Physics. — "*Termnames of some Multiplets in the Ironspectrum*" [1]).
By S. GOUDSMIT. (Communicated by Prof. P. ZEEMAN).

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F. M. WALTERS [2]) succeeded to find so called "multiplets" [3]) in the complicated ironspectrum. These multiplets show the same structure as the combination between two threefold spectral-terms, e.g. the *p-d* combinations in the spectra of the alkaline earths. Such a group of lines is called multiplet if one of the combining terms is more than threefold.

The structure of a multiplet is governed by the restriction-rules for the "inner" quantumnumbers j, which were first used by SOMMERFELD [4]). The relations between these quantumnumbers and the azimuthal quantumnumbers k, were found by LANDÉ [5]), who also got formulae for the ZEEMANeffects of the spectrallines.

LANDÉ uses the following quantumnumbers:

1st. The azimuthal quantumnumber K, which gives the moment of momentum of the emitting electron. $K=\frac{1}{2}$ for the s-terms, $\frac{3}{2}$ for the p-terms, $\frac{5}{2}$ for the d-terms, etc.

2nd. The quantum R, which gives the moment of momentum of the atom without the emitting electron. $R=\frac{1}{2}$ for all the spectral-

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1) Since the original publication, several articles have been published on the same subject:
O. LAPORTE, ZS. f. Phys. 23, 135 and 26, 1, 1924.
M. A. CATALÁN, Anal. Fis. y. Quim. XXI, 464.
H. GIESELER u. W. GROTRIAN, ZS. f. Phys. 22, 245, 1924.
O. LAPORTE's articles are by far the completest, for the rest the absorption experiments of H. GIESELER and W. GROTRIAN are of great importance.

2) F. M. WALTERS JR. Journ. Wash. Ac. of Sc. **13**, 1923, p. 242.

3) M. A. CATALÁN, Phil. Trans A 223, 1922, p. 125.
H. GIESELER, Ann. der Phys. 69, 1922, p. 147.

4) A. SOMMERFELD, Atombau u. Spektrallinien 3. Aufl., p. 446.

5) A. LANDÉ, ZS. f. Phys. 15, p. 189.

terms of a singlet-termsystem, $\frac{2}{2}$ for a doublet-system, $\frac{3}{2}$ for a triplet-system, etc.

3rd. The quantumnumbers J for a complex-term, which is determined by R and K, are given by the formula

$$|K-R|+\frac{1}{2} \leqq J \leqq |K+R|-\frac{1}{2} \quad . \quad . \quad . \quad . \quad (1)$$

Using the rules of SOMMERFELD and LANDÉ, it is not difficult to determine the values of R, K and J for the terms, which form the iron-multiplets found by WALTERS; this means, that it is possible to determine the termsymbols and the term-system.

The following scheme contains the frequencies $\nu_{vac.}$ and the term-differences of an iron-multiplet, called provisionally XY. Each of the terms X and Y has a certain quantumnumber J. The fact that not all the 25 combinations XY are appearing, is easily explained by the restriction-rule for J, being:

$$J \rightarrow \begin{matrix} J+1 \, . \\ J \quad\;\; . \\ J-1 \, . \end{matrix}$$

J	X_1 (7) $4\frac{1}{2}$	X_2 (6) $3\frac{1}{2}$	X_3 (5) $2\frac{1}{2}$	X_4 (4) $1\frac{1}{2}$	X_5 (3) $\frac{1}{2}$
	415.9	288.1	154.1	89.9	
Y_1 (7) $4\frac{1}{2}$	25900.00	25484.03			
240.2					
Y_2 (6) $3\frac{1}{2}$	26140.19	25724.24	25436.14		
199.5					
Y_3 (5) $2\frac{1}{2}$		25923.77	25635.67	25451.45	
139.7					
Y_4 (4) $1\frac{1}{2}$			25775 35	25591.23	25501.3
71.7					
Y_5 (3) $\frac{1}{2}$				25662.35	

If we give to the X-terms successive quantumnumbers J, chosen quite arbitrarily, increasing, according to LANDÉ's interval-rule, in the direction of the growing term-differences, then by the restriction-rule, these numbers are also determined for the Y-terms.

