Physics. — *"On the Radiation and the Temperature of the External Photospheric Layers"*. By Dr. J. SPIJKERBOER. (Communicated by Prof. W. H. JULIUS).

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There recently appeared an article [1] "On the Radiation and Temperature of the External Photospheric Layers" by RAGNAR LUNDBLAD. There some conclusions are arrived at with regard to molecular scattering, absorption, and radiation in the outer photospheric layers of the sun, and with regard to the temperature in these layers. The conclusions are decisive and of a far-reaching nature. An introductory sentence as "Starting from the observations on the distribution of the energy over the sun's disk, the optical properties of the photospheric layers and the state of radiation within them are examined as closely as possible with a minimum of a priori assumptions" might, however, possibly lead one to impose too great confidence in the results, at least on a first perusal.

I am of opinion that very serious objections may be raised to the way of treatment of this problem by LUNDBLAD, and to the conclusions communicated by him. It is my intention to discuss the following points in this paper:

1. the differential equation from which LUNDBLAD starts, is that for a *plane* layer, and the same as that of SCHWARZSCHILD; 2. the limiting conditions are put so that it is a priori assumed that if solely molecular scattering causes the change of intensity over the sun's disc, this distribution of intensity must be independent of the wave-length; hence LUNDBLAD's conclusion that the influence of molecular scattering is very small, rests entirely on an arbitrary supposition; 3. no sufficient grounds are advanced to support this supposition.

As the further conclusions of LUNDBLAD are based on the supposition that molecular scattering alone cannot be the cause of the distribution of intensity over the sun's disc, there is the same arbitrariness in these conclusions as in the supposition.

But these conclusions should also be rejected (as I will show in

[1]) R. LUNDBLAD, Astrophysical Journal, 58, 113, 1923.

a fourth paragraph) because, when the question is treated if the sun's atmosphere cannot consist of deeper layers which absorb and scatter, and an outer layer which only scatters, neither the limiting condition to which I referred under 2, may be combined with the supposition of a plane layer mentioned under 1, nor the solution of the integral equations can be considered sufficient. And that when considering absorption alone or absorption with scattering within definite limits LUNDBLAD is led to conclusions which should be entirely eliminated, will also be discussed in the fourth paragraph.

1. In his above cited article LUNDBLAD writes on page 115 "strictly speaking, the quantity ξ is a function of r, because the angle of incidence against the successive layers slowly varies as the beam traverses distances which cannot be neglected in comparison with the radius of the sun. But since we need not take such enormous distances into consideration, we are allowed to treat ξ as a constant".

If one does not want to venture a priori on a supposition on the *depth* of the layers in the sun, which either through molecular scattering, or through absorption, or through their own radiation also exert an influence on the emitted radiation, it is erroneous to treat ξ as a constant in the differential equation of the problem. By not considering ξ as a function of r, the problem which would have to be put without special suppositions for a *spherical shell*, is reduced to that for a *plane layer*.

Accordingly LUNDBLAD's equation is perfectly equivalent to the equations drawn up by SCHWARZSCHILD for a plane layer, and which have also been used by me.

By writing: $J_\lambda(r, \xi) = b(x,i)$; $\alpha = \varkappa$; $\beta = \sigma$; $dr = -dx$; $\xi = \cos i$; $E = E$; $G = \frac{1}{2} A$, LUNDBLAD's equation (1) passes for $\xi > 0$ into the second of SCHWARZSCHILD's equations (3) [1]):

$$\cos i \frac{db}{dx} = (\varkappa + \sigma)\, b - J,$$

whereas, for $\xi < o$, by assuming: $J_\lambda(r, \xi) = a(x, i)$; $\alpha = \varkappa$; $\beta = \sigma$; $dr = -dx$; $+\xi = -\cos i$; $E = E$; $G = \frac{1}{2} A$, the same equation (1) passes into the first of SCHWARZSCHILD's equations (3):

$$\cos i \frac{da}{dx} = -(\varkappa + \sigma)\, a + J,$$

in which $J = \dfrac{\sigma A}{2} + \varkappa E$, and closely related to the function H of LUNDBLAD.

