Chemistry. — Equilibria in systems in which phases, separated by a semi-permeable membrane. XIX. By F. A. H. SCHREINEMAKERS.

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Influence of the pressure on osmotic systems.

Of the many cases, which may occur, if we change the pressure of an osmotic system in which also occur solid phases, we only shall discuss some cases. If we assume that the saturation curve wv and the isotonic curves wm and an of fig. 1 (communication III) are valid for the pressure P then we have a.o. the osmotic equilibria:

$$[Y+L_w \mid L_e]_P \qquad [Y+L_a \mid L_n]_P \qquad \text{fig. 1 III } . . . (1)$$

We now bring the pressure on the right side of the membrane on the higher pressure P_1 . As, like we have seen in the previous communication, then the O.W.A. of the liquid at the right side of the membrane decreases, we get the osmotic systems:

$$(Y+L_w)_P \stackrel{\downarrow}{\leftarrow} (L_e)_{P_1} \qquad (Y+L_a)_P \stackrel{\downarrow}{\leftarrow} (L_n)_{P_1} \quad . \quad . \quad . \quad (2)$$

in which the water diffuses in the direction of the arrows. We now assume that a sufficient quantity of Y is present, so that the system at the left side of the membrane remains. Then the systems (2) pass into the osmotic equilibria:

$$(Y + L_w)_P \mid (L'_e)_{P_1} \qquad (Y + L'_a)_P \mid (L'_n)_{P_1} \qquad . \qquad . \qquad . \qquad (3)$$

With the isotonic curve of the pressure P is conjugated an isotonic curve of the pressure P_1 , which must be situated further from the point W than curve wm; we imagine this to be represented by curve bo. (fig. 1. III). Consequently liquid L'_e will be represented by the point of intersection l of the line We with this curve bo. The first of the osmotic equilibria (3) becomes, therefore:

$$(Y+L_w)_P \downarrow (L_l)_{P_1}$$
 fig. 1 III

The result of increase of pressure is, therefore, that from liquid L_e so much water diffuses to the left, till it passes into the liquid L_l .

The composition of the liquids L'_a and L'_n in the second of the systems (3) depends on the ratio of the phases. However, it is sure that L'_a will be represented by a point of the saturationcurve between w and a or by the point w itself. In the latter case L'_n is represented by point o; the second of the osmotic equilibria (3) becomes then:

$$(Y + L_w)_{P_1} | (L_o)_{P_1}$$
 fig. 1 III

If, however, L'_a is represented by a point a' between w and a, then L'_n will be represented by a point n' on the side WX between o and X. Those points a' and n', however, are situated not arbitrarily with respect to one another, but the isotonic curve of the pressure P going through point a' and the isotonic curve of the pressure P_1 going through point n' must be conjugated with one another.

We imagine in fig. 1 III two points q and r anywhere within the region wmXv; as the O.W.A. of each of those liquids L_q and L_r is larger than that of the solid substance Y under the same pressure, consequently an osmotic complex

$$[L_q \mid Y \mid L_r]_P \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

exists, in which no diffusion of water occurs. [For systems with two membranes, compare Comm. X]. We are able now, by bringing the pressure of the left liquid to a definite value P_1 and that of the right liquid to a definite value P_2 , to make the O.W.A. of both liquids equal to that of the solid substance Y under the pressure P. Then the osmotic complex (4) passes into an osmotic equilibrium, which, if we raise the pressure of both liquids still with $\triangle P$, passes into the osmotic system:

in which the water diffuses from left and right towards the solid substance Y. The double-membrane $\begin{vmatrix} Y \end{vmatrix}$ in system (4) impermeable for water, becomes permeable, therefore, if we raise the pressure of both liquids sufficiently only.

If a sufficient quantity of solid Y is present, then between the two membranes the system $Y + L_w$ is formed, and the right and the left liquid lose water as long as their O.W.A. becomes equal to that of the liquid L_w under the pressure P.

We take under the pressure P an equilibrium

$$E = (F + L)_P \quad \dots \quad (6)$$

in which F is an arbitrary liquid. We now bring the pressure to P + dP; keeping the total composition of this system E constant; then we get the new system:

$$E' = (F + L')_{P+dP}$$
 (7)

in which the liquid L' differs a little in composition from L; this difference in composition is completely defined by the change in pressure dP. With the transition of system E into E' the O.W.A. changes, as well on account of the change in pressure itself, as on account of the

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change of the composition of the liquid; both influence may intensify or debilitate each other. In the previous communication we have seen that this total change $d\xi$ of the O.W.A. is defined by:

if viz. $\triangle V_E$. δw is the change in volume, which occurs in the system E under the constant pressure P, if we add δw quantities of water. As $\triangle V_E$, like we will see further, can be positive, negative and also zero, it follows from (8):

the O.W.A. of an equilibrium E = F + L of constant composition decreases or increases at increase of pressure, according as ΔV_E is positive or negative; if $\Delta V_E = 0$, then the O.W.A. does not change with a small increase of pressure.

