Meteorology. — *"A non-tangent infralateral arc"*. By Dr. S. W. VISSER. (Communicated by Prof. E. VAN EVERDINGEN Jr.).

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On 24th June 1923 I saw at the Astronomical Observatory at Lembang a beautiful halo, which I will describe in the following pages.

Already early in the morning a mock-sun was visible on the right of the sun. Direct measurements of its distance were impossible, as the sun itself was hidden by thick clouds. About twelve o'clock a very bright lower tangent arc appeared, which after a few minutes became so intensely luminous as to be visible from time to time through the lower clouds. Soon this arc spread and developed into a complete circumscribed halo within which a weak ordinary ring became also visible. I succeeded between $12^h 17^m$ and $12^h 49^m$ in taking some 26 measurements of both rings by means of the cloud theodolite, mounted at the Observatory expressly for observations of halo's. To these measurements I will refer afterwards. In the mean time I kept a keen lookout for other halo's. Not before $12^h 49^m$ my effort was rewarded by the apparition of a spot of light on the left below the sun, near the place where the mayor ring (46°) was to be expected. This spot soon grew more intense and developed into a short, oblique arc. Colours (red and green) were visible. On the other side of the sun nothing could be observed, because there the Cu-cloud around the Tangkoehan Prahoe shielded the Cirrus layer from our vision. I now concentrated my full attention on this arc and obtained 12 measurements until $1^h 4^m$. Sometimes clouds prevented the observation. Moreover between $12^h 16^m$ and $1^u 2^m$ fourteen control-observations of the sun were made. At $1^h 4^m$ the lower cloud had so much increased, that the measurements had to be finished. At half past seven in the evening the Ci-St proved to be still present, there was a bright lunar halo, but without any particularities.

The same halo's were seen by M.M. VOûTE and RIJKEN RAPP during their railway journey between Tjimahi and Bandoeng. However they saw the small arc not on the left (west) of the sun, but on the right (east). Though on the left hand side the sky presented an equally smooth Cirrus-veil as on the right, nothing was to be seen there. According to RIJKEN RAPP the arc was intensely coloured and bent like a portion of the greater ring. I have not been able to note any curvature at Lembang.

Before discussing my measurements I give here a short review of the theory of the infralateral arc.

BRAVAIS explains the arc by the refraction of light in ice-crystals with a horizontal principal axis, the light entering at a vertical basal plane (the hexagonal terminal plane of the crystal) and leaving at a sideplane of the prism. The refracting angle is 90° then. For a definite position of the principal axis (defined f. i. by its azimuth) we get a circular arc perpendicular to this axis and at a distance from the sun, depending on the sun's height. In a simple way we may imagine this circle by drawing the case of the circumzenithal arc and rotating the drawing then over 90°, so that the axis which at first was vertical, now gets a horizontal position. To each azimuth of the axis such a circle belongs. The envelope of all these minor circles is the infralateral arc. One among these circles is tangent to the greater ring. For the rest, this arc does no more then the circumzenithal arc fulfil the conditions for minimum deviation of the refracted rays of light.

PERNTER (Meteorologische Optik, 1st Edition) sticks to these conditions. He considers the arc as a "Lowitz arc of the greater ring" and deduces the form and position of the lateral arcs to the smaller and greater rings in an exactly analogous way. Without going into the details of the calculations, we may state, that the arc according to PERNTER in consequence of the conditions for minimum deviation which he imposes, generally will be less distant from the ring than BRAVAIS's arc.

BESSON (Sur la Théorie des Halos, Paris 1909, p. 51, p. 70) has shown, that PERNTER's theory is not very satisfactory. EXNER (PERNTER-EXNER, Meteor. Optik, 2nd Edition 1922 p. 405) concurs in this opinion and develops a new theory. During the normal fall of an ice-prism the principal axis and one of the bigger diagonals of the hexagon are placed horizontally. An infralateral arc may than be formed by light, entering the basal plane and emerging from one of the oblique prism-planes. The plane perpendicular to the refracting edge is inclined to the horizon at an angle of 30°. For one definite height of the sun (27° 45′) the lateral arc is tangent to the ring. For all other suns-heights the arc deviates towards the outer side. If we allow rotations about the principal axis, minimum deviations are possible up to a suns-height of 80° 50′. According te EXNER (f.i. pag. 402) measurements are lacking. However there exists one by BESSON (l.c. pag. 71). 23rd April 1908 with a suns-height of 53° he saw an infralateral arc on the left below the sun at a height of 19°, whereas from BRAVAIS's theory a height of 18° 57′ would follow.

This case bears some resemblance to that of Lembang. "Three minutes afterwards" BESSON writes "the ring of 22° and the circumscribed halo appeared, complete but scarcely visible". In both observations the same forms of halo's appear. [1])

For the measurements at Lembang as a rule the red of the arc was vised at. Once green was measured. Two times the left- and righthand ends of the red were determined.

