Mathematics. — "*Inner Limiting Sets*". By Prof. J. WOLFF.
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HOBSON has been the first to prove the following theorem: [1])

An enumerable set of points which has no part that is dense in itself, is an inner limiting set, i.e. the common part of an enumerable set of open sets each of which we may assume to contain the following one.

BROUWER has given an extremely short proof, but just as HOBSON he makes use of the transfinite ordinal numbers [2]).

In the proof which follows here, no use is made of these numbers.

1. If $E_1, E_2, \ldots$ are inner limiting sets, if further each E_k is a part of an open set Ω_k, while no two Ω_k have any points in common, also the sum $E_1 + E_2 + \ldots$ is an inner limiting set.

For we may write:

$$E_k = \Omega_{k1}\, \Omega_{k2} \ldots\ldots \quad , \quad k = 1, 2, \ldots\ldots$$

which means that E_k is the set of points lying in Ω_{ki} for every i. The Ω_{ki} are open sets of which we may assume that they all lie in Ω_k. The set

$$(\Omega_{11} + \Omega_{21} + \ldots)\,(\Omega_{12} + \Omega_{22} + \ldots) \ldots\ldots$$

contains $E_1 + E_2 + \ldots\ldots$, but no point outside them, as $\Omega_{ki}\, \Omega_{lj} = 0$ for $k \neq l$. Now the auxiliary theorem has been proved.

2. We call a set of points E an inner limiting set *in a point* P if there exists an open set containing this point, so that the part of E lying in this set is an inner limiting set. This holds also good for the part of E lying in an arbitrary open set which is a part of the above mentioned one.

3. If an enumerable set E is an inner limiting set in each of its points, E is an inner limiting set.

We call the points of E: $P_1, P_2, \ldots$

[1]) Proc. London M.S. (2) 2, p. 316—323.
[2]) These Proceedings, Vol. XVIII p. 48 (1915).

Round P_k as centre we take an interval I_k (a quadrangle, a cube, etc. according to the number of dimensions of the space in which E is given), so that $E I_k$ is an inner limiting set, taking care that the boundary of I_k contains no point of E, which is possible on account of E being enumerable.

By I_k we understand the open interval, by $\overline{I}_k$ we shall indicate the closed one, by an accent, the complement of a set. Now

$$E = EI_1 + EI_2 (\overline{I}_1)' + EI_3 (\overline{I}_1)' (\overline{I}_2)' + \ldots$$

From N°. 1 there follows now immediately that E is an inner limiting set.

4. Let E be enumerable and not an inner limiting set. In this case according to N°. 3 the set D of the points E in which E is not an inner limiting set, is not empty. Let P be a point of D and I an interval with P as centre. EI is according to N°. 2 not an inner limiting set, hence neither is $EI—P$; according to N°. 3, $EI—P$ contains a point Q in which $EI—P$ is not an inner limiting set, hence E is not an inner limiting set in Q, so that Q lies in D. From this there follows that D is dense in itself and from that the theorem which was to be proved.