Astronomy. — *"On a change with the declination in the personal equation in meridian-observations."* By C. H. HINS. (Communicated by Prof. W. DE SITTER).

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In the preliminary reduction of the R.A. in the current programme of the Leyden meridian circle, the question arose as how to deal with the clock rate.

The first difficulty that arises is that the registering clock KNOBLICH is not the same as the principal clock of the observatory, HOHWÜ 17.

We shall therefore first give a brief account of how the reduction was made in former observations.

Before the beginning and after the end of the observations the beats of HOHWÜ 17 were registered for one and a half minute, with intervals of three seconds, on the slips upon which the KNOBLICH clock registers every two seconds. In this way the value of the difference of time Kn.—H. 17 was obtained at two epochs. Even when these comparisons were made three or more times in one evening, they were always represented by a straight line. (See PANNEKOEK, Annals of the Leyden Observatory Vol. X part I).

At regular intervals of about ten days determinations of time were made which gave the daily rate of the clock HOHWÜ 17. This "rough" rate observed in the interval between two determinations of time was reduced by the formula:

$$p = \textit{daily rate} - 0^s{,}0153\,(b - 760^{\mathrm{m.m}}) + 0^s{,}0263\,(t^\circ - 10^\circ) - 0{,}37\,(t - t')$$

to the so called *reduced daily rate* p at $t = 10^\circ$, $b = 760^{\mathrm{m.m}}$ and $t - t' = 0$.

In the above formula

b represents the mean barometer in the interval.

t the mean temperature in the interval.

$t - t'$ the mean difference of the temperatures at the top and the bottom in the case of the clock.

daily rate: the rough rate as explained above.

From this *reduced daily rate* the actual rate of H. 17 during the observations was derived by the above formula and the observed values of b, t and $t - t'$.

All transits expressed in the time of the Kn. clock could be reduced to H. 17 time by means of the interpolated difference Kn.—H. 17, or, which comes to the same thing, from the observed relative rate

Kn.—H. 17 and the calculated absolute rate of H. 17 the absolute rate of the Kn. clock was derived.

By this method the observed transits of the fundamental stars give only the mean clock correction at a mean sidereal time and did not give any data for finding the rate of the clock.

The question now arose, whether it would not be possible, excluding the clock H. 17, to derive from the registered transits of the fund. stars not only the mean clock correction, but also the rate, both expressed in time of the Kn. clock.

In order to arrive at a solution of this question I here give the postulates underlying each of the methods.

I: (by means of H. 17).

1. The times of transits of the fund. stars must be sufficiently accurate to give a reliable clock correction by their mean.

2. The relative rate Kn.—H 17 obtained must be accurate, from which it follows that:

a. The rate of the Kn. clock must be constant during the observations.

b. The registering of the beats of H. 17 must be accurate.

c. There must not be a systematic difference between the registration of these beats before and after three or four hours of observations.

3. The rate of the H. 17 clock must conform to the formula given above, not only in the mean for several days, but must do so without any retardation, in other words it must react immediately to every change of temperature and barometer.

II: (without the H. 17 clock).

1. The transits of the fundamental stars must have sufficient accuracy to give by their mean a reliable clock correction and at the same time a reliable clock rate.

2. The rate of the Kn. clock must be constant during the observations.

A comparison of the two groups of postulates shows that II, 2 and I, 2*a* are the same.

The only difference between II, 1 and I, 1 is that in the second method the observations of the transits of fund. stars require a higher degree of refinement and the question is reduced therefore to the following:

What conditions must be laid down for observations to make them suitable for the deduction of a clock rate?

In the first place it is desirable that the stars of the observation programme should all be included between fundamental stars, so that nowhere an extrapolation of the clock rate takes place; the night's work therefore should always begin and end with a number of fund. stars.

In the second place it is necessary that the fund. stars, or rather their groups, be at a sufficient distance from each other, so that the accidental errors in the times of transits shall have as little influence as possible.

Theoretically it is always possible to fulfil these requirements by a well chosen programme of observations. Only on nights interrupted by clouds it lies outside the power of the observer to fulfil them.

It is, therefore, necessary to examine more closely the accuracy of the times of transits obtained.

The clock corrections as derived from the separate stars are affected by two kinds of errors, accidental and systematic.