1) K. SCHWARZSCHILD, Sitzungsberichte der Kön. Preuss. Akad. der Wiss., **47**, 1183, 1914.

That LUNDBLAD's function G is equivalent to $^1/_2\,A$ of SCHWARZSCHILD is implied in the transformations introduced above.

For [1])

$$G = \tfrac{1}{2}\int\limits_{-1}^{+1} J_\lambda\,(r,\xi)\,d\xi = \tfrac{1}{2}\int\limits_{-1}^{0} J_\lambda\,(r,\xi)\,d\xi + \tfrac{1}{2}\int\limits_{0}^{+1} J_\lambda\,(r,\xi)\,d\xi =$$

$$= \tfrac{1}{2}\int\limits_{0}^{\frac{\pi}{2}} a\,(x,i)\,sin\,i\,di + \tfrac{1}{2}\int\limits_{0}^{\frac{\pi}{2}} b\,(x,i)\,sin\,i\,di = \tfrac{1}{2}\,A.$$

2. LUNDBLAD's integral equations (7) and (8) (also LUNDBLAD has been obliged to split up the solution for the cases $\xi > 0$ and $\xi < 0$, and has accordingly practically introduced SCHWARZSCHILD's b-radiation and a-radiation later) are not entirely equivalent to SCHWARZSCHILD's integral equations (6) and (7).

This is owing to the fact that LUNDBLAD, before he writes down the solution, makes a *second supposition*, i. e. that the effective optical mass is infinitely great. Consequently the term that indicates in SCHWARZSCHILD what is still present in the emitted radiation of the same direction, of the intensity incident on the effective mass that is not thought infinitely great, after it has penetrated the atmosphere, is not found in LUNDBLAD ($\xi > 0$). If in SCHWARZSCHILD $H = \infty$ is taken, the integral equations are again perfectly equivalent, as appears pretty easily when the above-given transformations are taken into consideration, and in connection with the significance of the "optical mass" introduced by LUNDBLAD (cf. also SCHWARZSCHILD loc. cit. p. 1187).

This second supposition of LUNDBLAD ($\mu = \infty$; loc. cit. p. 117) already includes that if the particular case of a merely scattering, but not absorbing, nor itself radiating layer is considered (LUNDBLAD; loc. cit. p. 126), an intensity *must* be found that has the same ratios for all wave-lengths for the different places on the disc. For then the wave-length plays a part in m and as the optical mass has been put ∞, the layer is *thicker* for the great wave-lengths than for the small wave-lengths.

The distribution of intensity over the disc very certainly *not* being the same for different wave-lengths according to the observations, the second supposition could not but lead to the conclusion that the significance of the molecular scattering must be eliminated.

SCHWARZSCHILD has made the observation (loc. cit. p. 1197) that

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[1]) J. SPIJKERBOER, Arch. néerl., III A, V, 1, 1918.

already for $H = \infty$ (hence for an "optical mass" 8) the distribution of the emitted intensity over the different directions will be equal, except for a few percentages, to the distribution which would be found for the limiting case $H = \infty$.

3. It is clear that this second supposition has a far-reaching influence, particularly in connection with the first premise. For if in a consideration on the phenomena of radiation in the photospheric layers the theory of a plane layer, without corrections, is to be premised, it must be borne in mind that the thickness of the layer is small with respect to the radius of the sun (cf. my thesis for the doctorate); if after this a merely scattering atmosphere is to be treated, for which the optical mass is infinitely great, at any rate greater than 8, this implies that this optical mass must be thought crowded in this layer of relatively slight thickness; if besides the supposition of the infinitely great optical mass for *all* wave-lengths is to be premised, it must be assumed that even for the *least* strongly scattered kinds of radiation, hence for the infra-red, the mass can be so tremendous in that layer of comparatively limited dimension, while for the strongly scattered wave-lengths (violet) it is then, of course, a fortiori infinitely great.

