Mathematics. — *"A Congruence* (1,0) *of Twisted Cubics".* By Prof. JAN DE VRIES.

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1. The twisted cubics through four points C_1, C_2, C_3, C_4 cutting the straight line b twice, form a linear congruence $[\varrho^3]$; for through any point there passes one ϱ^3. The base points C are the *cardinal points*, b is a *cardinal chord*.

If d is a chord of one of the ϱ^3, $d(C_1\,C_2\,C_3\,C_4) = b(C_1\,C_2\,C_3\,C_4)$. The chords d form therefore a *tetrahedral complex*; a ray l not belonging to this complex, is not cut twice by any ϱ^3: the *class* of the congruence is *zero*.

Together with C_k and b a chord d defines a hyperboloid; on this there lie ∞^1 curves ϱ^3 and these define on d an involution; d is consequently a *tangent* to *two* curves.

The tangents meeting at a point P, lie on the complex cone of P; their *points of contact* form a twisted curve of the 5$^{\text{th}}$ order, ϱ^5, passing through P.

2. Let B_4 be the point of intersection of b with the plane $\gamma_{123} \equiv C_1\,C_2\,C_3$. Each *conic* ϱ^2 through the points C_1, C_2, C_3, B_4 is a component part of a degenerate ϱ^3; the transversal t_4 through C_4 resting on b and ϱ^2 is the second component part. The straight lines t_4 form the *pencil of rays* through C_4 in theplane $C_4\,b$. There are therefore *four* pencils of rays formed by *singular straight lines.*

The pairs of lines of the pencil (ϱ^2) produce three figures each consisting of three straight lines, e.g. the combination of $C_1\,C_2$, $C_3\,B_4$ and the straight line t_4 resting on $C_1\,C_2$. There are evidently *twelve* figures consisting of *three straight lines.*

3. With a view to finding the order of the surface Λ formed by the ϱ^3 cutting a straight line l, we determine the intersection of Λ with the plane γ_{123}. It consists of two conics of the pencil (ϱ^2); the former cuts l, the latter is a component part of the ϱ^3 which is defined by the transversal through C_4 of b and l. Hence Λ is a *surface* of the 4$^{\text{th}}$ order; the cardinal points C are apparently *double points* of Λ^4. A ϱ^3 not lying on Λ^4, can only cut this surface in the points C and on the cardinal chord b; from this there follows that b is a *double straight line.*

On Δ^4 there lie 9 straight lines and 8 conics.

The straight lines resting on b and l, determine a representation of Δ^4 on a plane.

A straight line l_1 through a point C cuts Δ^4 in two more points outside C; from this follows that the ϱ^3 cutting l_1, lie on a hyperboloid: this is entirely defined by l_1, b and C_k. Analogously the ϱ^3 resting in a fixed point on b or on a straight line intersecting b, form respectively a quadric cone or a hyperboloid.

4. A plane λ through l cuts Δ^4 along a curve λ^4 which has a double point on b. In each of the three points of intersection of λ^4 with l, λ is touched by a ϱ^3. Hence the curves ϱ^3 touching a plane σ, have their *points of contact* on a *curve* σ^3.

Let B be a point of b; the ϱ^3 through the five points B and C_k touching σ, form a surface of the 10^th^ order with sextuple points in B and C_k [1]). There are accordingly 4 ϱ^3 through B and C_k which have b as a chord; consequently b is *quadruple* on the locus Δ of the ϱ^3 touching the plane σ and belonging to the congruence (1,0). Also it appears that Δ has *quadruple points* in C_k. Accordingly an arbitrary ϱ^3 of the (1,0) has 24 points in common with Δ, i. e. Δ is a *surface* of the 8^th^ order.

5. Δ^8 has the curve of contact σ^3 and a conic σ^2 in common with the plane δ. The curves σ^3 and σ^2 touch each other in 3 points; there are therefore *three* curves ϱ^3 which *osculate* the plane σ.

If revolves round l, σ^3 describes a surface of the fourth order with the single straight line l.

On the curve ϱ^3 cutting l in R, the pencil of planes (σ) defines an involution; l is therefore cut by two tangents of ϱ^3. Consequently through l there pass two planes in which R is a point of the "complementary" curve σ^2. Hence σ^2 describes a surface of the fourth order with the double straight line l.

Let us now consider the relation between the points P and Q which the curves σ^3 and σ^2 in a plane σ have in common with l. Through P there passes one ϱ^3; the tangent at P defines the plane σ, hence two points Q. Through Q there pass two ϱ^3, hence two curves σ^2, and two planes σ each containing a curve σ^3; six points P are therefore associated to Q. If two homologous points P and Q coincide, there arises a double coincidence of the (6,2), for at that point a ϱ^3 is osculated by the plane σ. On l there lie therefore four points N for which the plane of osculation ν passes through l.

[1]) This is easily seen from the intersection of this surface with γ_{123}, which consists of 2 conics and 3 double straight lines.

6. If we consider N as the null-point of v, there arises a *null-system* with the characteristic numbers $\alpha = 1$, $\beta = 3$, $\gamma = 4$ (§ 5).

If v continues to pass through a point P, the locus of N consists of a surface $(P)^5$ and the four pencils of rays round the points C_k in the planes $C_k b$ (§ 2).

If v revolves round the straight line l, v describes a curve λ^7 and the four singular rays through C_k which rest on l.

The surfaces $(P)^6$ and $(Q)^6$ have in common the curve λ^7 corresponding to PQ, and the 18 singular straigth lines $C_k C_l$ and $C_k B_l$.

With a ϱ^3 $(P)^5$ has in common the 3 points of which the planes of osculation pass through P; the remaining 12 common points lie in the cardinal points C; these are therefore triple points of $(P)^5$. The planes of osculation in C_k envelop accordingly a cone of the third class.