Of the influence of the first an idea can be obtained by computing the mean error of the time of transit derived from the transits of the separate threads (in Leyden numbering eleven). Generally speaking this error will not exceed $\pm 0^s.024$. If we had two groups of four fund. stars separated by an interval of two hours, the mean error of the clock rate, due exclusively to the accidental errors would thus be about

$$\pm \frac{\sqrt{0.012^2 + 0.012^2}}{120} = \pm 0^s.00014.$$

The second category of errors plays, however, at least as important a part and it is therefore necessary to make the times of transit as completely independent of them as possible.

Under these errors we include:

1. The magnitude equation.

By a suitable use of gratings it is possible to confine this error within narrow limits. What remains must be removed as much as possible by deriving the magnitude equation of the various observers and correcting for it.

2. The errors in the assumed positions of the fundamental stars. Since it is practically impossible to confine oneself to the small number of accurately established fundamental stars, it is necessary to use as fund. stars some that are of doubtful value, so that these errors must have a great influence not only on the mean clock correction, but even more on the rate of the clock as derived from the observations (see the large probable errors given by Boss' P. G. C. for various stars for the epoch 1910,0).

3. Personal errors of the different observers.

Assuming that the error mentioned under 1. be removed, I shall endeavour to avoid as much as possible those mentioned under 2. and 3., beginning with 3. From the whole of the observations of all the observers, freed from their individual personal errors, it will then perhaps be possible to estimate those given under 2.

A brief account of the Leyden programme may precede. The stars of the programme are, as far as possible, divided into zones, whence in view of the somewhat restricted material (1600 stars distributed over 24 hours of right ascension and declination —2°. to 52°, and the distribution in R.A. still very irregular) it was impossible to take the zones very narrow.

In general the zones are chosen with the limiting declinations of 0°—20°, 20°—30°, 30°—40° and 40°-zenith, but it was often necessary to include in a zone stars of slightly different declination. The fund. stars are so chosen that they lay below, in and above the zone so that their mean declination was as much as possible at the middle of the zone.

As, therefore, fundamental stars differing by 25° in declination are sometimes observed on the same night, the question arises, whether the different observers registered stars with such different declinations, i. e. of different velocities, all in the same way.

If a systematic difference of this kind existed, it would be desirable to reduce the times of transit of the separate stars to a hypothetical star with a declination equal to the mean declination of the fund. stars used. The times of transit of the programme stars could then be reduced also to the same hypothetical star, and by a purely differential reduction the results of the programme stars would be freed from this systematic error.

For this purpose the separate nights of the different observers are treated in the following way:

Let observer **X** on one night observe n fund. stars.

Times of transit $T_1 \ldots T_n$, mean T

Declinations $\delta_1 \ldots \delta_n$, mean δ

Clock correction derived from each star $a_1 \ldots a_n$, mean a.

As unknowns may be taken the rate of the clock $= x^s$ per minute and the possible influence of the declination on the time of transit $= + y^s$ per 1° deviation from the mean declination.

Every night gives n equations of the form:

$$(T - T_i)\, x^s + (\delta - \delta_i)\, y^s = (a^s - a_i^s) \qquad i = 1 \ldots\ldots n$$

From these n equations the unknowns x and y have been solved by least squares.

The x as found in this solution can be regarded as a first approximation to the rate of the clock.

We will now consider the values of y, which are obtained from different nights for the same observer.

Observer H.

The table below gives the results for y from 39 nights during the years 1920 and 1921. The list is arranged according to increasing mean declination of the fund. stars used.

1st column date of observation.
2nd „ mean declination of fund. stars used.
3rd „ result for y.

1920 March 21	3°.9	$+0^{s}.00147$	1920 Febr. 8	21°.8	$+0^{s}.00065$
„ „ 29	3°.9	+0.00123	„ „ 15	23°.9	+0.00274
„ May 23	10°.0	+0.00263	„ March 21	27°.6	+0.00151
„ „ 13	11°.8	−0.00160	„ Febr. 21	29°.3	+0.00304
1921 Febr. 23	13°.2	+0.00235	1921 March 3	29°.4	+0.00319
1920 „ 14	14°.4	−0.00163	1920 Febr. 11	29°.9	+0.00268
„ July 14	16°.3	+0.00250			
„ „ 15	16°.3	+0.00307			
„ May 18	18°.8	−0.00030			