The readings and some distances and angles calculated from these have been entered in the following table.

Nr.	M. J. T.	$\odot$ az.	$\odot$ h′	Az_w	h_w	Δ_w	A_w	h_b	Δ_b	A_b	h_{w-b}	Δ_{w-b}	A_{w-b}
1	12u 50m	N 17.2W	58.2	N 60.1W	22.8	46° 44′	58° 19′	20.6	45.7	48.0	+1.9°	+0.9	10.1°
2	"	"	"	58.3	22.4	46 17							
3	12 51	17.6	"	60.3	22.4	47 0							
4	"	"	"	58.9	22.5	46 17							
5	12 52	18.0	58.1	60.4	22.9	46 23	56 10	20.5	45.5	48.4	+1.4	+0.9	7.8
6	"	"	"	56.4	20.9	46 24							
7	12 54	18.8	57.9	58.2	21.5	46 10	54 57	20.5	45.3	48.8	+1.0	+0.9	6.2
8	12 55	19.3	57.8	56.4	20.3	46 11	54 10	20.5	45.2	49.1	+0.5	+1.0	5.1
9	12 56	19.7	57.7	60.3	21.8	46 19							
10	12 57	20.1	57.6	60.2	20.3	47 22	55 13	19.4	46.2	50.5	+0.4	+1.2	4.8
11	12 58	20.5	57.5	59.6	21.1	46 10	54 39	20.4	45.3	49.6	+0.7	+0.9	5.1
12	1 4	23.0	57.1	59.5	20.0	45 46	51 17	20.3	45.2	50.3	−0.3	+0.6	1.0

The observations 5 and 9 refer to the lefthand end, 6 and 8 to

[1]) See also: E. VAN EVERDINGEN. Halo's in April, Hemel en Dampkring 21, 1923, p. 216, 217.

the righthand end, 10 to the green. The time is Middle-Java time. The suns-height and azimuth were calculated and with these the readings of the theodolite were reduced. Az_w and h_w stand for the observed azimuth and height of the arc; Δ_w is the distance of the observed points from the sun calculated from the 4 foregoing columns; A_w is the angle between the suns vertical and the radiusvector from the sun to the arc, deduced from the observations.

The column under Δ_w shows, that the points measured deviate sensibly from the ring, for the red of the ring is formed at a distance of 45°6′ from the sun. The mean deviation is 1.1°. Gradually the distance decreases, but for Nr. 12 it is still 0.6° larger than that of the ring. This deviation is so big and so systematic, that it is impossible to think of observational errors. Indeed there is question here of a *non tangent* arc. The position of the tangent-point of the arc was calculated according to BRAVAIS's theory. The results have been entered under h_b, Δ_b and A_b. The calculation was carried through for the 10th observation for green ($n = 1.3115$) for the rest for red ($n = 1.307$). In taking the differences between observation and calculation the first four points, which in consequence of the initial weakness of the arc happened to be less accurate than the others, were combined to a mean value. The observations 5 and 6, 8 and 9, which refer to the ends of the arc, were substituted by their mean values.

Almost all the observed points are too high (column h_{w-b} gives the difference observation and calculation), but they approach the height calculated from theory. The angle A, which according to theory should increase for a sinking sun, in reality rapidly decreases. In consequence the difference between observation and calculation decreases from 10° tot 1°. Finally, the distance from the sun remains almost constantly 0.9° too big, hardly showing any tendency to decrease.

During the whole time of observation the arc remains outside of Bravais's arc; the position with respect to the sun approaches more and more that of the theoretical tangent point.

This arc deviates from that of BRAVAIS and hence still more from that of PERNTER. No more is it in harmony with EXNER's theory. For in this case we have to assume a normal plane inclined at an angle of 30°. In our case the rays of the sun are in their turn inclined to this plane at an angle of at least 57.1°—30° = 27.1°. The smallest distance from the arc to the sun is then 57.6°, which is quite out of question for the observed arc.

As was explained above, crystals showing various orientations of the principal axis in the horizontal plane contribute to the formation of the infralateral arc.

That is why I calculated what position in space the axis ought to present in order to give rise to the phenomenon as it was observed. I supposed, that the refraction took place in the normal plane — for in this case the deviation is a minimum and the intensity of light a maximum.

We consider the spherical triangle ZSN, formed by the zenith Z, the sun S and the vanishing point of the crystal-axis N. We know ZS, the complement of the suns-height, $\angle$S, the supplement of the angle A we already determined, and arc SN. The latter is the angle of incidence i of the rays of light and is to be deduced from the observed Δ. Arc ZN and $\angle$Z may then be calculated, ZN gives the height of the vanishing point, $\angle$Z is the difference in azimuth with the sun. From this follows the azimuth of the axis, as the sun's azimuth is known.

