Physics. — "*A moving coil galvanometer of high sensitivity*". By Prof. F. ZERNIKE. (Communicated by Prof. H. HAGA).

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Introduction. The problem to determine the conditions for which a moving coil galvanometer reaches maximum sensitivity, has been frequently discussed in the literature of the subject [1]).

The result attained is in short as follows: for any fixed period of oscillation the sensitivity varies inversely as the root of the *moment of inertia* K of the moving system.

Now the *torque* D of the suspension cannot be decreased beyond the limit determined by the smallest dimensions of suspension strip available, which limit until recently was 0,2 C.G.S. Hence K, which is proportional to D, cannot be decreased indefinitely.

In recent years several galvanometers have been constructed with much smaller torques. Even if there was no limit to the smallness of D, the sensitivity would still be restricted as K cannot be indefinitely decreased because of the presence of the *galvanometer-mirror*. Indeed it is very remarkable that in the above mentioned discussions this important detail of the instrument has hardly been taken into account. Only EINTHOVEN [2]) has laid stress upon the fact that by judging the sensitivity of galvanometers the size of the mirror ought to be taken into account. As is well known, his studies led to the construction of another type, the stringgalvanometer, hence do not answer the question which we put: *how to make a reflecting galvanometer with moving coil as sensitive as possible.*

To solve this question I will start with a given mirror. Up till now the mirror was considered to be a detrimental though a necessary addition, because it increases the moment of inertia. Indeed I found the moment of inertia of the mirror to be from 1 to 3 °/₀ of the whole system in different commercial instruments. I will invert this

[1]) See i.a. W. JAEGER, Z f. Instrumentenk. **23,** 261 en 533 (1903).
W. P. WHITE, Phys. Rev. **19,** 305 (1904).
W. J. H. MOLL, these Proc.
Discussed at length by W. JAEGER, Elektrische Messtechnik, Leipzig 1917, pg. 204 sqq.

[2]) W. EINTHOVEN, Ann. d. Physik **12,** 1062 (1903).

and state: the coil is a detrimental though a necessary addition to the mirror, and one should take care that the total moment of inertia does not become much greater than that of the mirror.

The following formulae will show clearly what may be attained in this respect. Afterwards I shall prove that for the technical construction according to these principles one can calculate every detail of construction about in the same way as an engineer calculates a dynamo, at the same time I shall give the data of actually constructed galvanometers, as they are put on the market by KIPP and Sons Ltd., Delft.

Calculation. For the voltage sensitivity we have

$$P = \frac{Hf}{Dr} \quad \ldots \ldots \ldots \ldots \quad (1)$$

as condition for the limit of aperiodicity

$$\frac{H^2 f^2}{2r} = \sqrt{DK} \quad \ldots \ldots \ldots \ldots \quad (2)$$

and for undamped oscillations

$$D = \frac{4\pi^2}{T^2} K \quad \ldots \ldots \ldots \ldots \quad (3)$$

Here and further on the letters have the following meaning:
P rotation in consequence of unit e.m.f. in the circuit,
H intensity of the magnetic field,
f winding surface of the coil,
D torque for unit angular displacement,
K moment of inertia of the whole system,
K_0 moment of inertia of the coil,
r resistance of the whole circuit,
r_0 resistance of the coil,
T complete period of the undamped oscillations,
$m = H/H_{min}$,
in which all quantities are to be expressed in electromagnetic C.G.S. units.

Eliminating Hf and D from (1), (2) and (3) we get:

$$P^2 = \frac{T^3}{4\pi^3 Kr} \quad \ldots \ldots \ldots \ldots \quad (4)$$

for the voltage sensitivity. Now it is well known that the current sensitivity of any galvanometer is proportional to $\sqrt{r}$, the voltage sensitivity inversely proportional to this. Therefore the power sensitivity (Watt-sensitivity) is independent of the resistance. It is apparent from (4) that we can only increase this power sensitivity by reducing K or by increasing T. The latter alternative, however, would soon render the galvanometer less fit; therefore I prefer to introduce at once the maximum value of T which we will allow in any special case. So we must try to find the maximum of (4) for fixed T, mirror and external resistance. Moreover I'll assume H to be given. From the result it will then be clear in which way the resulting sensitivity depends upon H. From (2) and (3) we find:

$$H^2 = \frac{4\pi}{T} \cdot \frac{Kr}{f^2} \cdot \qquad (5)$$

Now suppose the coil to be short circuited and the mirror removed by which $Kr = K_0 r_0$, thus assuming its minimum value for the coil in use.

