Physics. — "*Dispersion of Light by Irregular Refraction and by Molecular Scattering*". By Dr. J. SPIJKERBOER. (Communicated by Prof. W. H. JULIUS).

(Communicated at the meeting of December 29, 1923).

1. *Introduction.* In the explanation of the general distribution of light over the sun's disc as well as in the inquiry into the distribution of intensity in the solar spectrum the scattering by irregular refraction and the molecular scattering must be taken into account.

If molecular scattering can to a great extent be the cause through which the diminution of intensity [1]) from centre towards limb of the sun's disc, also for different wave-lengths, is as it is observed, it is the irregular refraction which, as Professor JULIUS has shown, can also play a part in this, and can also account for the origin of the sharp solar limb.[2])

As both the irregular radial curvature and the molecular scattering become very considerable for light from the immediate neighbourhood of absorption lines [3]), it must be assumed that the FRAUNHOFER lines are absorption lines which are enveloped by dispersion bands.

Both in the study of the "structure of the solar radiation" [4]) and in that of the "relation between the broadening and the mutual influence of dispersion lines in the spectrum of the sun's limb" [5]) we are confronted by the question of the greater or smaller influence of the molecular scattering or of the scattering by refraction. So far the influence of these two causes of scattering on each other has not been taken into account.

The purpose of this paper is to examine how we must imagine this mutual influence to be, and what conclusions can be derived from it. It seems to me that we are not justified in leaving these conclusions out of consideration.

2. *Scattering through irregular refraction.* In a paper on "Regular Consequences of Irregular Refraction in the Sun" [6]) Professor JULIUS

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[1]) J. SPIJKERBOER, Verstrooiïng van licht en intensiteitsverdeeling over de zonneschijf. Proefschrift. Utrecht 1917; Arch. néerl., IIIA, V, 1, 1918.

[2]) W. H. JULIUS, Astroph. Journ., **38**, 129, 1913.

[3]) W. H. JULIUS, These Proc., Vol. XII, p. 266, 1909; Vol. XIII[1], p. 2, 1910; Vol. XIII[II], p. 881, 1911; Handwörterbuch der Naturwissenschaften, VII, 832.

[4]) P. H. VAN CITTERT, These Proc., Vol. XXII, p. 73, 1919.

[5]) W. H. JULIUS and M MINNAERT, These Proc., Vol. XXVI, p. 329, 1923.

[6]) W. H. JULIUS, These Proc., Vol. XII, p. 266, 1909.

decided the question in 1909 what must be the influence of anomalous dispersion on the width of the so-called absorption lines in the sun's spectrum, if throughout the sun's atmosphere there are gradients of density which in many places are supposed to be of the same order of magnitude as in the spots, but which repeatedly reverse their signs.

If in the sun's atmosphere there is a spherical region C (fig. 1),

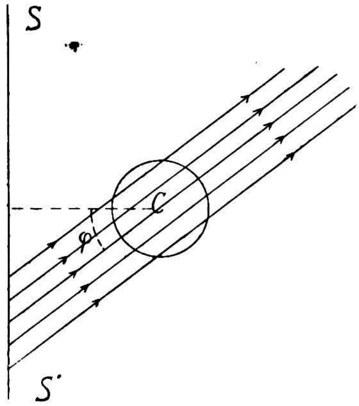

Fig. 1.

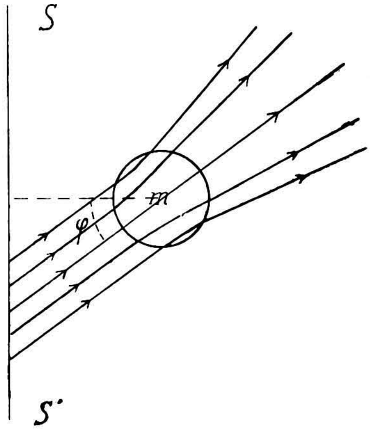

Fig. 2.

inside which the density does not deviate from that of the surroundings, the radiation, which leaves the surface SS' [1]) at an angle φ with the normal, will permeate this region C without change of direction.

