allows it to force its way into the canal. It should be noticed that, if this supposition is correct, the hernial sac must enter the canal from the medial side, on the medial side of the vessels, consequently precisely at the spot where the septum femorale is usually localized, so that my view does not clash with daily experience. However, further investigations may throw more light upon the matter.

Mathematics. — *"Explanation of some Interference Curves of Uniaxial and Biaxial Crystals by Superposition of elliptic Pencils".* By J. W. N. LE HEUX. (Communicated by Prof. HK. DE VRIES).

(Communicated at the meeting of February 26, 1921).

In LISSAJOUS' "Etude optique des mouvements vibratoires" [1]), the name of "Unisson" is given to the curve, resulting from the composition of two vibrations, which only differ in amplitude and in phase.

When the amplitudes are supposed to be equal and not diminishing with continued movement, when the directions are at right angles and the difference of phase increases from 0° to 90° — the unisson may be considered as a pencil of ellipses, whose envelope is a square [2]).

The null-ellipse of this pencil is a diagonal d_1 of the square, the end-ellipse is the circle, inscribed in the square. Let the other diagonal be d_2.

Two equal unissons, U_1 and U_2, partially covering each other, produce certain "watered curves" (moiré), which may be divided into two sets:

1°. those similiar to hyperbolas, when the exact covering of U_1 and U_2 may be obtained by moving the centre of the pencil along d_1 (fig. 1) and

2°. those, similiar to lemniscates, when the exact covering may be obtained by moving the centre along d_2 (fig. 2).

The "watered curves" [3]), above mentioned, bear a strong resemblance to the interference curves of some crystals — it will be examined, whether these interference curves may be explained by superposition of two pencils of ellipses.

Therefore, the image of the hyperbolas will be compared to the interference curves of a uniaxial crystal in convergent light, the crystal-plate being cut parallel to the optic axis and the image of the lemniscates to the interference curves of a biaxial crystal, the plate being cut perpendicular to the first diameter.

1) Annales de Chimie et de Physique, 3ième série. t. LI. Octobre 1857.
2) Proc. Kon. Acad. v. Wet. pp. 857—870 March 1914.
Mathésis, 3ième série t. X pp. 209—212, 1910.
3) On stereoscopic curves, see Comptes Rendus t. 130 p. 1616.
Also: Harmonic Vibrations and Vibration Figures by J. GOOLD, C. E. BENHAM, R. KERR and Prof. L. R. WILBERFORCE, Newton, London.

The "isophase" surface of BERTIN, by means of which interference curves usually are explained, is the locus of points with a constant difference of retardations:

$$\vartheta = d V \left(\frac{1}{V_1} - \frac{1}{V_2} \right).$$

In this formula d means the length of the way in the crystal, supposing, with BERTIN [1]), that the bifurcation of the ray is neglected, V the velocity of light in the medium, V_1 the velocity of the ordinary and V_2 that of the extra-ordinary ray in the crystal.

V being constant, we may write $\frac{\vartheta}{V} = \frac{d}{V_1} - \frac{d}{V_2} =$ constant.

The centre of the ellipsoid of polarisation is supposed to be in the centre of light in the lower side of the plate.

Let P be a point of the image of interference curves in the upper side of the plate, where $d = d_1$.

Suppose $\frac{d_1}{V_1} = m$, then $d_1 = m V_1$, so P lies on a surface $\varrho = m V_1$ ($m =$ constant), homothetic with the blade $\varrho = V_1$ of the surface of the wave.

Suppose $\frac{d_1}{V_2} = n$, then $d_1 = n V_2$ and P lies also on the surface $\varrho = n V_2$ ($n =$ constant), homothethic with the blade $\varrho = V_2$ of the surface of the wave.

The surfaces $\varrho = m V_1$ and $\varrho = n V_2$, each being cut by the upper side of the crystal plate in a pencil of curves C_1 and C_2, when m and n are variable, it is evident, that each curve of the image is the locus of the points of intersection of those curves of the pencils, which correspond to $m - n =$ constant.