From (5) we then derive the minimum value of H with which the galvanometer can be made aperiodic. The importance of this minimum magnetic field, which I shall represent by H_{min}, lies in the fact that this quantity appears to be independent of the dimensions of the coil, the diameter of the wire etc. Indeed, taking only the vertical part of the windings into account, it will be easily found that:

$$\frac{K_0 r_0}{f^2} = s\sigma$$

i.e. the product of the density and the specific resistance of the metal. The horizontal part of the circuit of the coil, the insulation etc. can only *increase* the value found here and consequently H_{min}. The following relations therefore hold:

$$H_{min} = \frac{4\pi}{T} \frac{K_0 r_0}{f^2} = \frac{4\pi s\sigma}{T} \qquad (6)$$

so that H_{min} must be considered as a constant in finding the maximum sensitivity.

(5) and (6) give:

$$\frac{H^2}{H_{min}^2} = \frac{Kr}{K_0 r_0} = m^2 \qquad (6a)$$

in which thus m is a known number > 1. Representing K/K_0 by k, $r/r_0 = m^2/k$. Instead of (4) we can write:

$$P^2 = \frac{T^2}{4\pi^3 (K - K_0)(r - r_0)} \cdot \frac{1}{m^2} (k-1)\left(\frac{m^2}{k} - 1\right)$$

In this expression only the two last factors are variable. Their product is a maximum for $k = m$. Hence the conditions for maximum sensitivity are:

$$\frac{K}{K_0} = \frac{r}{r_0} = \frac{H}{H_{min}} \qquad (7)$$

and thus the maximum sensitivity:

$$P^2 = \frac{T^2}{4\pi^2 (K-K_0)(r-r_0)} \cdot \frac{(m-1)^2}{m^2}.$$

From this we derive that m should be made as large as possible. $K-K_0$ is the moment of inertia of the mirror, $r-r_0$ is the given external resistance (rather + the resistance of the flexible leads which is a known quantity in any special case). The greatest though in practice unattainable sensitivity is thus:

$$P_{max}^2 = \frac{T^2}{4\pi^2 K_{mir}\, r_{ext}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

whilst the ratio P/P_{max} might be called the efficiency of the galvanometer. Hence one generally finds for this efficiency:

$$\frac{P}{P_{max}} = \frac{1}{m} \sqrt{(k-1)\left(\frac{m^2}{k} - 1\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

As m can be e.g. 10 the conditions (7) mean that one should not only — as is known from former researches — reduce the resistance of the galvanometer compared with the external resistance, *but that also the moment of inertia of the coil should be small in comparison with that of the mirror.*

Technical construction. We are going to make use of the above mentioned formulae for the further calculation of galvanometers with two different periods of 3 resp. 8 sec. For the circular mirrors which may be used we have:

Diameter	12	10	8	millimeters
Moment of inertia	0,0055	0,0026	0,0011	

for a thickness of 0,20 mm. Mirrors thinner than this are mostly insufficiently plane, besides they warp too easily in mounting.

The attainable value of H depends not only on the size of the permanent magnet which is used but also on the dimensions of the airgap. For various existing galvanometers I found for H values near 700; once I found 1100. The small coils with only few turns of wire, which are needed according to our calculations, allow to increase H considerably, provided one places an iron core inside the coil. I use for example a core of 6.8 mm. diameter and 15 mm. height, and an airgap of 1.2 mm. round it. The coil then consists of rectangular turns of wire of 8×16 mm. With a simple steel-magnet the magnetic field proved to be

$$H = 2000$$

I am going to accept these values for the following. From (6) we derive for copper with $T = 3$ resp. 8 sec., $H_{min} = 250$ resp. 150. Taking however the horizontal pieces of the wire into account, the resistance increases $^{3}/_{2}$ times, the moment of inertia $^{7}/_{6}$ times, whence:

$$H_{min} = 330 \text{ resp. } 200$$
$$m = 6 \quad ,, \quad 10$$

Not to make the coil too thin and mechanically too weak, I'll not take in accordance with (7) $K/K_0 = m$, but $= 3$. Then according to (9) the efficiency will still be 78 resp. 80 % whereas it is 83 and 90 % in the most favourable case.

