Physics. — *"The Propagation of Light in Moving, Transparent, Solid Substances".* III. *Measurements on the Fizeau-Effect in Flint Glass.* By Prof. P. ZEEMAN, W. DE GROOT, Miss A. SNETHLAGE and G. C. DIBBETZ.

(Communicated at the meeting of April 23, 1920.)

More accurate than the results obtained with quartz, an account of which was given in communication II [1]); are those of the Fizeau-effect in moving flint glass.

Six cylindric rods of a length of 20 cm. and a circular cross-section of a diameter of 25 mm. were made for us by the firm ZEISS at Jena. The kind of glass is the ordinary silicate flint glass of the type 0.103 of the firm SCHOTT und GENOSSEN. The endplanes are plane-parallel in close approximation. The clearness of the interference fringes appeared to be excellent with stationary glass column, while, when the necessary precautions were taken, also when the column was in rapid motion, the fringes remained still very good. The photos taken were much better than those that had been obtained before (II) with quartz. This is partly owing to the excellent material [2]), to the greater cross-sections of the rods (now 25 mm. as against 15 mm. before for quartz), and to the smaller number of internal reflections [2]). It appeared finally possible to observe also the Fizeau-effect for moving flint glass directly in a telescope, as clearly as it is possible for moving water, and we had the privilege to demonstrate the effect before several physicists.

The perfect sureness with which the rather complicated apparatus worked at last, was not obtained until some improvements had been made in the arrangement as it had been used for quartz. We will discuss the principal of them.

2. Through different causes the interference fringes can take an oblique position during the movement of the column of glass cylinders. It is, however, necessary that the fringes remain parallel to

1) These Proc. Vol. XXII, N⁰. 6, p. 512.
2) Compare II, 2.

the horizontal or vertical cross-lines. Else no photos are obtained on which measurements can be made.

Already in the experiments with quartz a *compensator* was inserted in one of the interfering beams of light, consisting of a plane-parallel circular glass plate of a thickness of 5 mm. and a diameter of 25 mm., to which every desired position could be given. The inclination of the interference fringes can be modified by rotation round a horizontal axis; a simple arrangement was, therefore, applied through which the observer, sitting at the eye-piece of the telescope, could bring about the desired rotation. Besides a plane-parallel plate was placed before the object glass of the telescope in such a way that an image of the interference fringes could be observed in a small telescope placed on one side, while at the same time after removal of the eye-glass a photo of the fringes was made with the large telescope. Thus the observer at the small telescope could at once observe an error in the position of the fringes, and if necessary, redress it during the photographing. This proved to be but rarely necessary when an experiment had been properly prepared.

3. As was set forth before (I, 4), it was necessary to superpose 20 to 30 photographs of the interference fringes, each with an exposure of a hundredth second, because otherwise the photographic image was too faint. This number could be greatly reduced by working without filters, hence directly with the white arc-light.

Diminution of the number of exposures increases the sharpness of the photos, and renders it possible to take more in succession, before the disturbances through fluctuations of the temperature in the glass-rods, which inevitably occur in consequence of the movement of the apparatus, become troublesome.

For the interpretation of the photo obtained it is then necessary to know what is the effective wave-length λ of the white arc-light, with which the fringes have been photographed.

The accuracy in the determination of λ need not be very great, as will appear presently (see 5).

4. *Determination of the effective wave-length of the light used.*

The effective wave-length of the operative light, had to be measured *after* it had left the last mirror of the interferometer, and of course for that kind of plates that was used in the experiments.

The beam from the interferometer was focussed with a cylinder lens on the slit of the collimator of a HILGER spectroscope with constant deviation, from which the prism had been removed, and replaced by a small totally reflecting prism.

By placing a grating replica before the object glass of the photographic camera, a spectrum of the source of light could be photographed.

The most active part in this spectrum could be made directly visible by putting a wedge of smoked glass with the side horizontal before the slit of the collimator (KENNETH MEES method).

The prismatic action of the wedge was counteracted by a second wedge of clear glass.

The result for the effective wave-length λ was 4750 A°, with an uncertainty of $\pm$ 25 A°.