The forms, into which the wave is found to diverge, are a sphere and an ellipsoid for uniaxial crystals; so the sections with the upper side of a plate, cut parallel to the optic axis, are a circle and an ellipse, having the same tangent in the extremities of the minor axis of the ellipse (the section in the upper side is an approximate form of that in the lower side of the plate).

For the wave in a biaxial crystal, we find two surfaces, which are in fact one continuous surface. A plate, cut perpendicular to the first diameter, gives two ellipses, one of which is wholly surrounded by the other.

Thus, it may be said generally, that interference curves may be considered as "watered figures" of two concentric pencils of ellipses E_1 and E_2.

[1]) Ann. de Chim. et de Phys. (3). 63 pp. 57—92, 1861 en Sér 2. T 63. 1861.

J. W. N. LE HEUX: "Explanation of some Interference-Curves of Uniaxial and Biaxial Crystals by Superposition of Elliptic Pencils".

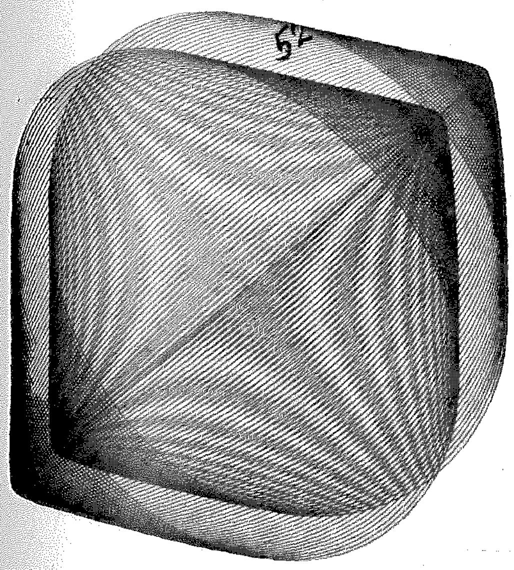

Fig. 1.

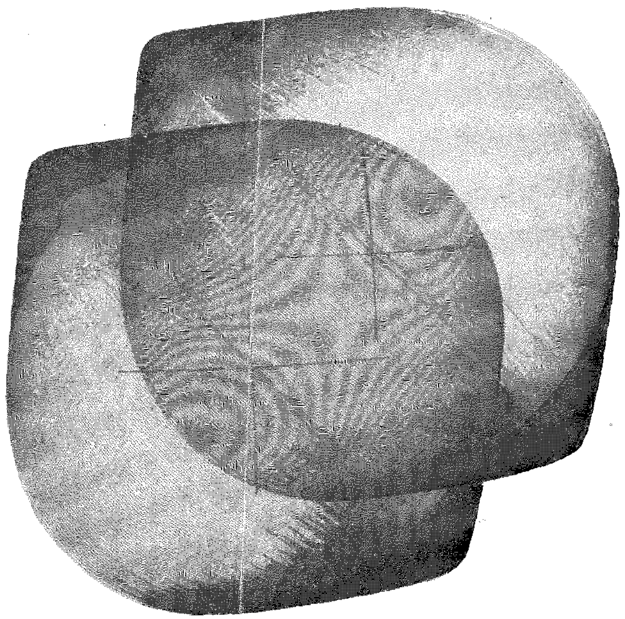

Fig. 2.

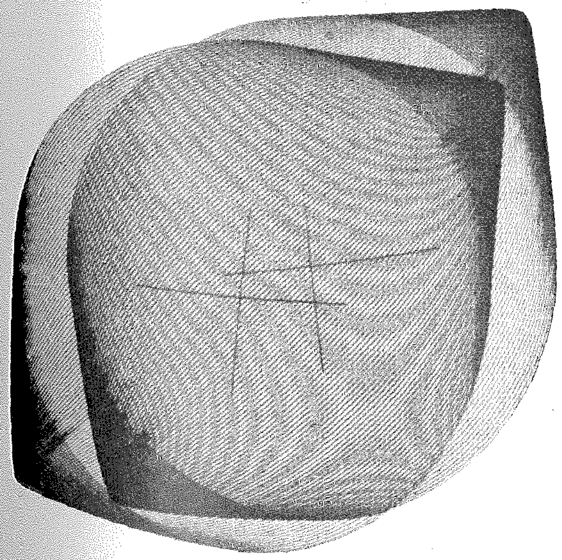

Fig. 3.