We will determinate for some cases this change of the O.W.A. Let us firstly take the equilibrium:

$$E = (Y + L_w)_P$$
 fig. 1 III (9)

On increase of pressure this passes into the equilibrium:

$$E' = (Y + L_w)_{P+dP}$$

in which L'_w is represented by a point w' which we must imagine in fig. 1 III in the vicinity of point w on the side WY. In order to apply formula (8) we have to determinate $\triangle V_E$ viz. the change in volume which occurs in system E (consequently under the constant pressure P) on addition of water. If we represent the composition of L_w , which remains unchanged on addition of water to system E, by $y \operatorname{Mol} Y + (1-y) \operatorname{Mol} W$, then, on addition of water the reaction:

$$(1-y)$$
 Mol $W + y$ Mol $Y \rightarrow 1$ Mol L_w

occurs. If we represent the molecular volumina of L_w and the solid substance Y by V and V_y , then we find for $\triangle V_E$ and also with the aid of (8) for the change of the O.W.A.:

As we have seen in the previous communication, we can deduce the change of the O.W.A. yet also in another way. The O.W.A. of the liquid L_w of equilibrium E is viz. defined by:

For the O. W. A. : $\xi + d\xi$ of liquid L'_w of equilibrium E is valid then :

$$d\xi = \left(-V + y \frac{\partial V}{\partial y}\right) dP + yt \, dy \quad . \quad . \quad . \quad . \quad (12)$$

in which, however, dy is defined by dP. For the equilibrium E is true viz:

from which follows for the equilibrium E':

$$\left[V + (1-y)\frac{\partial V}{\partial y} - V_y\right]dP + (1-y)t\,dy = 0 \quad . \quad . \quad (14)$$

If we substitute this value of dy in (13) then we find (10^b) .

As 1-y is always positive, the sign of $\triangle V_E$ is defined by the sign of the numerator of (10^a) . If $V > V_y$ then the numerator is always positive; if $V < V_y$ then the sign depends on the value of y. With values of y, which do not differ much from 1, the numerator shall be negative then. We may say, therefore, that in general $\triangle V_E$ shall be positive, unless L_w should be a concentrated solution, which is formed from its components with contraction of volume.

In order to give another form to $riangle V_E$ in (10^a) we put:

$$V + (1-y)\frac{\partial V}{\partial y} - V_y = \triangle V_y \qquad V - y\frac{\partial V}{\partial y} = \triangle V_w \quad . \quad (15)$$

so that $riangle V_y$. δw represents the change in volume of the system E, when δw quantities of solid Y dissolve in the liquid; $riangle V_w$. δw is the change in volume of the liquid, if this takes in δw quantities of water. We now can write for (10^a):

Of the many considerations to which this relation can give rise, we only take the case $\triangle Vy = 0$. Instead of (10^a) and (10^b) we then get

$$\Delta V_E = \Delta V_W \quad . \quad . \quad (17^a) \qquad d\xi = -\Delta V_W \cdot dP \quad . \quad . \quad (17^b)$$

The change of the O.W.A. of system E on change of the pressure is then the same as if no solid Y but only the liquid L_w is present. This also follows from (14); if herein $\triangle V_y = 0$, then follows dy = 0, so that the composition of the liquid L_w remains constant with a change of pressure dP; consequently the quantity of solid Y remains unchanged also.

As the O.W.A. of the solid substance Y under the pressure P is equal to that of the liquid L_w of system E, and under the pressure P + dP equal to that of the liquid L'_w of the system E', (10^b), therefore, defines also the change of the O.W.A. on a change of pressure of the solid substance Y.

Of course similar considerations are valid also for the change of the O.W.A. on a change of pressure of the equilibria:

$$H+L_{w}$$
 $H+L_{v}$ fig. 1 IV
 $D+L_{w}$ $D+L_{v}$ fig. 2 IV $X+L_{b}$ fig. 1 V
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and consequently also for the change of the O.W.A. on a change of pressure of the solid substances H, D and X.

Hence follows, therefore:

the O.W.A. of a solid substance F can as well increase as decrease on increase of pressure; this depends on the $\triangle V_E$ of the system E = F + L, in which L is the liquid, which is formed from the solid substance F by taking in (losing) water.

We now take under the pressure P the equilibrium:

$$E = (Y + L_a)_P$$
 fig. 1 III (18)

and we bring this, while we are keeping constant the total composition, to the pressure P + dP; we then get an equilibrium:

$$E' = (Y + L_a)_{P+dP}$$
 (19)

in which L'_a is represented by a point a', which we must imagine in fig. 1 III in the vicinity of a on the line Ya (within or out of the region Ywv). In order to apply formula (8) we have to define ΔV_E , viz. the change in volume, which occurs in system (18) on addition of δw quantities of water. However, with this, the composition of the liquid L_a does not remain constant, as it was the case with liquid L_w in (9); L_a viz. changes its composition along curve wv starting from a in the direction towards w.