1920 Jan. 21	30°.6	$+0^{s}.00181$	1920 May 16	40°.4	$+0^{s}.00312$
1921 Febr. 20	30°.7	+0.00283	„ „ 13	41°.0	+0.00387
1920 „ 17	32°.5	+0.00544	1919 Dec. 15	41°.6	+0.00863
1921 March 10	32°.8	+0.00169	1920 Jan. 25	41°.8	+0.00597
1920 Aug. 14	34°.4	+0.00162	„ March 21	43°.0	+0.00519
1921 Febr. 14	34°.8	+0.00317	„ Febr. 23	43°.2	+0.00620
1920 April 23	35°.9	+0.00430	„ Jan. 14	43°.5	+0.00642
„ Febr. 6	35°.9	+0.00343	„ Febr. 20	43°.6	+0.00638
„ May 3	35°.9	+0.00145			
„ „ 21	36°.5	+0.00149			
„ April 16	37°.9	+0.00247			
„ March 29	38°.4	+0.00356			
„ „ 18	38°.9	+0.00203			
„ April 10	39°.2	+0.00322			
„ Febr. 26	39°.7	+0.00465			
„ „ 19	39°.9	+0.00504			

It is clear by the predominating of the positive sign, that the systematic error looked for exists, but also, that the error is not constant in the different declinations, and shows a marked increase for increasing declination.

The following table gives the mean values of y, for the zones according to which the programme stars are divided:

Zône	0°—20°	12°,1	$+0^s,00108 \pm 61$
,,	20°—30°	27°,0	$0,00230 \pm 42$
,,	30°—40°	35°,9	$0,00301 \pm 33$
,,	40°—zenith	42°,3	$0,00572 \pm 60$

As the velocity of the stars is proportional to sec. δ, I have endeavoured to represent the resulting values of y by a formula of the form:

$$y = A \frac{d}{d\delta} \sec \delta = A \tan \delta \sec \delta .$$

The four means given above, give four equations of condition for solving the quantity A. Each equation has been given a weight equal to the number of nights which have contributed to the equation. I find $A = + 0^s.00401$.

y is thus represented by the formula:

$$y = + 0^s.00401 \tan \delta \sec \delta .$$

The residuals between the observed and calculated values of y for the four zones are as follows:

$-0^s,00020$, $-0^s,00001$, $+0^s,00057$. $-0^s,00079$

The correction for the time of transit of a star with decl. δ_i, when the mean decl. of the fund. stars is δ, now becomes:

$$z = \frac{0^s.00401}{bg\ 1^\circ} \int_{\delta}^{\delta_i} \tan \delta \sec \delta \, d\delta = 0^s.230 (\sec \delta_i - \sec \delta)$$

If $\delta_i > \delta$ the time of the transit must be corrected by a positive quantity.

If $\delta_i < \delta$ the time of the transit must be corrected by a negative quantity.

in other words:

Stars with high decl. are registered too early.

,, ,, small ,, ,, ,, ,, late.

Observer Z.

List of results for y, obtained and arranged in the same way as for observer H.

1920 March 22	5°.0	$+0^{s}.00159$	1920 June 10	31°.2	+0 .00339
„ Febr. 23	11°.1	+0.00167	„ Febr. 3	32°.2	+0.00222
„ May 12	11°.3	−0.00013	„ June 8	32°.5	+0.00314
„ Febr. 18	14°.0	+0.00077	„ Aug. 27	32°.6	+0.00547
„ May 15	18°.4	+0.00095	„ Febr. 24	34°.3	−0.00017
„ „ 21	18°.4	+0.00041	„ „ 23	36°.8	+0.00535
1920 Febr. 12	22°.4	−0.00275	1920 June 21	40°.0	+0.00132
„ „ 5	23°.6	+0.00377	„ Aug. 17	42°.0	+0.00315
„ Oct. 28	25°.6	+0.00288	„ Jan. 9	42°.2	+0.01195
„ Nov. 19	28°.0	+0.00209	„ Febr. 19	43°.2	+0.01142
„ „ 8	28°.6	+0.00505			

The separate results give the following means:

Zone	0°—20°	13°,0	$+ 0^s,00088$	± 28
,,	20°—30°	25°,6	$+ 0^s,00221$	± 133
,,	30°—40°	33°,3	$+ 0^s,00324$	± 86
,,	40°—zenith	41°,8	$+ 0^s,00696$	± 276

Owing to the smaller number of nights, the mean error is much greater, although the same influence is fairly marked.