Suppose e.g. the terms X_1, X_2 etc. have respectively $J = 7,6,5,4,3$, then the terms Y_1, Y_2, etc. will necessarily get the quantumnumbers $J = 7,6,5,4,3$, in order to explain the observed combinations. But than the missing of the combination X_5 Y_5 remains still unexplained.

SOMMERFELD and LANDÉ however gave the condition, that the combination

$$J = \tfrac{1}{2} \rightarrow J = \tfrac{1}{2}$$

is not allowed, as an addition to the restriction-rule for J. Obviously the combination X_5 Y_5 must represent such a case. Thus both X_5 and Y_5 must get the value $J = \frac{1}{2}$. This result enables us to calculate the absolute values of J, being:

$J = 4\frac{1}{2}$ for X_1, $3\frac{1}{2}$ for X_2, $2\frac{1}{2}$ for X_3, $1\frac{1}{2}$ for X_4 and $\frac{1}{2}$ for X_5.

The Y-terms have the same quantumnumbers.

Formula (1) gives us R and K:

$$R = 2\tfrac{1}{2} \quad K = 2\tfrac{1}{2}.$$

Consequently we have here the case of two d-terms of a quintet-system.

Owing to the structure of formula (1), R and K are not always given unambiguously. In such a case it will be sufficient to calculate the ZEEMANeffect for a single line of the multiplet by the rules of LANDÉ and to compare with the experiments. On the other hand the ZEEMANeffects form a confirmation for the whole.

As the same termdifferences, in other words the same complex-terms, appear several times in these ironmultiplets, it is quite possible to derive from the above mentioned multiplet the values of J for all other terms.

It resulted that the 20 iron-multiplets are combinations of 13 different multiple terms of a triplet- and a quintet-termsystem. Two of them are irregular (x and y), probably the connected multiplets were not completely observed [1]).

Some new combination-multiplets of these terms could be found and also a new term [2]).

Table I contains the termsymbols and the termdifferences.

Table II compares the calculated ZEEMANeffects with those indicated by F. M. WALTERS, according to observations of A. S. KING [3]).

Table III shows the observed combinations and

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[1]) According to the ZEEMANeffect of multiplet (12) the x-term is probably partially a D-term.

[2]) The multiplets 1—20 are found by F. M. WALTERS, the others are new.

[3]) A. S. KING, Contr. Mount Wilson, Vol. III, p. 82.

Table IV the frequencies of the multiplets in the usual schematic form.

It deserves attention, that the multiplet-lines given here, coincide, almost totally with the "low-temperature lines" given by A. S. KING [1]), in so far as they are comprised in the spectral-region examined by him.

TABLE I.

	Termname	k	Termdifferences $J = 5^1/_2$	$4^1/_2$	$3^1/_2$	$2^1/_2$	$1^1/_2$	$^1/_2$
Tripletsystem	d′ (P″ ?)	3 (2 ?)			390.6	252.0		
	d″ (P′ ?)	3 (2 ?)			412.9	263.8		
	f′	4		584.7	407.6			
	f″	4		476.5	358 5			
	g′	5	388.4	311.8				
Quintetsystem	D′	3		415.9	288.1	184.1	89.9	
	D″	3		344.0	261.5	173.2	86.6	
	D‴	3		240.2	199.5	139.7	71.1	
	D⁗	3		384.3	272.6	175.2	86.0	
	F′	4	448.5	351.3	257.8	168.9		
	F″	4	344.1	289.2	218.4	144.9		
	F‴	4	292.3	227.9	164.9	106.8		
	y	?	474.9	354.3	244.8			
			$J = 4^1/_2$	$3^1/_2$	$2^1/_2$	$2^1/_2$	$1^1/_2$	
	x	(3 ?)	411.2	294.4	145.4	70.2		

[1]) A. S. KING, Contr. Mount Wilson, Vol. VI, p. 351.

TABLE II.