5. This accuracy is, however, sufficient. This can be verified by numerical calculation, or by the following consideration.

We know (II, 9) that the optical effect is given by the formula:

$$\Delta = \frac{4\,lv}{\lambda c}\left(\mu - 1 - \lambda \frac{d\mu}{d\lambda}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By deriving from this the value of $\frac{1}{\Delta}\frac{d\Delta}{d\lambda}$, we can see how great the influence is of an error in the determination of the effective wave-length on the calculated effect.

Instead of λ we introduce the frequency ν, which brings about a slight simplication, and we put $\varphi = \mu - 1 + \nu \frac{d\mu}{d\nu}$. Then:

$$\frac{1}{\Delta}\frac{d\Delta}{d\lambda} = \frac{1}{\varphi \cdot \nu}\,\frac{d\varphi \cdot \nu}{d\nu} \times -\frac{1}{\lambda^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now:

$$\frac{d\varphi}{d\nu} = 2\,\frac{d\mu}{d\nu} + \nu\,\frac{d^2\mu}{d\nu^2},$$

for which we may write in approximation:

$$\frac{d\varphi}{d\nu} = 2\,\frac{d\mu}{d\nu},$$

because μ depends almost linearly on ν.

Equation (2) then becomes:

$$\frac{1}{\Delta}\frac{d\Delta}{d\lambda} = \nu\left(\frac{2\nu\frac{d\mu}{d\nu}}{\varphi} + 1\right),$$

$\nu\frac{d\mu}{d\nu} : \varphi$ becomes about $\frac{1}{8}$ for the ordinary flint glass, so that:

$$\frac{1}{\Delta}\frac{d\Delta}{d\lambda} = \frac{1}{\lambda} \times \frac{5}{4}$$

For $\lambda = 5000$ A°, $d\lambda = 25$ A° becomes $\frac{d\Delta}{\Delta} = {}^{3}/_{4}\ {}^{0}/_{0}$.

6. *Ribbon-shutter*. The shutter which acts periodically and is worked electromagnetically, described in I, 4, repeatedly gave cause for disappointment, because it was never certain that the light was transmitted at the very moment that the cradle passes a chosen point of the path.

This is perfectly certain with the ribbon-shutter, which is diagrammatically represented in fig. 1.

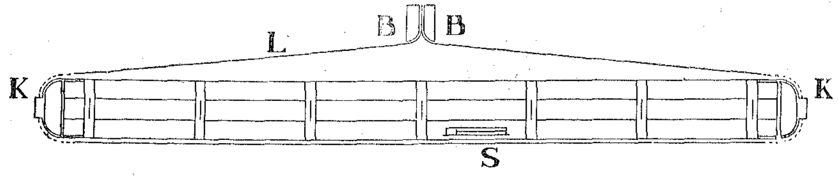

Fig. 1.

A band of ribbon L of lancaster linen is clasped between two blocks B, which are firmly fastened at a chosen place of the bed of the apparatus. It is passed round the beam with the glass column. The beam can execute its usual movement to and fro without being hindered by the ribbon, for this can easily slide over the copper pieces K, the length of the ribbon remaining constant. There have been made two openings in the ribbon of 10 or 15 cm., which at a certain position of the beam, but only then, allow the light to pass through circular holes in the pieces K, and during the time that corresponds with the length of the openings in the ribbon. By displacing BB along the bed, the moment at which the light is transmitted, may be chosen. The edges of the ribbon are provided with a hem to obtain greater firmness, and prevent fraying.

The copper pieces K are smoothly polished, and the friction of the ribbon is therefore very slight. Sometimes it was still diminished by some talc-powder.

The electric shutter, which was used in the experiments described in II, was now used after a small modification to admit the light only in one of the movements to and fro of the beam. For this purpose the movable arm is placed before the arc-lamp. The phase is adjusted so that the arm turns when the beam has just passed the middle of its course.

7. *Improvement of the Velocity Measurement.* In the experiments with quartz rods the velocity was measured directly according to a method which has been described in II, § 11. We have made this method simpler and more delicate, and also arranged it so that the velocity could be immediately read in every experiment.