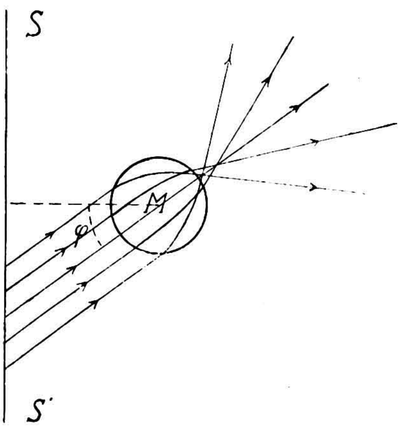

Fig. 3.

In a region with a density gradient such that the density in m (fig. 2) is a minimum, the incident beam is broadened like a plume; the same thing holds if there exists a gradient of density in this spherical region with a maximum of density in M (fig. 3) [2]). This plume-like broadening will be dependent in a great degree on the way in which the density

[1]) We may imagine the surface SS' lying so deep within the sun's atmosphere that the radiation follows the cosine law there; we may also assume that the surface SS' is imagined so that outside it the radiation is so small that it is a negligible fraction of the total energy emitted; we further call the surface SS' the surface of the nucleus.

[2]) Compare also a paper by L. S. ORNSTEIN and F. ZERNIKE, These Proc., Vol. XXI, p. 115, 1917.

varies within the regions m and M; in different cases the rays of light will present a course entirely different from that represented in fig. 2 and fig. 3, but this broadening, both in the case that there occurs a maximum and in the case that there occurs a minimum of density, will always vary with the refractivity in the atmosphere, and it will, therefore, depend on the density and the refraction-constant. Hence the broadening will become considerable for light of a wave-length differing little from the wave-length of an absorbed vibration, at least if the density of the component that produces the absorption line, is not too small.

Thus if the angle φ becomes greater or if the refractivity is more considerable than in the cases drawn in fig. 2 and fig. 3, part of the incident radiation can, for a definite frequency, return to the surface of the nucleus SS', and consequently not leave the sun.

By integration round the normal from C on SS' (over 2π) and over φ (from 0 to $\frac{\pi}{2}$) is found the radiation which penetrates the region C, and which would also leave the sun if there was no radial curvature.

If radial curvature *is* taken into account, part of the incident energy returns from m or M to the surface of the nucleus, and this part is the greater as the refractivity is the more considerable, variations of density being the same.

By considering only the effect of the radial refraction, and by assuming that there are several of these regions m and M in the sun's atmosphere, we must conclude that into the regions lying nearer SS' there can also penetrate radiation which was curved within the regions lying further from the nuclear surface so, that it returned to SS'; of this energy striking M or m at an angle $\varphi > 90°$ part will recede once more from the sun on account of the refraction in those regions lying nearer. The part of the returned radiation that does leave the sun's atmosphere in this way will, however, be of little importance, at any rate only a small fraction of the intensity which of the radiation was sent back by the first curvature.

3. *Molecular scattering.* Let us now consider the case that only molecular scattering in the sun's atmosphere is taken into account.

Again SS' be the surface of the nucleus; EE' a boundary layer, outside which no scattering particles are present.

We choose a coordinate x normal to SS' measuring the scattering mass, and for which the increase, as long as the density in the direction perpendicular to SS' does not change, is equal to the product of the coefficient of scattering s in the region under consideration, and the displacement t normal to SS'. Hence, for a definite point P inside the atmosphere x indicates the sum of the values st between the boundary layer and this point. As the density, and consequently s, will increase towards SS', a point for which $x = \frac{1}{2}H$, must be imagined to lie nearer A than B, when $x = 0$ is chosen for EE' and $x = H$ for SS'. This supposition has no influence on the further consideration. The place of the layer under consideration can every time be chosen so as to be suitable. For radiation forming an angle i with the normal AB we distinguish radiation b emanating from SS', and radiation a, returning to SS'. Besides on i, a and b depend on x.

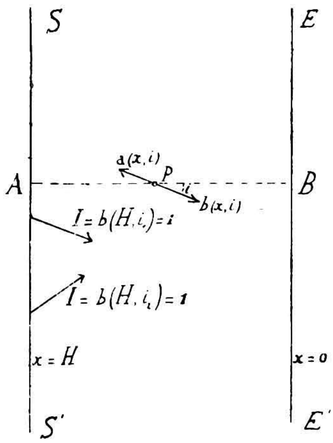

Fig. 4.