Combination J	J	ZEEMANeffect Calculated [1])	Observed (multipletnumber)		
f	*f*		18		
$4^1/_2$	$4^1/_2$	$\frac{(0)5}{4}$	$\frac{(0)\ 6}{5}$		
$3^1/_2$	$3^1/_2$	$\frac{(0)13}{12}$	$\frac{(0)\ 12}{11}$		
$2^1/_2$	$2^1/_2$	$\frac{(0)2}{3}$	$\frac{(0)\ 2}{3}$		
f	*g*		19		
$4^1/_2$	$5^1/_2$	$\frac{(\overline{0}\ 1\ 2\ 3\ 4)\ \overline{20}\ 21\ 22\ 23\ 24\ 25\ 26\ 27\ 28}{20}$	$\frac{(0)\ 5}{4}$		
$3^1/_2$	$4^1/_2$	$\frac{(\overline{0}\ 2\ 4\ 6)\ \overline{57}\ 59\ 61\ 63\ 65\ 67\ 69}{60}$	$\frac{(0)\ 7}{6}$		
$2^1/_2$	$3^1/_2$	$\frac{(\overline{0}\ 1\ 2)\ 7\ 8\ 9\ 10\ \overline{11}}{12}$	$\frac{(0)\ 7}{8}$		
D	*D*	Every line $\frac{(0)3}{2}$	Every line	3: $\frac{(0)\ 3}{2}$	16: $\frac{(?)\ 3}{2}$
D	*F*		9	17	12
$4^1/_2$	$5^1/_2$	$\frac{(\overline{0}\ 1\ 2\ 3\ 4)\ \overline{10}\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18}{10}$	$\frac{(0)\ 6}{5?}$	$\frac{(0)\ 5}{4}$	
$4^1/_2$	$4^1/_2$	$\frac{(3\ 6\ 9\ \overline{12})\ 18\ 21\ 24\ \overline{27}\ \overline{30}\ 33\ 36\ 39}{20}$			$\frac{(1)\ 3?}{2}$
$4^1/_2$	$3^1/_2$	$\frac{(\overline{0}\ 1\ 2\ 3)\ 3\ 4\ 5\ 6\ 7\ 8\ \overline{9}}{4}$			(0) 2
$3^1/_2$	$3^1/_2$	$\frac{(1\ 2\ \overline{3})\ 3\ 4\ \overline{5}\ \overline{6}\ 7\ 8}{4}$			$\frac{(2)\ 4}{3}$
$2^1/_2$	$2^1/_2$	$\frac{(1\ 2)\ 1\ 2\ 3\ 4}{2}$	$\frac{(12)\ 1234}{2}$		$\frac{(12)\ 1234}{2}$
$1^1/_2$	$2^1/_2$	$\frac{(0\ 1)\ 1\ 2\ 3\ 4}{2}$	$\frac{(01)\ 12}{2}$		$\frac{(01)\ 123}{2}$
$2^1/_2$	$1^1/_2$	$\frac{(3)\ 0\ 3\ 6}{2}$	$\frac{(03)\ 036}{2}$		$\frac{(3)\ 03}{2}$
$1^1/_2$	$1^1/_2$	$\frac{(3)\ 0\ 3}{2}$	$\frac{(3)\ 3}{2}$	$\frac{(3)\ 03}{2}$	$\frac{(3)\ 03}{2}$
$^1/_2$	$1^1/_2$	0	0	0	0

[1]) The maxima of intensity are overlined.

TABLE II (*Continued*).