As I think I have shown, on such premises it is utterly useless to examine whether the results of the observations of the distribution of light, as they have been made among others at Mount Wilson, are in concordance with the theoretical results. It is known beforehand that this *cannot* be the case.

4. When LUNDBLAD (loc. cit. p. 128) treats the case that the sun's atmosphere would consist of deeper layers, which absorb (and radiate as a black body) *and* scatter, and an external layer which only scatters, the limiting condition $\mu = \infty$ is certainly as questionable as regards the supposition that the layer is to be considered as plane as in the case considered in paragraph 2.

I may be allowed to refer here to the latter part of my thesis for the doctorate (p. 162—166) [1]), where I have come to the conclusion by different ways that layers of the sun lying *very deep* (roughly calculated lying down to $^1/_{10}$ of the radius of the sun below the photospheric limit) contribute to the total radiation for red and infra-red. And yet there is no question there of an optical mass $= \infty$; it is put no more than 8.

But it seems to me that the treatment of this special case by

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[1]) J. SPIJKERBOER, Proefschrift, Utrecht, 1917; Arch. néerl., III A, V, 108—112, 1918.

LUNDBLAD must besides be considered as incompatible with the scheme of his solution of the integral equations.

For LUNDBLAD puts (loc. cit. p. 117) that his "emissivity-function" H is a polynomium of the N^{th} degree, $H(m) = \sum_{i=0}^{N} a_i m^i$, and says that it follows from the physical meaning of $H(m)$ that H is a continuous function of m with a limited number of maxima and minima. He is of opinion that an approximation will therefore probably prove very good even for a comparatively small number N. Though this is in general correct in my opinion, it should be borne in mind that in the special case considered here for *small* values of m (i. e. in the outer regions) the layer may only be scattering, whereas for *greater* values of m (i. e. in the deeper layers) scattering and absorption must co-operate.

This condition can only be thought to be satisfied when in the polynomium above mentioned the coefficients a of the first terms (of the lower powers of m) chiefly express the influence of the scattering and perhaps in a small degree the influence of the absorption, and if the coefficients of the higher powers of m indicate the influence of absorption and auto-radiation or perhaps are also partly determined by the influence of the scattering.

According to the equations of 9, 14, and 15 of LUNDBLAD (loc. cit. p. 118 and 119) there exists a simple relation between the coefficients a_i of the polynomium under consideration and the coefficients b_i of a second polynomium which indicates what the distribution of intensity is over the sun's disc in its dependence on the place on the disc. It seems impossible to me to come to another conclusion than that in the special case considered in this paragraph where the outer layer is considered to be merely scattering, *less* or *nothing* will be found about the coefficients a which determine the influence of the absorption and auto-radiation according as the optical mass of the outer merely scattering layers remains more or less far below SCHWARZSCHILD's critical value 8 mentioned in paragraph 2. And then also this part of LUNDBLAD's train of reasoning contains, therefore, an arbitrary supposition, viz. that the optical mass in the merely scattering layer must be comparatively *small* (much less than 8).

That from the observations on the distribution of intensity over the disc, as they have been made at Mount Wilson, only the four coefficients of the lower powers of m can be determined, renders the whole treatment for this special case hazardous even apart from this arbitrary supposition.

When LUNDBLAD has calculated, for different wave-lengths, the values of the coefficients of the four powers of m in question, he derives from this the function H and the function G for the same wave-lengths and for different values of m. And from the values of these functions he then concludes to an upper limit for the coefficient of scattering, if the whole atmosphere were absorbing and scattering. As the numerical results for the absorption and scattering coefficients have been obtained only after the cases of scattering alone or scattering alone in the outer layers have been erroneously eliminated, while the calculations of the functions H and G are founded on the supposition of an infinite optical mass and a plane layer, no practical value can be assigned to these numerical values either.

Before long I hope once more to discuss the frequently still incorrect views of the nature of the scattering in extensive gas masses, also in connection with an article by ABBOT, which appeared already earlier and which treats, besides the scattering in the sun's atmosphere the sharp solar limb.

It is not superfluous to point out here that LUNDBLAD does not take the irregular refraction into account at all.

Bussum, February 1924.