

Mathematics. — “*Perfect sets of points with positively-irrational distances*”. By Prof. L. E. J. BROUWER.

(Communicated at the meeting of February 23, 1924).

The following construction of a set of points of the kind mentioned in the title is perhaps somewhat simpler than that which was developed by Prof. WOLFF in these Proceedings Vol. XXVII, p. 95–96.

In the closed unity interval we determine a fundamental series of sets of points s_1, s_2, s_3, \dots , in which each s_n consists of 2^n closed intervals having distances greater than zero from each other, whilst within each interval of s_{n-1} two intervals of s_n are situated, and the distance of two positively-different points of s_n differs positively from each fractional number $\frac{a}{n}$ ($n \leq \nu$). The greatest common divisor $\mathfrak{D}(s_1, s_2, \dots)$ forms a perfect set of points two arbitrary positively-different points of which possess a positively-irrational distance.

In the above fundamental series the possibility of determining s_n when disposing of s_{n-1} follows from the property that in the closed unity interval for each positive integer ν and for each positive ϵ a finite set of closed intervals can be defined approaching each point of the closed unity interval at a distance $< \epsilon$, and two arbitrary positively-different points of which have a distance positively differing from each fractional number $\frac{a}{\nu}$.

This definition can be given as follows: Let m be a positive integer such that the greatest common divisor of m and ν be unity, whilst $\frac{1}{m} \leq \epsilon$, then the distance of two arbitrary points of the finite set of points π_m consisting of the points $\frac{1}{m}, \frac{2}{m}, \dots, \frac{m-1}{m}$, differs at least an amount of $\frac{1}{m\nu}$ from each non-vanishing fractional number $\frac{a}{\nu}$. So, if on each point of π_m as centre we lay a closed interval of length $\frac{1}{2m\nu}$, a finite set of closed intervals arises approaching each point of the closed unity interval at a distance $< \epsilon$, whilst two arbitrary positively-different points of it have a distance differing at least an amount of $\frac{1}{2m\nu}$ from each non-vanishing fractional number $\frac{a}{\nu}$.