Combination J	J	ZEEMANeffect Calculated	Observed (multipletnumber) 10	11
F	F		10	11
$5^1/_2$	$5^1/_2$	$\frac{(0)\ 7}{5}$	$\frac{(0)\ 3}{2}$	$\frac{(0)\ 7}{5}$
$5^1/_2$	$4^1/_2$	$\frac{(\overline{0}\ 1\ 2\ 3\ 4)\ 24\ 25\ 26\ 27\ 28\ 29\ 30\ 31\ \overline{32}}{20}$	$\frac{(0)\ 3}{2}$ $\frac{(0)\ 3}{2}$	$\frac{(?)\ 7}{4}$ $\frac{(0)\ 8}{5}$
$4^1/_2$	$4^1/_2$	$\frac{(0)\ 27}{20}$	$\frac{(0)\ 3}{2}$	$\frac{(?)\ 4}{3}$
$4^1/_2$	$3^1/_2$	$\frac{(\overline{0}\ 2\ 4\ 6)\ 21\ \ 23\ 25\ 27\ 29\ 31\ \overline{33}}{20}$	$\frac{(0)\ 3}{2}$ $\frac{(0)\ 3}{2}$	$\frac{(0)\ 3}{2}$ $\frac{(?)\ 03}{2}$
$3^1/_2$	$3^1/_2$	$\frac{(0)\ 5}{4}$	$\frac{(0)\ 5}{4}$	$\frac{(?)\ 6}{4}$
$3^1/_2$	$2^1/_2$	$\frac{(\overline{0}\ 1\ 2)\ 3\ 4\ 5\ 6\ \overline{7}}{4}$	$\frac{(?)\ 3}{2}$	$\frac{(?)\ 8}{5}$ $\frac{(0)\ 4}{3}$
$2^1/_2$	$2^1/_2$	$\frac{(0)\ 1}{1}$	$\frac{(0)\ 1}{1}$	$\frac{(?)\ 1}{1}$
$2^1/_2$	$1^1/_2$	$\frac{(0\ 1)\ \ 0\ 1\ 2}{1}$	$\frac{(0\ 1) 0\ 1\ 2}{1}$	
$1^1/_2$	$1^1/_2$	0	0	0

TABLE III

Observed combination multiplets [1])			
1 D'—d'	7 F'—d'		
2 D'—D''	8 F'—D''		
3 D'—D'''	9 F'—D'''	16 D'''—D^{IV}	
4 D'—F''	10 F'—F''		24 f'—F''
5 D'—F'''	11 F'—F'''	17 F'''—D^{IV}	
6 D'—x	12 F'—x		
	13 F'—f''		18 f'—f''
22 D'—g'	14 F'—g'		19 f'—g'
21 D'—y	15 F'—y		20 f'—y
24 D'—d'' (P' ?)			

[1]) The combinations D'—f'', f'—d' and f'—D'' seem to be also present in the iron-spectrum.

TABLE IV.

1	D'_1 4½	415.9	D'_2 3½	288.1	D'_3 2½	184.1	D'_4 1½	89.9	D'_5 ½
d'_1 3½	36766.84		36351.00		36062.94				
390.6									
d'_2 2½			36741.55		36453.53		36269.68		
252.0									
d'_3 1½					36705.50		36521.39		36431.44

2	D'_1 4½	D'_2 3½	D'_3 2½	D'_4 1½	D'_5 ½
D''_1 4½	39625.63	39209.92			
344.0					
D''_2 3½	39969.72	39553.82	39265.71		
261.5					
D''_3 2½		39815.25	39527.20	39343.20	
173.2					
D''_4 1½			39700.36	39516.46	39426.34
86.6					
D''_5 1½				39602.98	

3	D'_1 4½	D'_2 3½	D'_3 2½	D'_4 1½	D'_5 ½
D'''_1 4½	25900.00	25484.03			
240.2					
D'''_2 3½	26140.19	25724.24	25436.14		
199.5					
D'''_3 2½		25923.77	25635.67	25451.45	
139.7					
D'''_4 1½			25775.35	25591.23	25501.35
71.1					
D'''_5 ½				25162.35	

4	D'_1 4½	D'_2 3½	D'_3 2½	D'_4 1½	D'_5 ½
F''_1 5½	33695.37				
344.1					
F''_2 4½	34039.53	33624.72			
289.2					
F''_3 3½	34328.72	33912.84	33623.60		
218.4					
F''_4 2½		34131.31	33843.18	33659.02	
144.9					
F''_5 1½			33983.12	33803.97	33714.06

TABLE IV (*Continued*).