When we choose for the faster galvanometer a mirror of 8 mm., for the other a mirror of 10 mm. we get:

$$K_0 = 0{,}0005 \text{ resp. } 0{,}0013$$

and

$$D = 0{,}0070 \quad ,, \quad 0{,}0025$$

These are about the utmost values which we can use, so that smaller mirrors would hardly produce a greater sensitivity. We have namely up till now neglected *the airdamping* in our calculations. This appears to be already quite perceptible here. By further reducing the product KD which is 10.10^{-6} here, we should cause the galvanometer to be already aperiodically damped on open circuit. By using a smaller coil one could get somewhat further.

To complete the calculation the resistance must be known. As an example I choose 100 Ω for the total resistance. From (6a) we derive:

$$\frac{r}{r_0} = 12 \text{ resp. } 33 \quad \text{and} \quad r_0 = 8{,}3 \text{ resp. } 3{,}0 \ \Omega$$

and f from

$$\frac{K_0 r_0}{f^2} = \frac{7}{4} s\sigma = 2{,}6 \,.\, 10^{-5}$$

$f = 13$ in both cases.

As one winding has an area of 1,2 cm^2 we must take 11 windings with a length of wire of 51 cm. From the resistance and the length we find for the diameter of the wire 0.035 resp. 0.06 mm. The first of these two values is rather too small. We can remedy this 1st. by taking a smaller coil of 5×12 mm. e.g. thus increasing the number of turns and the length of the wire. The diameter then becomes 0.043 mm. 2nd. By not taking $k = 3$ but e.g. $k = 2$. Then we find $K_0 = 0.0011$, $r_0 = 5{,}6 \ \Omega$ $f = 15.3$ diameter 0.046. A combination of both methods gives the appropriate thickness of 0.055 mm.

From (4) we derive the voltage sensitivity. For the deflection P'

per microvolt in scale divisions at a distance of 1000 divisions we find the general formula

$$P' = 0{,}57\ T^{3/4}\ K^{-1/2}\ r^{-1/2}$$

and in the two cases which have been calculated:

$$P' = 8 \text{ resp. } 21\ ^{\mathrm{mm}}/_{\mu \mathrm{V}}$$

I have actually attained sensitivities of this order, also the threefold sensitivity in the case of 10 Ω total resistance. By providing the galvanometer with a magnetic shunt, which can be moved by a screw, I was able to make H continuously variable, between the greatest value mentioned and $^1/_3$ of it. By the id of this shunt we are able to use one single instrument for resistances from 11 to 100 Ω e.g. and to bring the instrument at once in the aperiodic condition for any resistance within these limits. For the weaker magnetic fields got in this way m will be reduced to 2 or 3. All the more reason not to take K/K_0 greater than 3.

Remains a very important matter viz. how it is possible to realize the required very small restoring torque. For a strip with rectangular section of sides a and b and length l, which is twisted, we have

$$D = \frac{G}{3l} ab^3$$

when b is small with respect to a. G is the torsion modulus.

One might try to cut small strips of metal foil in order to get b very small. The ordinary silver foil for instance has a thickness of 0,2 μ. Experiments showed that these strips cause a many times greater torque than that calculated from their dimensions. Apparently the beaten metal departs too much from the simple shape supposed in calculating the formula, its thickness being very uneven and its surface very rugged.

Therefore I have made silver foil of 0.4 to 0.7 μ thickness by electrolysis, for instance by precipitating the metal on zinc and afterwards dissolving the zinc in a weak acid. By means of a razor attached to a dividing engine, one is able to cut narrow strips of this foil, if necessary even of 0,02 mm. width.

For a silver strip 0.5 μ thick, 0.1 mm. wide and 10 mm. long the formula gives $D = 0.00012$. Two of these strips would thus give a torque of only one tenth of the smallest one required. Repeated experiments showed that by very careful treatment one gets with such strips a torque which is 2 or 3 times the calculated one. From the dimensions mentioned one finds further for the resistance 3 Ω

which is low enough even for galvanometers of small resistance. Of course one can also use wider strips.

It is not possible to use these thin strips as a suspension, for they show the small torsional rigidity only when unstretched. Therefore I use a quartz fibre suspension taking care that this produces 80 or 90 % of the required torque. As a consequence the elastic after-effect of the strips which is otherwise considerable, is of little or no importance. The metal strips — probably copper is in some respects to be preferred to silver — hang loosely on both sides of the quartz fibre.

Finally I will mention the fact that the very light moving systems of my galvanometers appear to be little sensitive to tremors. Surely this is at least partly due to the relatively strong air-damping which makes the vibrations about horizontal axes decay pretty quickly.

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