In main lines the arrangement is still the same as represented in II § 11. The screen with two slits S_1 and S_2 used formerly was however, replaced by a screen S (cf. figure of the preceding §), the construction of which will be further explained by referring to figure 2.

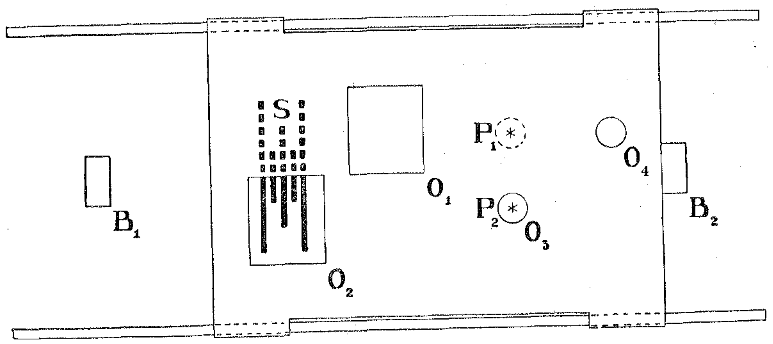

Fig. 2.

Our earlier slit S_1 is replaced by a glass scale, slit S_2 by a small aperture.

The graduated glass scale was obtained by covering a glass plate with a soot-layer, and by drawing by the aid of a chisel of the width of exactly 1 mm., five lines in the soot-layer at mutual distances of exactly 1 mm.

Then the soot-layer was fixed with a drop of varnish, and the first line was covered with red glass, the others with blue glass. All this was cemented on the beam. In fig. 2 the alternate long and short lines are indicated under S; the colours render errors of front and back in the observation through lenses impossible. In the figure the lines are partially dotted, as they are half covered by a screen, which can slide to and fro. During the movement of the beam in one sense the lower, during the movement in opposite sense the upper half of the scale is automatically covered through the inertia of the screen. The blocks B_1 and B_2 define the extreme positions of the screen.

As we said, our former slit S_1 has been replaced by the scale with the coloured lines; instead of the slit S_2 there are two fine

apertures P_1 and P_2 on either side of a horizontal line through the middle of the scale. In the position drawn in the figure the light can leave through P_2, because the opening O_3 in the moving screen allows this. When the screen rests against B_1, P_2 is covered, and O_3 comes in the position, in which it is possible that light is emitted through P_1.

As was explained in II § 11 an image of the slit arrangement is projected on the rotating disc R, which is provided with radial slits. With a weakly magnifying telescope the image projected on the rotating disc is observed with intervals of 0,001 sec. In this time the beam moves about 1 cm. at the velocity used in the neighbourhood of 10 $^{M}/_{sec}$. The observer sees the coloured scale S in the field of vision, and then the "star" P_2, or rather the after-image of the light emitted by this star at a former transmission. The place of the star on the scale can be read accurately down to $\frac{1}{2}$ mm., and the distance from P_2 to S being about 50 mm., the velocity can be determined certainly accurately down to $1\,^0/_0$. All the changes in the velocity of the beam are immediately visible, and the velocity corresponding to every photo taken can at once be noted down.

It is necessary that the "star" moves at the same level at which the axis of the rotating disc has been placed, for else a small correction must still be applied to the velocity.

Thanks are due to Mr. W. M. Kok, assistant at the Physical Laboratory, for his valuable help in the execution of the arrangement for the velocity measurement.

8. *Results.* The extreme values of the velocity which were directly measured in our experiments, were 918 and 994 cm/sec.

There were made two series of measurements, which were distinguished by the way in which the velocity was found. In the first series, A, the method of communication II, in the second, B, that described above in § 7, was followed. All the results for the effect were reduced to a velocity of 1000 cm./sec.

Series A.

When the measurements of the 34 separate photos obtained on 11 plates are combined, the effect is found to be $0{,}247 \pm 0{,}006$.

When first the observations on each plate are combined, and the mean is taken of the results of 11 plates, the effect is found to be $0{,}247 \pm 0{,}009$.

Series B.

gives for the effect derived from 49 observations divided over 13 plates $0{,}238 \pm 0{,}006$, for the effect derived from the mean of the results of the 13 plates $0{,}240 \pm 0{,}008$.