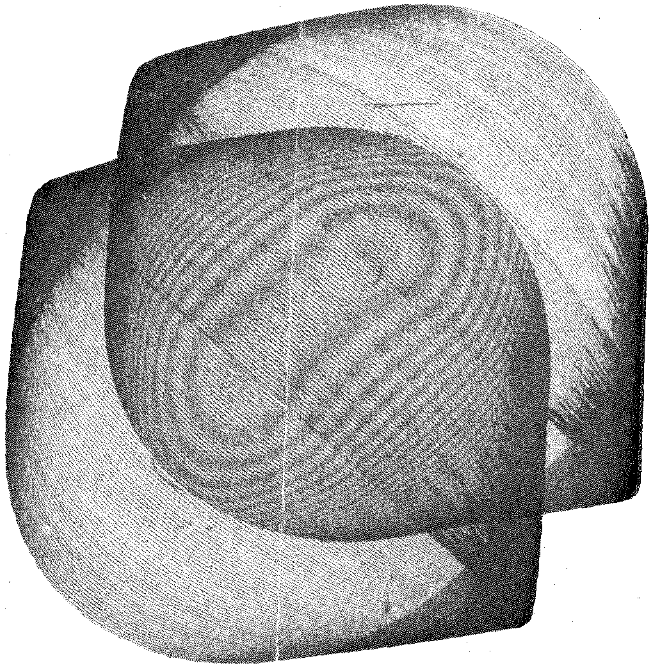

Fig. 4.

Now, it is possible by means of a simple algebraïc transformation, to derive these concentric pencils E_1, and E_2, from the excentric pencils U_1, and U_2.

The curves of the new pencils however are composed of four vibrations, which differ only in phase and whose directions two by two are parallel to those of the compounding vibrations of the unissons.

To support these conclusions by experiment, I have made use of an instrument [1]) planned by myself, with four pendulems, two of which describe a LISSAJOUS-curve on a plane, which describes such a curve itself. In our case, those two curves are unissons. The resulting movement is a spiral, which may be considered as a pencil of concentric circles. By altering the difference of phase, the circles are converted into ellipses.

It is shown now by experiment, that always hyperbolas are obtained by superposition of two pencils, which have for null-curves a circle and an ellipse, having the same tangent in the extremities of the minor axis, and lemniscates, when the null-curves are two ellipses, one of which is surrounded by the other.

Because the pencils U_1 and U_2, by inversion, may be derived from the pencils E_1, and E_2, it is evident, that the "watered curves" of the unissons are approximate images of interference curves of uniaxial and biaxial crystals.

This theory is supported by further experiments. The centre of the hyperbolas is displaced by a small rotation of one of the unissons. The image resembles the interference-curves of a uniaxial crystal, cut non-parallel to the optic axis (fig. 3).

With a small number of curves, the image of the lemniscates has both poles surrounded by the inner curve, just as is taught in crystal-optics. (fig. 4). Two concentric pencils of circles (each compounded of four vibrations), show the phaenomenon of Newton's rings.

Probably it is also possible to obtain AIRY's spirals in this manner, and the way, in which a certain image appears by superposition of two pencils of ellipses may throw more light upon the phaenomena of refraction and polarisation of light in crystals.

[1]) Other instruments are described in: Harmonic Vibrations and Vibration Figures by GOOLD etc.

Also: A. C. BANFIELD. The Photo-Ratiograph, Illustrated London News, Sept. 25th 1920. pg. 470.

Comptes Rendus t. 130. pg. 1616.

H. J. OOSTING. Hand. XIV Ned. Nat. en Geneesk. Congres March 1913. Ann. d. Phys. u. Chem. N. F. 33. p. 415 1888; Maandbl. voor Natuurwetenschappen 1898; Zt. f. d. Physik. u. Chem. Unterr. 8 p. 190 1895 and 11, p. 221 1898.