5	D'_1 4½	D'_2 3½	D'_3 2½	D'_4 1½	D'_5 ½
F'''_1 5½	26874.53				
292.3					
F'''_2 4½	27166.82	26750.88			
227.9					
F'''_3 3½	27394.67	26978.76	26690.69		
164.9					
F'''_4 2½		27143.66	26855.57	26671.45	
106.8					
F'''_5 1½			26962.43	26778.22	26688.31

6	D'_1 4½	D'_2 3½	D'_3 2½	D'_4 1½	D'_5 ½
x_1 4½	33095.93	32679.98			
411.2					
x_2 3½	33507.13	33091.17	32803.10		
294.4					
x_3 2½		33385.54	32097.53	32913.43	
145.4					
x_4 2½		33530.98	33242.94	33058.78	
70.2					
x_5 1½			33313.08	33128.96	33039.01

7	F'_1 5½ 488.5	F'_2 4½ 351.3	F'_3 3½ 257.8	F'_4 2½ 168.9	F'_5 1½
d'_1 3½		29390.18	29038.87	28781.22	
390.6					
d'_2 2½			29429.48	29171.76	29002.73
252.0					
d'_3 1½				29423.76	29254.81

8	F'_1 5½	F'_2 4½	F'_3 3½	F'_4 2½	F'_5 1½
D''_1 4½	32697.50	32249.05	31897.78		
344.0					
D''_2 3½		32593.04	32241.78	31983.99	
261.5					
D''_3 2½			32503.25	32245.56	32076.65
173.2					
D''_4 1½				32418.71	32249.70
86.6					
D''_5 ½					32336.56

TABLE IV (*Continued*).

9		F'_1 5½	F'_2 4½	F'_3 3½	F'_4 2½	F'_5 1½
D'''_1	4½	18981.72	18523.21	18171.92		
240.2						
D'''_2	3½		18763.41	18412.11	18154.40	
199.5						
D'''_3	2½			18611.62	18353.90	18184.98
139.7						
D'''_4	1⅓				18492.59	18353.66
71.1						
D'''_5	½					18395.76

10		F'_1 5⅓	F'_2 4½	F'_3 3½	F'_4 2½	F'_5 1⅓
F''_1	5½	26767.12	26318.65			
344.1						
F''_2	4½	27111.27	26662.76	26311.47	26053.71	
289.2						
F''_3	3½		26952.00	26600.71	26342.98	
218.4						
F''_4	2½			26819.15	26561.42	26392.53
144.9						
F''_5	1½				26705.35	26537.43

11		F'_1 5½	F'_2 4½	F'_3 3½	F'_4 2½	F'_5 1½
F'''_1	5½	19946.28	19497.77			
292.3						
F'''_2	4½	20238.55	19790.04	19438.75		
227.9						
F'''_3	3½		20017.93	19657.61	19408.90	
164.9						
F'''_4	2½			19831.51	19573.77	19403.86
106.8						
F'''_5	1½				19680.55	19511.63

12		F'_1 5½	F'_2 4½	F'_3 3½	F'_4 2½	F'_5 1½
x_1	4½	26167.66	25719.16	25367.84		
411.2						
x_2	3½		26130.35	25779.05	25521.31	
294.4						
x_3	2 2/2			26073.50	25815.77	25646.86
145.4						
x_4	2½			26218.89	25961.16	25792.20
70.2						
x	1½				26031.30	25863.38

TABLE IV (*Continued*).

13	F'_1 5½	F'_2 4½	F'_3 3½	F'_4 2½	F'_5 1½
f''_1 4½	29757.84	29309.36	28958.11		
476.5					
f''_2 3½		29785.88	29434.64	29176.87	
358.5					
f''_3 2½			20793 13	29535.23	29366.20

14	F'_1 5½	F'_2 4½	F'_3 3½	F'_4 2½	F'_5 1½
g'_1 5½	28450.93	28002.43			
388.4					
g'_2 4½	28839.29	28390.79	28039.50		
311.8					
g'_3 3½		27702.58	28351.32	28003.55	

15	F'_1 5½	F'_2 4½	F'_3 3½	F'_4 2½	F'_5 1½
y_1 5½	27854.20	27405 65			
474.9					
y_2 4½	(?)	27880.57	29529.29		
354.3					
y_3 3½		28234.85	27883.56	27625.84	
244.8					
y_4 2½			28128.34	27879.61	27701.70