Finally all the 83 observations may be combined; then 0,242 $\pm$ 0,004 is found.

When a result is calculated for each of the 24 plates separately, and then the mean is taken, the effect is found to be 0,243 $\pm$ 0,006.

The number after the $\pm$ sign indicates the mean error, and as was stated before all the results have been reduced to the velocity 1000 cm./sec.

Theoretical value of the effect for flint-glass.

The firm SCHOTT und GENOSSEN, Jena, gives the following indices of refraction for the flint-glass 0.103 used:

$\mu_{A}' = 1.6099$	$\lambda = 7677$ $A°$
$\mu_{C}' = 1.6153$	6563
$\mu_{D}' = 1.6202$	5893
$\mu_{F}' = 1.6324$	4862
$\mu_{G}' = 6428$	4341

The effect can be calculated according to the formula derived before (II, 9)

$$\Delta = \frac{4\,lw}{\lambda c}\left(\mu - 1 - \lambda\frac{d\mu}{d\lambda}\right).$$

By the aid of the given data, values for μ and $\frac{d\mu}{d\lambda}$ are derived for the effective wave-length 4750. The final result is:

$$\Delta = \frac{4\,.\,120\,.\,1000}{4750 \times 10^{-8} \times 3.10^{10}}\,(1,634 - 1 + 0,084) = 0,242.$$

This value is almost in perfect agreement with the value yielded by our experiments.

It is of interest to note that the dispersion term contributes to the value of the effect $= 0,242$ by an amount of 0,028.

If the dispersion had not been taken into account, 0,214 would have been found for the effect, which is incompatible with the experiments.

APPENDIX.

1. The publication of the above communication, which was already laid before the meeting of the Academy of April 23rd 1920, has been delayed through particular circumstances.

This affords me an opportunity to add a few remarks to the paper. Our collaborator, Mr. W. DE GROOT, phil. docts. orally informed me of another derivation of the formula for the optical effect (II, 9),

and independent of this at about the same time Prof. F. ZERNIKE at Groningen did the same in a letter dated November 11th 1919. The short, elementary derivation, which in the two communications is founded on the same idea, will follow here.

In principle the experiment with the moving glass rod can be classed under the following scheme (fig. 3). From the source of

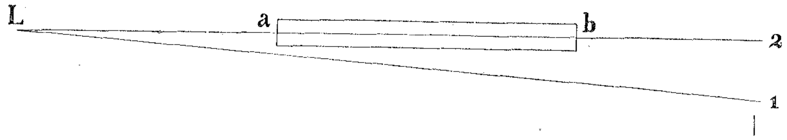

Fig. 3.

light L issue two beams of light, one through the glass ab, the other through the air. With the aid of the necessary devices the phases 1 and 2 are compared in two cases, first when the glass ab is at rest, secondly when it moves e.g. to the left with the velocity w. According to the principle of relativity the glass may as well be at rest, and the room with the other parts may be made to move to the right with the velocity w. Whether 1 and 2 are received with a moving or a stationary apparatus, makes no difference in the relative phase of the beams of light. Therefore only L is made to move to the right with the relative velocity w, and approaches the glass rod. This only gives a DOPPLER effect equal for the two beams, in which the wave-length varies from λ to $\lambda - \lambda\frac{w}{c}$. When everything is at rest, the phase-difference between 1 and 2 is $\frac{(\mu-1)}{\lambda}\,l$, when l is the length of the glass rod ab.

Hence the change due to the movement is:

$$-\frac{(\mu-1)\,l}{\lambda^2}\,d\lambda + \frac{l}{\lambda}\frac{d\mu}{d\lambda}\,d\lambda = \frac{wl}{\lambda c}\left(\mu - 1 - \lambda\frac{d\mu}{d\lambda}\right)$$

To get the total effect the formula should be multiplied by 4 — i.e. a factor 2 for the movement to and fro of the rays, and a factor 2 on account of the reversal of the direction of the movement — so that the formula given before, appears.

Also FIZEAU's experiment with the moving water and stationary glass end-plates may be treated by the method sketched above, but then the calculation is not so simple.

