Physics. — *"Gravity and Pressure of Radiation."* By H. GROOT. (Communicated by Prof. W. H. JULIUS.)

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§ 1. In 1910 LEBEDEW succeeded in experimentally showing pressure of radiation on *gases*, and measuring the value of the pressure. Since then attempts to take this force into account in astro-physical researches have not been wanting.

Particularly EDDINGTON [1]) and JEANS [2]) have reached highly remarkable results concerning the structure of "giant stars" by introducing besides gravity, also pressure of radiation into their equations. They come, among other things, to the conclusion that through the influence of this pressure, the gravity in the interior of a star can be considerably diminished, and this the more as the density is smaller.

One is naturally led to extend this investigation to states as will probably be met with in nebulae. And this the sooner as different authorities advocate the hypothesis that the law of NEWTON is not valid during the nebulous stage of a star, i.e. during the period that the star is being formed from primitive nebular matter.

KAPTEYN [3]) and CAMPBELL [4]) tried to account in this way for the surprising fact that the proper motions of the stars increase as a more advanced spectrum type is examined. The latter indicates a possible pressure of radiation as a force that might partially neutralize gravity.

Also F. NÖLKE [5]) in his cosmogonic considerations has recourse in numerous places to the pressure of radiation to render the not being constant of gravity plausible.

An estimation of the *extent of the possible effect* is, however, nowhere found. And so long as this is wanting all conclusions which are exclusively based on qualitative speculations, remain unreliable — as but too clearly comes to light in the different cosmogonies.

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[1]) M. N. **77**, (1916—17), p. 16 and p. 596; Astrophys. J. **48**, (1918).

[2]) M. N. **79**, (1919), p. 319.

[3]) J. C. KAPTEYN, Astrophys. J. 1910 (April).

[4]) CAMPBELL, Lick-Observ., Bulletin. Vol. VI N°. 196.

[5]) „Das Problem d. Entwicklung unseres Planetensystems", 2te Ausgabe (1919) Berlin; A. N. **188**, (4509).

What follows may be taken as an attempt to get some insight into the quantitative relations.

Here three different problems present themselves, which we will discuss successively:

A. To what pressure of radiation is a nebula subjected from the stars scattered around it (system nebula-star)?

B. What pressure does an absorbing body (planet) experience from the radiation of an extensive nebula in the neighbourhood?

C. Can the parts of a nebula repel each other appreciably through mutual radiation?

§ 2. *The system: nebula-star.*

When we wish to make an estimation of the relation between the attractive forces, to which a nebula is subjected from the surrounding stars, and the repulsive forces caused by the radiation of these same stars, we may begin by remarking that it is independent of the scattering in space of the stars considered. For the two forces are in the ratio r^{-2}, hence their ratio is not influenced by the distance. As not all the stars of the same absolute magnitude have the same mass, it would practically be necessary for the determination of the resultant of the active forces to know the nature of each of the stars concerned accurately. This is, of course, impossible. In our investigation we shall assume that on an average all the stars have an equally large mass, and radiate equally strongly as our sun. On this simplified supposition the ratio of the attraction of the whole stellar system to the repulsion caused by the radiation of the same system, is equal to that of the same forces exerted by one star at any distance.

With a view to the hypothesis of KAPTEYN and CAMPBELL mentioned before [1]) we will examine the following case more closely.

A star with a mass equal to that of the sun may be at 1 parsec. distance from a spherical [2]) nebulous mass of a radius of 15000 astronomical units. Seen from the star, the nebula occupies the 0.0014^{th} part of the sky.

Let us assume in order to find an *upper limit* of the pressure of radiation to which the nebula is subjected, that *all* the radiation received from the star, is absorbed. (We know that in reality the absorbed fraction is exceedingly small). The star emits as much

[1]) Compare also the view of H. SHAPLEY, Astrophys. J. **50**, (373), 1919.

