The results mentioned above would not have been attained, if I had not been sustained by the confidence in the success of the undertaking, which the director of the Laboratory, Prof. E. VERSCHAFFELT, has always shown, and by the aid in word and deed, I experienced from Mr. J. VAN DER ZWAAL, instrumentmaker, and Mr. J. MESSIAS, clockmaker, in the technical execution of the plans.

October 1920.

(*From the Laboratory of Plant-Physiology of the University of Amsterdam*).

Hydrology. — "*On the Motion of Ground Water in Frost and Thawing Weather.*" By Prof. EUG. DUBOIS.

(Communicated at the meeting of January 29, 1921).

From small pools, from detached ditches, especially with high sides, from wheel tracks, water is seen to disappear on prolonged frost from under the ice formed, so that beneath the ice there are air-filled spaces. The vanished water-layer can be from a few centimeters to some decimeters thick. The phenomenon is universally known, but the question what happens to the water, has not been answered as yet.

Other, equally common phenomena, are observed in thawing weather. Before the frost the soil may have been fairly dry near the surface, but without previous snow or rain it is found to be muddy on the still frozen substratum, as soon as thaw has set in. Not until the frost has quite gone from the ground, the superficial soil resumes its former, less wet condition, because then the excess of water sinks away. Whence this excess of water?

When the frost has gone from the ground, new-set plants that had not yet properly taken root, may be found "frozen up", that is partly, in some cases of small plants entirely, uprooted. By what cause?

Some winters, especially that of 1917/18, I had an opportunity to make observations in the "sand-diluvium" of central Limburg, which, I think, can throw some light on the causes of these phenomena.

The most important fact found, was that *in thawing weather the ground water rises*. Without snow or rain, and without superficial inflow of water, the level of the water, among others in ditches, after the ice in them had melted, is seen to rise in the district mentioned to such an amount as 1 cm. per twenty-four hours.

Hence there is displacement of ground water, during frost upwards, and during thaw downwards, and I imagine this to take place as follows.

The pressing action of the surface tension of the water that surrounds the ground grains, decreases with increasing diameter. Hence in the state of equilibrium the coarse ground grains are covered with thicker water layers than the fine ground grains. And just as

the liquid moves from a small soap-bubble towards a large one, in the same way water that surrounds the ground grains moves from the fine towards the coarse grains when the state of equilibrium has not yet been reached. So far as I know, E. RAMANN was the first to point out this important influence of the grain-size on the moisture of the soil.[1]) In the capillary spaces between the grains the height to which the water rises is also in inverse ratio to the cross-section of those interstitial spaces.

Now it seems to me that the same influence manifests itself in these phenomena of freezing and thawing of the ground.

For as soon as the frost penetrates into the ground, the ground grains become larger in consequence of their water-envelopes getting frozen, and then suck the water to them from the neighbouring, still unfrozen grains; this water freezes again, and thus the diameter of the solid grains gets greater and greater. In the same way the capillary spaces get narrower, so that ground water rises in them. The quantities of water that thus can be retained in the frozen parts of the ground, must be very considerable.

This appears in thawing weather from the muddy state of the ground at the surface, which thaws first. When also the lower layers are thawed, the water that has risen during the frost, can sink away, and return to the ground water.

Plants are not found uprooted through frost until it thaws. This may be explained in this way: when the ground thaws, differences of tension arise directed from below upward, through which the plants that have not yet firmly taken root, are ejected.

[1]) In the third edition of his "Bodenkunde", p. 332, (Berlin 1911).

Mathematics. — *"Two Representations of the Field of Circles on Point-Space."* By Prof. JAN DE VRIES.

(Communicated at the meeting of January 29, 1921).

1. In 1917 these Proceedings (Vol. 19, p. 1130) contained a paper of Dr. K. W. WALSTRA on the representation of the circles of a plane on the points of space. In this representation a pencil of circles is replaced by a point-range, a net of circles by a field of points, and two orthogonal circles are represented by two points that are harmonically separated by a paraboloid of revolution, the points of which are the images of the point-circles of the field of circles.

Lately this representation has been investigated more closely and applied further by Dr. J. SMIT in his thesis entitled: "A Representation of the Field of Circles on Point-Space" (Utrecht 1920). We arrive also at this representation in the following way. Let A be a point outside the plane Φ of the circles c; through c and A a sphere is passed. If we consider its centre as the image of c the representation defined in this way shows all the above mentioned peculiarities.

2. In order to arrive at another representation of the field of circles we transform in the first place the plane Φ by inversion with centre N into a sphere β; the circles c are in this way replaced by circles c' of β. Now we consider the pole C of the plane γ' of c' as the *image* of the circle c. The *point-circles* P of Φ are, evidently, represented by the points P' of β. A *straight line* l of Φ is transformed by the inversion into a circle λ through N, is therefore represented by a point L of the plane ν touching β at N. N is apparently the image of the straight line at infinity of Φ.

3. A *pencil of circles* (c) is transformed by inversion into a "pencil" (c'), i.e. a system of which there passes one circle through any point of β, so that the planes γ' of the circles c' form a pencil, pass therefore through a straight line r'. But then the poles C lie in a straight line r (the polar line of r' with respect to β). Also in this representation a *pencil of circles* is therefore transformed into a *point-range*.