Mr. ZERNIKE still points out that an actual experiment might be taken with the two beams of light running in opposite directions and stationary glass rod, as is supposed in the calculation. It would then only be required in the experiment described before to make the glass prism slide to and fro, and keep the glass rod at rest. For different optical and mechanical reasons the execution seems to me attended with greater difficulties than the experiment made.

2. In a letter of October 22nd 1919 Prof. M. VON LAUE had the kindness to draw my attention to a thesis for the doctorate of P. HARRESS of 1912 [1]), which he sent me, and in which a subject is treated closely related to our investigations. In HARRESS's experiment the light runs to the right and to the left in a cycle of glass prisms, which as a whole is in rotatory motion.

The comparison of the observed displacement of the interference-fringes and the theory elaborated by HARRESS gave a very unsatisfactory agreement. This is chiefly owing to the theory, in which the absolute and the relative velocity of the light are mixed up. VON LAUE has redressed this error, which greatly improves the agreement between theory and observation. HARRESS's experiment closely resembles SAGNAC's experiment of 1913, of which, very remarkably, VON LAUE gave the relativistic theory already in 1911 [2]).

In SAGNAC's and HARRESS's experiments the displacement Δ of the interference fringes expressed as fraction of their distances is:

$$\Delta = \frac{2\omega}{c\lambda} \Sigma rl,$$

in which l is the length of the path passed over in the rotating apparatus, r the distance of this to the axis of rotation, ω the angular velocity, the sum extended to all the different paths.

Index of refraction and dispersion do not occur in this formula, which in itself is already a difference with our experiments.

It seems unnecessary to enter into a fuller discussion of HARRESS's work, as in an interesting paper by VON LAUE [3]) the experiments by FIZEAU, SAGNAC, HARRESS, and those made by us are discussed and compared, and as with exclusion of the influence of the dispersion the two last-mentioned experiments have also already been treated in the fourth edition of VON LAUE's Relativitätstheorie [4]).

3. I pointed out on an earlier occasion that it might be interesting to examine substances in which $\frac{d\mu}{d\lambda}$ is great.

[1]) Re-edited in O. KNOPF, Ann. d. Phys. **62**, 389, 1920.
[2]) Münchener Sitz. Ber. 1911, 404.
[3]) VON LAUE, Ann. d. Phys. **62**, 448, 1920.
[4]) Cf., p. 23, 25, 185—189.

In particular substances with strong absorption-bands or lines deserve attention, such as *didymium* compounds, which can be obtained as solid solutions in glass, and the vapour of *sodium*.

When these substances are chosen to work with, horizontal interference lines must be thrown on the slit of the spectroscope. Horizontal lines are observed in the telescope, which diminish in distance from red to violet. Through the FIZEAU-effect the lines would move up and down in case of rapid motion to and fro of selectively absorbing substances, and at those places in the spectrum where $\frac{d\mu}{d\lambda}$ assumes large values the amplitude of the movement might become considerable.

From preliminary experiments on the dispersion of didymium glass at the ordinary temperature and at that of liquid air it appeared that the value of $\frac{d\mu}{d\lambda}$ assumes nowhere great values in the visible spectrum. Though rods of didymium glass of excellent quality are to be had, I am yet of opinion that it would not be worth the trouble to make experiments with them on the FIZEAU-effect.

Nor can results on the FIZEAU-effect be expected with sodium vapour. Close to the absorption D-lines $\frac{d\mu}{d\lambda}$ can, indeed, become very great, but at the most interesting place near the D-lines the absorption too becomes very great. On continuation of the experiments it might perhaps have deserved recommendation to work with a stationary tube with sodium vapour and moving prism (see § 1 of this appendix). It appeared however, clearly enough from some experiments that the observation of the FIZEAU-effect with sodium vapour was out of the question.

Though these experiments did not yield the result for which they were undertaken, they gave occasion to the observation of an interesting interference phenomenon in sodium vapour, about which a separate communication will shortly follow [1]).

P. Z.

[1]) This communication has been published already. These Proceedings Vol. **24**, p. 206, 1922.