[2]) I choose the spherical form to simplify the calculations; one should not think here of a *planetary nebula*, which is known to show on the other hand very quick proper motion.

radiation as our sun, i.e. $4{,}2 \,.\, 10^{33}$ ergs per second. The energy absorbed by the nebula per second is $5{,}1 \,.\, 10^{29}$ ergs. The pressure of radiation can be calculated in this case with the well-known formula:

$$D = \frac{S}{c} \quad \ldots\ldots\ldots\ldots \quad (1)$$

in which S = quantity per second of received radiation in ergs, $c = 3 \,.\, 10^{10} \frac{\text{cm}}{\text{sec.}}$, D = pressure of radiation in dynes.

This yields in our case:

$$D = 1{,}7 \,.\, 10^{19} \textit{ dynes.}$$

Putting the mass of the nebula, like that of the star, about equal to that of the sun, i.e. $2 \,.\, 10^{33}$ gr., we find for the maximum acceleration through pressure of radiation:

$$a = 0.8 \,.\, 10^{-14} \frac{cm}{sec^2},$$

for that of the attraction:

$$a' = -1{,}4 \quad 10^{-11} \frac{cm}{sec^2}.$$

Accordingly by the side of the attraction the pressure of radiation, even when calculated on exceedingly favourable suppositions, is almost negligible. As the same ratio must be valid with regard to the whole stellar system, we conclude:

The attraction of the stellar system on a nebula is not appreciably modified by pressure of radiation. Deviations from the law of Newton in such nebulae as we have considered, cannot be accounted for by the counteraction of the pressure of radiation.

Of course this consideration no longer holds when the dimensions of the nebulae become hundreds of times greater. But for the problem in question we were obliged to assume that the nebula from which the new star is being formed, had already conglomerated to the stated dimensions.

§ 3. *The system: nebula-planet.*

In view of some cosmogonic considerations on the origin of the solar system, it may be of interest to examine how great the pressure of radiation is which can be exerted on a newly formed planet by the mother nebula.

We begin by solving the question: what is the pressure of radiation which a spherical nebula of *constant* density ϱ and radius R

exerts on a planet P, which absorbs all the received radiation, and which is at a distance Δ from the centre of the nebula.

An element $d\tau$ of the nebula emits in the direction OP a quantity of radiation given by:

$$\frac{Se^{-\mu\rho s} \cos \psi}{x^2} d\tau$$

when $\mu =$ absorption coefficient, $x =$ distance from $d\tau$ to planet, $s =$ length of the path passed over by the radiation inside the nebula, $S =$ the intensity of radiation, which we shall assume to be constant inside the nebula (see figure).

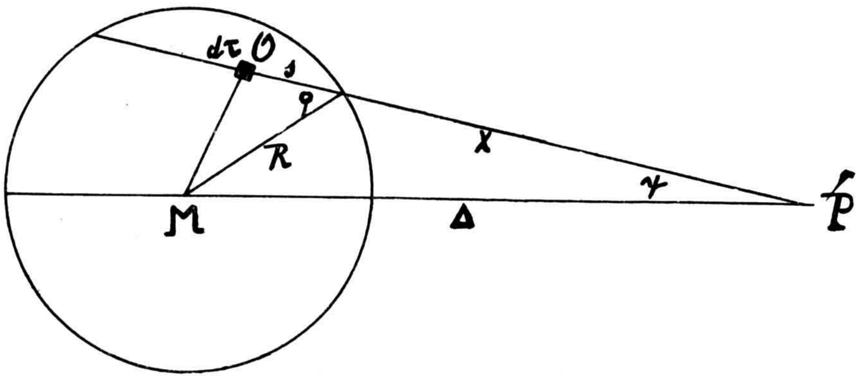

Fig. 1.