Treated as above, y can be represented by the formula:

$$y = + 0^s.00497 \tan \delta \sec \delta .$$

The residuals $O—C$ for the four zones are:

$$—0^s,00030, \quad —0^s,00043, \quad —0^s,00068, \quad + 0^s,00100$$

The correction z becomes:

$$z = + 0^s.285 \, (\sec \delta_i — \sec \delta).$$

For observer Z. we find the same phenomenon, that stars of high decl. are registered too early, those with small decl. too late.

Observer G.

List of the separate results, arranged as before.

1920 Febr. 17	4°.9	$+0^{s}.00022$
1921 Jan. 14	6°.7	−0.00158
1920 March 4	9°.6	+0.00260
„ „ 26	9°.7	−0.00018
„ Febr. 6	11°.3	−0.00261
„ „ 15	11°.9	+0.00070
1921 „ 24	13°.3	+0.00090
„ „ 26	13°.3	+0.00143
1920 May 14	13°.4	−0.00075
1920 Febr. 18	22°.4	+0.00113
„ May 28	27°.2	−0.00174
„ June 9	30°.1	+0.00068
1921 March 15	30°.7	−0.00461
„ Febr. 22	31°.2	−0.00201
1920 „ 3	32°.0	−0.00269
„ May 19	33°.8	−0.00072
1921 March 11	34°.2	−0.00263
1920 Jan. 23	33°.9	+0.00297
„ July 17	34°.4	−0.00043
„ March 18	34°.9	+0.00036

1920 Febr. 7	35°.7	$+0^{s}.00123$
„ „ 26	35°.9	+0.00095
1921 „ 19	36°.2	−0.00207
1920 May 10	36°.7	+0.00105
„ „ 9	40°.3	+0.00285
„ June 15	41°.0	$+0^{s}.00012$

As the different declinations are spread more irregularly over the separate nights, I have arranged the y's in somewhat different groups.

Zone	0°—20°	$10°,5 + 0^{s}.00008 \pm 53$
„	20°—35°	$31°.3 - 0^{s}.00088 \pm 64$
„	35°— zenith	$37°.6 + 0^{s}.00069 \pm 50$

Since these means are all nearly equal to or smaller than their m.e., we may conclude that for observer G. no influence of the declination on the time of transit exists.

The explanation of the effect may perhaps be sought in the fact, that the observers H. and Z. endeavour to make the contact of the signal key coincide with the moment that the star passes a thread, which is in agreement with their own statement as to their method of registering, while the observer G. begins with the movement of registering at the moment that he observes the star to be bisected by a thread.

That actually this difference in the method of registering exists between G. and Z. is confirmed by special observations, made for the determination of their relative personal equation, which is found to be positive in the sense G.—Z.

The observers H. and Z., therefore, will be influenced in their registering by the velocity $c. \sec. \delta$, with which the star *approaches* the thread, the observer G. will be independent of it.

As it is always possible in zone-observations with differential reduction to limit the influence of the magnitude-equation by using different gratings, while in practice it is often necessary to take the range of the zones relatively large, it will always be desirable for an observer who is aware that he follows the first method of registration, to test his observations of transits for a dependence on the declination. Perhaps we may see in this result once more an argument in favour of the impersonal micrometer.

Finally we give a table, which shows that the correction reaches quantities, which are by no means negligible.

Mean decl. fund. stars	Decl. programst.	Correction to the time of transit	
		Observer H.	Observer Z.
20°	10°	$-0^s.011$	$-0^s.014$
20°	15°	-0.007	-0.008
20°	25°	$+0.009$	$+0.011$
20°	30°	$+0.021$	$+0.026$
30°	20°	-0.021	-0.026
30°	25°	-0.012	-0.016
30°	35°	$+0.015$	$+0.019$
30°	40°	$+0.035$	$+0.043$
40°	30°	-0.035	-0.043
40°	35°	-0.019	-0.024
40°	45°	$+0.025$	$+0.031$
40°	50°	$+0.058$	$+0.071$

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