The limiting conditions are: 1. $b(H, i)$ is independent of i and equal to the radiation incident from the surface of the nucleus, which radiation we put 1 (thus finding a and b as fractions of this unit); [1])

2. $a(0, i) = 0$.

As solution for a and b is found:

$$a(x, i) = \int_0^x I(\xi)\, e^{(\xi - x)\sec i} \sec i \, d\xi, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$b(x, i) = e^{(x - H)\sec i} + \int_x^H I(\xi)\, e^{(x - \xi)\sec i} \sec i \, d\xi, \quad . \quad . \quad (2)$$

in which

$$I(\xi) = \tfrac{1}{2} \left\{ \int_0^{\frac{\pi}{2}} a(\xi, i) \sin i \, di + \int_0^{\frac{\pi}{2}} b(\xi, i) \sin i \, di \right\}.$$

[1]) If in connection with what was said in note 1 on page 166, SS' should be imagined so that the radiation outside it should be slight, the first limiting condition would not be fulfilled; though of great importance for the distribution of light over the sun's disc, this question is, however, of minor importance here.

According to an approximation which SCHWARZSCHILD calls "approximation according to SCHUSTER" [1]), I is found equal to $\frac{\xi+0,5}{H+1}$.

By insertion into (1) and (2), we get [2]):

$$a\,(x,i)=\frac{x+0,5-\cos i}{H+1}+e^{-x\sec i}\frac{\cos i-0,5}{H+1}, \quad . \; . \quad (1')$$

$$b\,(x,i)=\frac{x+0,5+\cos i}{H+1}+e^{(x-H)\sec i}\frac{0,5-\cos i}{H+1} \quad . \; . \quad (2')$$

Where before the value $b\,(0,i)$ was my chief point of interest, I will now in particular direct the attention to the intensities of radiation a and b for different values of x and i, for some values of H. (Compare the tables on the next page 170).

SCHWARZSCHILD has shown that the approximation obtained in this way, is a very good one. Though the values $b\,(0,i)$ are more accurately known in the different cases, I have given the approximated values also for them in the above mentioned tables.

The result of the calculations is clearly set forth in the figures 5—7 (p. 171) (fig. 5 for $H=8$; fig. 6 for $H=4$; fig. 7 for $H=1$). From O the radiation a and b is indicated on the radii vectores from O for the angles for which the cosine possesses the values used in the tables. As the figures show, the angles have been taken with OY as fixed leg.

The full lines apply to the b-radiation, the other lines to the a-radiation.

Besides the intensity of the a-radiation is plotted on the radii vectores from O which give the angles calculated from OY'.

In this way curves are obtained round O which give the intensities of radiation for points as P in fig. 4 in a plane through AB for directions between PA and PB.

On revolution of such a curve round YY' as axis the surface of irradiation of P is formed.

The lines 1, 2, 3 and 4 belong respectively to $x=H$, $x=\frac{1}{2}H$, $x=\frac{1}{4}H$ and $x=0$.

In the X-axis the broken line 1 should come together with the full line 1; the approximation, which was required for all the other lines, but not for the full line 1 is the cause that this does not take place.

1) Cf. K. SCHWARZSCHILD, Sitzungsberichte Kön. Pr. Ak. d. Wiss., **47**, 1183, 1914 and A. SCHUSTER, Astroph. Journal, **21**, 1, 1905.

2) Compare J. SPIJKERBOER, Dissertatie Utrecht, 45, 1917; Arch. néerl., IIIA V, p. 45, 1918.

TABLE I $H = 8$.

	$\cos i = 1$	$\cos i = 0.8$	$\cos i = 0.6$	$\cos i = 0.4$	$\cos i = 0.2$	$\cos i = 0.0$
$b\,(H,i) =$	1	1	1	1	1	1
$a\,(H,i) =$	0.83	0.86	0.88	0.90	0.92	0.94
$b\,(\frac{H}{2},i) =$	0.61	0.59	0.57	0.54	0.52	0.50
$a\,(\frac{H}{2},i) =$	0.39	0.41	0.43	0.46	0.48	0.50
$b\,(\frac{H}{4},i) =$	0.39	0.37	0.34	0.32	0.30	0.28
$a\,(\frac{H}{4},i) =$	0.17	0.19	0.21	0.23	0.26	0.28
$b\,(0,i) =$	0.17	0.14	0.12	0.10	0.08	0.06
$a\,(0,i) =$	0	0	0	0	0	0

TABLE II $H = 4$.