The results are as follows:

Nr	i	ZN	Z	az.ax.
1—4	74.6°	92.4°	55.2°	N 72.7°W
5—6	74.2	93.0	53.2	71.2
7	73.5	93.0	51.8	70.6
8—9	73.2	93.7	51.3	70.8
10	74.4	93.6	52.4	72.5
11	73.5	93.4	51.6	72.1
12	72.2	93.9	48.1	72.1

Hence in the mean the crystal-axis is inclined at an angle of 3°.3 to the horizon and its azimuth is N 71.8 W.

The position of the axis appears to be stationary. The differences with the mean value are as a rule below 1°. The conclusion is the more remarkable for the azimuth, as the difference in azimuth with the sun decreases more than 7° during the observations.

In trying to find an explanation of such a position by taking into account the influence of gravitation, wind [1]) and atmospheric

[1]) M. PINKHOF. Bijdrage tot de theorie der halo-verschijnselen. Verhandelingen Kon. Akademie van Wetenschappen 1e Sectie, Dl. 13, N°. 1, p. 21, 1919.

electricity on the position of the ice-crystals, I met among others with the difficulty, that the complete development of the circumscribed halo seemed at variance with the explanation proposed. Therefore I hope to come back to this point afterwards. For each explanation however the observations on the ring and its envelope may be wanted. They follow therefore as the concluding part of my remarks.

BRB = lower tangent arc; O.H = circumscribed halo; K = ring of 22°; l = left; r = right. The remaining symbols have the same meaning as in the other tables.

The mean of the 6 observations on the red of the ring is 21° 54′, only 2′ more than that found from the measurements on the top. The calculated Δ_b is meant for white light, the observed Δ_w for red. Leaving apart the 4 very discordant differences for the first 4 observations, the mean difference observation minus calculation is —0.3°, that means exactly the difference in distance for red and white. Hence these observations of the circumscribed halo are in harmony with the calculation for red. In the 15 measurements on the ordinary ring however, on the contrary a very distinct difference of +0.3° remains.

A. Measurements of the upper and lower top (red).

M. J. T.	⊙ h	height of the top		Δ	
		lower	upper	lower	upper
12u 18m	59.7°	37.5°	—	22.2°	—
21	59.6	—	81.3°	—	21.7°
22	59.6	—	81.7	—	22.1
24	59.5	38.0	—	21.5	—
31	59.4	—	81.1	—	21.7
32	59.3	37.7	—	21.6	—
35	59.2	37.1	—	22.1	—
36	59.1	—	80.9	—	21.8
48	58.4	36.3	—	22.1	—
			mean	21.9	21.8

Mean of all measurements 21° 52′. for red according to PERNTER 21°34′

B. Measurements of the ring and the circumscribed halo.

Nr.	Time	$\odot$ h	$\odot$ az	h_w	az_w		Δ_w	A_w	Δ_b	Δ_{w-b}
1	12u 17m	59.7°	N2.8° W	38.1°	N0°.0W	BRB r	22°37′	5°49′	21.9°	+0.7°
2	17	59.7	2.8	38.1	7.6	„ l	22 48	9 55	22.0	+0.8
3	19	59.7	3.5	39.3	—11.7	„ r	22 32	31 58	22.8	—0.3
4	20	59.7	4.2	39.3	17.2	„ l	21 59	27 42	23.0	—1.0
5	26	59.5	7.0	58.5	55.0	O.H l	24 11	108 34	24.4	—0.2
6	27	59.5	7.4	58.5	50.6	K l	21 53	106 20	—	—
7	29	59.4	8.3	59.4	56.0	O.H l	24 11	110 40	24.4	—0.2
8	29	59.4	8.3	59.4	52.3	K l	21 58	109 1	—	—
9	37	59.0	11.8	59.0	60.0	O.H l	24 15	110 47	24.5	—0.2
10	38	59.0	12.2	59.0	—36.7	O.H r	24 35	111 5	24.5	+0.1
11	41	58.9	13.5	58.9	56.8	K l	21 58	108 45	—	—
12	43	58.8	14.3	58.8	—28.8	K r	21 54	108 22	—	—
13	43	58.8	14.3	58.8	—33.0	O.H r	23 58	110 23	24.5	—0.5
14	46	58.5	15.6	43.7	—16.4	O.H r	24 34	69 11	25.0	—0.4
15	47	58.5	16.0	51.6	—20.8	K r	21 52	87 15	—	—
16	47	58.5	16.0	50.9	—25.7	O.H r	24 31	90 0	25.1	—0.6
17	49	58.3	16.8	41.1	44.8	K r	21 46	72 32	—	—

Weltevreden, July 1923.