When $d\tau$ is taken $= x^2 dx \sin \psi \, d\psi \, d\omega$, and $r_1 =$ radius of the planet, the radiation of the whole nebula on the planet is:

$$A = \pi r_1^2 S \iiint dx \, d\psi \, d\omega \, \sin \psi \cos \psi \, e^{-\mu\rho s} \quad . \quad . \quad . \quad . \quad (2)$$

When we introduce φ instead of ψ (see figure) through:

$$\Delta \sin \psi = R \sin \varphi$$

and when we take $dx = ds$, (2) passes into:

$$A = \frac{\pi r_1^2 SR^2}{\Delta^2} \int_0^{\frac{1}{2}\pi} d\varphi \int_0^{2R\cos\varphi} ds \int_0^{2\pi} \sin \varphi \cos \varphi \, e^{-\mu\rho s} \, d\omega \quad . \quad . \quad . \quad (3)$$

Integration yields [1]):

$$\left.\begin{aligned} A &= \frac{\pi r_1^2 \,.\, 2\pi R^2 S}{\Delta^2} \,.\, \frac{1}{\mu\rho} \left[\tfrac{1}{2} - P^{-2} + P^{-1} e^{-P} + P^{-2} e^{-P}\right] \\ P &= 2\mu\rho R \end{aligned}\right\} \quad . \quad (4)$$

[1]) Compare: Bottlinger, "Die Gravitationstheorie und die Bewegung des Mondes", Bayerische Akademie, 1912.

When $M =$ mass of the nebula, $m =$ mass of the planet, then by making use of:

$$M = \tfrac{4}{3}\pi R^3 \varrho \quad , \quad m = \tfrac{4}{3}\pi r_1{}^3 \varrho',$$

we may write:

$$A = \frac{Mm}{\Delta^2} \cdot \frac{9S}{4\varrho\,\varrho' r_1} \left[\tfrac{1}{2} P^{-1} - P^{-3} + (P^{-2} + P^{-3})\, e^{-P}\right] \quad . \quad . \quad (5)$$

The pressure of radiation experienced by the planet when it absorbs all the radiation received, is again calculated with the formula:

$$D = \frac{A}{c}.$$

The NEWTONIAN attraction must be diminished by this amount in order to find the resulting force K.

This becomes therefore:

$$K = f \cdot \frac{Mm}{\Delta^2}\left[1 - \frac{9S}{4fc\varrho\,\varrho' r_1}\left(\tfrac{1}{2} P^{-1} - P^{-3} + P^{-2} e^{-P} + P^{-3} e^{-P}\right)\right]. \quad (6)$$

Can this diminution be great enough to bring about appreciable disturbances?

To investigate this, the following hypothetical case may be considered:

We assume that the solar nebula, after the formation of Neptune, has withdrawn to within the orbit of Uranus. We suppose Neptune itself to be still gaseous, though considerably denser than the solar nebula, and with a radius a 100-times larger than at present.

We further disregard the fact that the solar nebula in all probability must have had already a pretty great central condensation, a circumstance which has an unfavourable influence on a possible effect of pressure of radiation.

We base our calculation on the following numerical values:

Radius of Uranus orbit	$R = 2{,}868 \,.\, 10^{14}$ (cm.).
Present solar radius	$R_0 = 6{,}96 \;\; 10^{10}$ (cm.).
„ solar density	$\varrho_0 = 1{,}4$.
„ Neptune radius	$r_0 = 2{,}8 \;\; 10^{9}$ (cm.).
„ density of Neptune	$\varrho_0' = 1{,}1$.
Gravitation constant	$f = 6{,}66 \,.\, 10^{-8}$.
Absorption exponent [1])	$\mu = 0.0002$.

Thus we find:

$$K = f\frac{Mm}{\Delta^2}(1 - 2800\, S).$$

[1]) Cf.: EMDEN, „Gaskugeln", p. 285.

The value of S is still a doubtful point.

If we should assume that the nebula emits black radiation, S would already be $= \frac{1}{2800}$ at a mean temperature of not quite 1500°, hence attraction and repulsion would be about equally great.

This is undoubtedly erroneous; the radiation has been smaller than is calculated on the strength of STEPHAN-BOLTZMANN's law. On the other hand the nebula will have emitted other radiation [1]) besides temperature-radiation, which again partly compensates the deficit.