	$\cos i = 1$	$\cos i = 0.8$	$\cos i = 0.6$	$\cos i = 0.4$	$\cos i = 0.2$	$\cos i = 0.0$
$b\,(H,i) =$	1	1	1	1	1	1
$a\,(H,i) =$	0.70	0.74	0.78	0.82	0.86	0.90
$b\,(\frac{H}{2},i) =$	0.69	0.66	0.62	0.58	0.54	0.50
$a\,(\frac{H}{2},i) =$	0.31	0.34	0.38	0.42	0.46	0.50
$b\,(\frac{H}{4},i) =$	0.50	0.46	0.42	0.38	0.34	0.30
$a\,(\frac{H}{4},i) =$	0.14	0.16	0.18	0.22	0.26	0.30
$b\,(0,i) =$	0.30	0.26	0.22	0.18	0.14	0.10
$a\,(0,i) =$	0	0	0	0	0	0

TABLE III $H = 1$.

	$\cos i = 1$	$\cos i = 0.8$	$\cos i = 0.6$	$\cos i = 0.4$	$\cos i = 0.2$	$\cos i = 0.0$
$b\,(H,i) =$	1	1	1	1	1	1
$a\,(H,i) =$	0.34	0.39	0.46	0.55	0.65	0.75
$b\,(\frac{H}{2},i) =$	0.85	0.82	0.78	0.71	0.61	0.50
$a\,(\frac{H}{2},i) =$	0.15	0.18	0.22	0.29	0.39	0.50
$b\,(\frac{H}{4},i) =$	0.76	0.72	0.66	0.58	0.48	0.38
$a\,(\frac{H}{4},i) =$	0.07	0.08	0.11	0.15	0.23	0.38
$b\,(0,i) =$	0.66	0.61	0.54	0.45	0.35	0.25
$a\,(0,i) =$	0	0	0	0	0	0

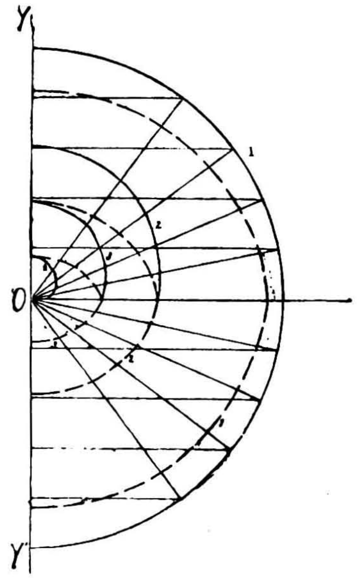

Fig. 5.

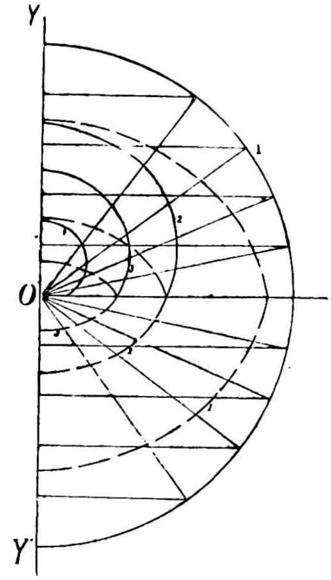

Fig. 6.

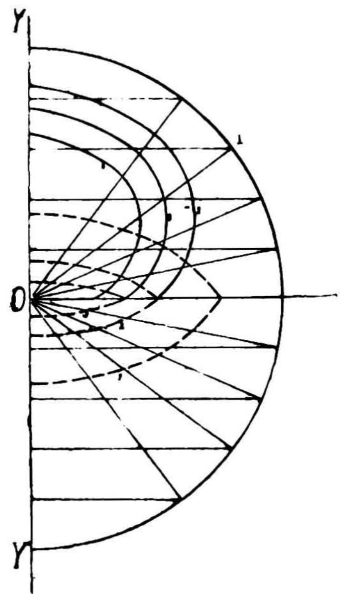

Fig. 7.

Also the molecular scattering causes part of the radiation leaving SS' to return to the surface of the nucleus. This is even an important part for great values of H.

