Mathematics. — *"The Triple Involution of Reye"*. By Prof. JAN DE VRIES.

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1. The congruence of the twisted cubics through five points B_k defines in an arbitrary plane V the triples of an involution that has been examined by REYE [1]). The tangent c at B_5 to the curve ϱ^3 which cuts V in a triple C_1, C_2, C_3, meets V in the point C, which we consider as the *image* of the group (C). For this representation the intersections S_{k5} of the lines $B_5 B_k$ and V are *singular*; for $B_5 B_k$ forms a degenerate ϱ^3 with any conic ϱ^2 of the pencil in the plane B_h B_l B_m that has the points B_h, B_l, B_m and the passage of $B_5 B_n$ as base points. Accordingly the point S_{k5} is the image of all the triples formed by S_{k5} and the pairs of the involution which are the intersections of these conics and the line $b_{hlm} \equiv S_{hl}\, S_{hm}\, S_{lm}$.

2. If C describes the straight line l, the curves ϱ^3 that touch the plane $B_5 l$ at B_5, describe a surface Λ that has only components of degenerate figures in common with the plane $B_1 B_2 B_3$. Through B_3, B_4, B_5 there passes a ϱ^2 that touches the plane $B_5 l$ at B_5 and cuts $B_1 B_2$. In accordance with this the intersection of Λ and the plane $B_1 B_2 B_3$ consists of the straight lines $B_1 B_2, B_1 B_3$, and $B_2 B_3$, and Λ is a quadrinodal cubic surface through the edges of the tetrahedron $B_1 B_2 B_3 B_4$ and the point B_5. Accordingly, as Λ is defined by two points C, hence by two triples (C):

"Any two triples of the involution of triples define with the passages D_{kl} of the edges of $B_1 B_2 B_3 B_4$ a cubic λ^3 that contains an infinite number of other triples".

The curves λ^3 form a net that has the angular points D_{kl} of a complete quadrilateral as base points. To this net belongs the configuration consisting of the line $D_{12} D_{34}$ and a definite conic λ^2. The corresponding surface Λ consists of the curves λ^3 that rest on $D_{12} D_{34}$ [2]).

3. If we define Λ^3 by a point on the line b_{123} and another point chosen at random in V, Λ^3 consists of the plane $B_1 B_2 B_3$ and a quadratic cone with vertex B_4. For the straight line l passes through the point S_{45}, so that Λ^3 contains all the conics ϱ^2 in B_1 B_2 B_3. Disregarding the groups

[1]) Geometrie der Lage, 3e Auflage, p. 225.

General considerations of triple-involutions in the plane are found in my paper on "cubic involutions". (These Proceedings **16**, p. 974—987).

[2]) The curves ρ^3 which cut an arbitrary straight line, form a surface of the 5th order with triple points in B_k. Here it is replaced by Λ^3 and the planes $B_1 B_2 B_5$, $B_3 B_4 B_5$.

(C) consisting of S_{45} and the pairs of the involution that lies on b_{123}, the point range on l is the image of an infinite number of triples lying on a ϱ^2.

"Any conic through the points D_{14}, D_{24}, D_{34}, S_{45} contains an infinite number of groups of the involution of triples."

As any of the five points B can be the vertex of the sheaf that produces the representation, there are *ten* pencils (λ^2) [1]).

4. If the image point C describes a conic γ^2, the corresponding curves ϱ^3 describe a surface Γ^6 with quadruple points B_1, B_2, B_3, B_4, double point B_5 and double lines in the edges of $B_1 B_2 B_3 B_4$. For the plane $B_3 B_4 B_5$ contains two conics ϱ^2 which touch a generatrix of the cone that projects λ^2 out of B_5; hence $B_1 B_2$ is a double line of Γ etc.

The intersection of Γ and V is a curve γ^6 with 6 double points D_{kl}, which contains an infinite number of triples (C).

If γ^2 passes through S_{45}, Γ contains the figures of which $B_4 B_5$ is a component part and we find a curve γ^5 with three double points, which contains an infinite number of triples. Etc.

Finally any γ^2 through the four singular points S_{k5} contains an infinite number of triples each of which has a point of γ^2 as image.

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[1]) Cf. W. VAN DER WOUDE, *The Cubic Involution of the First Rank in the Plane.* (These Proceedings **12**, p. 751).