In our opinion the result of this research may, therefore, be summarized as follows:

On account of the uncertainty which prevails with regard to the quantity of energy emitted by the nebula, it is difficult to make an accurate estimation of the amount by which the attraction of the mother nebula on a newly separated planet must be diminished in virtue of the pressure of radiation. In consequence of the contraction, both of planet and of nebula, the effect in question will continually diminish, and in general it will also have been greater for the larger and more remote planets. Taking everything together it is not excluded that the said diminution, had a quite appreciable amount, at least for the large planets.

If, therefore, in our solar system particularities should occur which can be accounted for as the result of such a change of gravitation, there is every reason to accept this explanation. And this seems actually to be the case, among others with the small inclinations and small eccentricities of the large planets. (See among others NÖLKE loc. cit.). To enter more fully into this, would lead us too far.

§ 4. *Gravitation and pressure of radiation in a nebula.*

Departing from our considerations in the preceding §§ we shall now consider the more irregular nebulae, which present two or more condensations, as e.g. the Dumbbell nebula. Most nebulae have dimensions which are probably to be measured in thousands of Neptune orbit radii. Not much is known about their masses. But when we assume that a multiple star will be formed out of such a nebula, we must assign to each of the parts of the nebula a mass of the same order of magnitude as our sun has. In order to effect a rough estimation of the acting forces, we shall more fully discuss the following system.

[1]) The light of the tails of comets, and probably of most nebulae, arises besides through temperature radiation, through other processes, which are not yet entirely known.

Two nebular spheres, each with a mass of 7.10^{33} gr. and a radius of 200 Neptune orbit radii have centres which are at a distance of 1000 Neptune orbit radii from each other. The density, like the intensity of radiation, is again considered constant inside the nebula.

The quantity P, which occurs in the formulae (5) and (6), may be written:

$$P = 2\mu\varrho_0 R_0 \left(\frac{R_0}{R}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which μ, ϱ_0 and R_0 have the same value as in the preceding §, in which R_0/R is, however, ± 300 times smaller. Then the value of P becomes so small that we may write instead of (5):

$$A = \frac{\pi r_1^2 \cdot \frac{4}{3}\pi R^3 S}{\Delta^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Assuming that of this quantity of radiation, which one nebula sends to another, the n^{th} part is absorbed by the latter sphere, this experiences a pressure of radiation which may be written after some reduction:

$$D = f\frac{M_1 M_2}{\Delta^2} \cdot \frac{S}{n} \cdot \frac{3R^6}{4fc\varrho_0^2 R_0^6} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

When numerical values are introduced, we get·

$$K = f\frac{M_1 M_2}{\Delta^2}\left(1 - 3.10^{15}\frac{S}{n}\right).$$

Even on the assumption that only $\frac{1}{100.000.000}$ of the received radiation is absorbed, the value of S need not be more than $0{,}3.10^{-7}$ to render the effect of the pressure radiation as great as that of gravitation. In the case of black radiation, a temperature of some tens of degrees above the absolute zero would already suffice to bring it about. The real temperature will on an average certainly be higher, besides in this case considerable luminiscence should also be taken into account, so that we come to the conclusion:

There is every reason to expect that in nebulae with some condensation nuclei the gravitation of the different parts on each other has greatly diminished, if it is not quite exceeded by the mutual pressure of radiation.

For the rest it should be pointed out that this diminution of gravitation only refers to the *internal* gravity. Towards the outside the ordinary Newtonian attraction remains valid.

S U M M A R Y.

After having adduced some grounds in § 1 why it seems desirable to study the effect of pressure of radiation in nebulae, we examined the *system nebula-star* in § 2. In this case the value of the pressure of radiation remains so small that even under favourable conditions no effect can be expected. Treating in § 3 the *system nebula-planet*, we saw that it is not excluded that in the first time of their formation from solar nebula the large planets were subjected to a strong pressure of radiation, and this may perhaps be responsible for some peculiarities in the system.

In the investigation of the system formed by *two or more nebulae* in § 4 we came to the conclusion that if anywhere, the pressure of radiation must manifest itself here.

Bussum, April 1921.