Scattering by refraction and molecular scattering both contribute to an economical consumption of the energy present inside the nuclear surface and radiated to the outside, which is emitted more slowly than would be the case without scattering[1]).

4. *An atmosphere with refractional scattering and molecular scattering.* Finally we raise the question what will be the consequences of irregular refraction in an atmosphere in which also the molecular scattering is thought of importance. For this purpose we again imagine the scattering atmosphere of § 3, but now with regions inside which the density differs appreciably from that of the surroundings, hence regions M and m. If such a region lies near SS', where x is about $= H$, we have to bear in mind that besides the b-radiation, also the a-radiation can now be of importance.

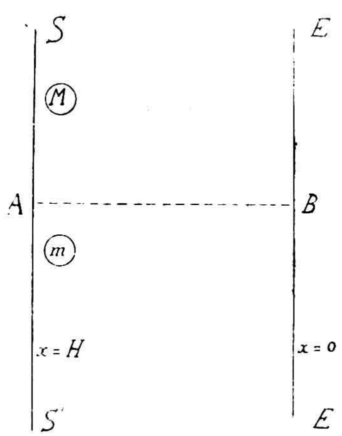

Fig. 8.

It follows from the tables of § 3 that for great values of H the a-radiation does not fall far below the b-radiation, that for smaller values of H the a-radiation may at any rate not be neglected compared with the b-radiation.

If the a-radiation was equal to the b-radiation (also for varying values of i) the scattering by irregular refraction would vanish.

We, therefore, conclude: in an atmosphere with molecular scattering regions where the density varies irregularly and which lie deep in this atmosphere, will be of very small or of less influence

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[1]) It is not impossible that the same question is of importance in the earth's atmosphere for the explanation of the decrease of the temperature of the layers of air further from the earth; the earth's atmosphere is irradiated with parallel radiation; in consequence of this the a-radiation will be of little importance for the incident radiation; for the energy emitted again by the earth, in which radiation of different direction is to be reckoned with, the a-radiation will be more important; besides, the density near the surface of the earth being greater, the plane $\frac{H}{2}$ will lie near the earth's crust; and moreover H is much greater for radiation of great wave-length; hence the process of retardation will have more influence for the outgoing than for the incident radiation. The meaning is, of course, that this question can also play a part.

on the distribution of light over the different directions, and on the intensity of the emitted radiation according as the scattering atmosphere is deeper or less deep (taking deep in the sense that $s \times t$ is great, i. e. that the scattering mass is great; 8 or more may be called great there).

If a region M or m lies at a depth $x = \frac{1}{2}H$, the importance of the a-radiation with respect to that of the b-radiation, gets into the background. The refractional scattering begins to gain influence. Yet even now the a-radiation remains important for great values of H, and the influence of the refraction remains small; for small values of H the a-radiation is still to be taken into account. Regions M or m lying near EE', leave the influence of the refractional scattering undiminished.

And in conclusion I will now consider the question whether, if strong irregular refraction is assumed in the outer layers, the part which the molecular scattering will play, may be considered of minor importance. This may certainly not be assumed for radiation which has not its place in the spectrum in the immediate neighbourhood of an absorbed vibration. For the irregular refraction is not anomalous for such radiation, and the irradiation of the regions M or m, which are situated near EE', is greatly controlled by the molecular scattering; on the regions M or m falls the radiation $b(0, i)$, which in a great degree varies in intensity for other values of i.

But also for radiation that has nearly the same wave-length as an absorption line (absorbent component within M or m), for which the irregular refraction becomes, therefore, anomalous, we cannot say that the molecular scattering does not act its part. For two reasons! For if the same absorbent component is present also in the deeper layers, radiation of a frequency of the immediate neighbourhood of the absorption line will, before it reaches the regions M and m, be already so much weakened in consequence of anomalous molecular scattering that the dispersion band (through molecular scattering) will already be present also in the spectrum of the radiation which must still permeate the regions M and m. And if the regions of irregular refraction arise through the motion of gas masses, it is also possible that in such a region M or m the absorbent component is present in a comparatively great degree, though much less outside it. Then the anomalous molecular scattering would also be considerable in those regions M and m, whereas outside these regions the layer is considered as little scattering.

Bussum, November 1923.