16	D'''_1 4½ 240.2	D'''_2 3½ 199.5	D'''_3 2½ 139.7	D'''_4 1½ 71.1	D'''_5 ½
D^{IV}_1 4½	18776.96	18536.78			
384.3					
D^{IV}_2 3½	19161.28	18921.12	18721.57		
272 6					
D^{IV}_3 2½		19193.65	18994.15	18854.45	
175.2					
D^{IV}_4 1½			19169.41	19029.71	18958.64
86.0					
D^{IV}_5 ½				19115.72	

TABLE IV (*Continued*).

17	F'''_1 5½	292.3 F'''_2 4½	227.9 F'''_3 3½	164.9 F'''_4 2½	106.8 F'''_5 1½
D^{IV}_1 4½	17862.37	17510.15	17282.23		
384.3					
D^{IV}_2 3½		17894.46	17666.58	17501.71	
272.6					
D^{IV}_3 2½			17939.15	17774.23	17667.50
175.2					
D^{IV}_4 1½				17949.53	17842.76
86.0					
D^{IV}_5 ½					17928.69
18	f'_1 4½	584.7 f'_2 3½	407.6 f'_3 2½		
f''_1 4½	24709.51	24125.21			
476.5					
f''_2 3½	25186.48	24601.78	24194.19		
358.5					
f''_3 2½		24950.20	24552.57		
19	f'_1 4½	f'_2 3½	f'_3 2½		
g'_1 5½	23402.96				
388.4					
g'_2 4½	23791.33	23206.58			
311.8					
g'_3 3½	24103.11	23518.41	23110.76		
20	f'_1 4⅓	f'_2 3½	f'_3 2½		
y_1 5½	22806.18				
474.9					
y_2 4½	23281.05	22696.40			
354.3					
y_3 3½	23635.39	23050.64	22643.06		
244.8					
y_4 2½		23295.48	22887.83		

TABLE IV (*Continued*).

21	D'_1 $4\frac{1}{2}$	415.9	D'_2 $3\frac{1}{2}$	288.1	D'_3 $2\frac{1}{2}$	184.1	D'_4 $1\frac{1}{2}$	89.9	D'_5 $\frac{1}{2}$
y_1 $5\frac{1}{2}$	34782.2								
474.9									
y_2 $4\frac{1}{2}$	35257.1		34841.2						
354.3									
y_3 $3\frac{1}{2}$	35611.4		35195.5		34907.5				
244.8									
y_4 $2\frac{1}{2}$			35440.3		35152.3		34968.2		

22	D'_1 $4\frac{1}{2}$	D'_2 $3\frac{1}{2}$	D'_3 $2\frac{1}{2}$	D'_4 $1\frac{1}{2}$	D'_5 $\frac{1}{2}$
g'_1 $5\frac{1}{2}$	35379.0				
388.4					
g'_2 $4\frac{1}{2}$	35767.4	35351.6			
311.8					
g'_3 $3\frac{1}{2}$	36079.3	35663.4	35375.4		

23	f'_1 $4\frac{1}{2}$	f'_2 $3\frac{1}{2}$	f'_3 $2\frac{1}{2}$		
F''_1 $5\frac{1}{2}$	21719.2				
344.1					
F''_2 $4\frac{1}{2}$	22063.3	21478.6			
289.2					
F''_3 $3\frac{1}{2}$	22352.5	21767.8	21360.2		
218.4					
F''_4 $2\frac{1}{2}$		21986.2	21578.6		
144.9					
F''_5 $1\frac{1}{2}$			21723.5		

24	D'_1 $4\frac{1}{2}$	D'_2 $3\frac{1}{2}$	D'_3 $2\frac{1}{2}$	D'_4 $1\frac{1}{2}$	D'_5 $\frac{1}{2}$
d''_1? $3\frac{1}{2}$	29057.2	28641.56	28353.35		
412.9					
d''_2? $2\frac{1}{2}$		29054.22	28766.22	28582.12	
263.8					
d''_3? $1\frac{1}{2}$			29029.75	28845.80	28755.79