In order to symplify the calculations, we express the composition of the liquid L_a in W, Y and an arbitrary phase Q. We write its composition :

x quant. W + y quant. Y + (1 - x - y) quant. Q . . (20) so that we have a system of coordinates with QW as X-axis and QYas Y-axis. If the equilibrium E consists of n quantities of L_a and mquantities of Y, then we have:

$$\Delta V_E \cdot \delta w = d \left(nV + mV_y \right) = V \, dn + n \, \frac{\partial V}{\partial x} \, dx + n \, \frac{\partial V}{\partial y} \, dy + V_y \, dm \quad (21)$$

in which V represents the molecular-volume of the liquid L_a ; dx and dy are the changes in concentration, which occur in the liquid L_a , by the addition of δw quantities of water. As the quantity of water increases with δw , but the quantity of Y and Q rests unchanged, it follows from this:

$$\delta w = d (n x) = x dn + n dx \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$0 = d(ny + m) = y dn + n dy + dm (23)$$

$$0 = dn (1 - x - y) = dn + dm - \delta w (24)$$

in which (24) is simplified with the aid of (22) and (23). As the liquid L_a is in equilibrium with solid Y, the equation:

$$\zeta - x \frac{\partial \zeta}{\partial x} + (1 - y) \frac{\partial \zeta}{\partial y} = \zeta_y \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

is valid.

Hence follows:

[-rx + (1-y)s] dx + [-xs + (1-y)t] dy = 0. . . (26)

Eliminating dn, dm, dx and dy from the equations (21)-(24) and (26) we find the value of $\triangle V_E$. This elimination becomes more simple if we let coincide the arbitrary phase Q in (20) with the point *a* in fig. 1 III; then aW is the X-axis and aY the Y-axis of the system of coordinates; we now have x=0 and y=0. Putting those values in (22)-(24) and in (26) and eliminating dn, dm, dx and dy then we find after dividing by δw :

$$\Delta V_E = V + \frac{\partial V}{\partial x} + \frac{s}{t} \left(V_y - V - \frac{\partial V}{\partial y} \right) \dots \dots \dots \dots (27)$$

so that the change of the O.W.A. is known with the aid of (8).

We are able to deduce this more plainly in the following way. We represent again the composition of the liquid L_a as in (20); its O.W.A. is then defined by

$$\xi = -\zeta - (1-x)\frac{\partial\zeta}{\partial y} + y\frac{\partial\zeta}{\partial y}$$

For an arbitrary liquid, which differs infinitely little from L_a then is under the pressure P + dP:

$$d\xi = -\left[V + (1-x)\frac{\partial V}{\partial x} - y\frac{\partial V}{\partial y}\right]dP + \left[-(1-x)r + ys\right]dx + \left[-(1-x)s + yt\right]dy$$
(28)

If this new substance under the pressure P + dP is always in equilibrium with solid Y, then follows from (25) that dx and dy must satisfy an equation, which we get by adding to (26) the term:

$$\left[V-x\frac{\partial V}{\partial x}+(1-y)\frac{\partial V}{\partial y}-V_{y}\right]dP \quad . \quad . \quad . \quad (29)$$

We call this (26^{a}) . Just as above we now let coincide the arbitrary phase Q with the point a. As then x and y become zero, (28) and (26^{a}) pass into:

$$d\xi = -\left[V + \frac{\partial V}{dx}\right] dP - r \, dx - s \, dy \quad . \quad . \quad . \quad (30)$$

$$0 = -\left[V + \frac{\partial V}{\partial y} - V_y\right] dP + s \, dx + t \, dy \quad . \quad . \quad (31)$$

We now must define dx and dy in such a way that liquid L_a of equilibrium (18) passes into L'_a of (19); as, like we have noted already above L'_a must be situated on the line aY, consequently dx = 0. If we substitute this in (30) and (31) and eliminating then dy we find:

$$d\xi = -\left[V + \frac{\partial V}{\partial x} + \frac{s}{t}\left(V_y - V - \frac{\partial V}{\partial y}\right)\right]dP \quad . \quad . \quad (32)$$

Consequently the same value for $d\xi$ as follows from (27).

In a similar way we can define the change of the O.W.A. in the systems:

$$H+L$$
 fig. 1. IV $D+L$ fig. 2. IV

and other systems in which a liquid, saturated with a solid substance.

We now take a liquid, saturated with two solid substances, viz. the equilibrium:

$$E = (F + F_1 + L)_P$$
 (33)

If we bring this, while its composition remains constant, under the pressure P + dP, then the liquid L gets a definite change in composition. If we take as simple example for E a ternary system, f.i. one of the equilibria:

$$\begin{array}{cccc} X+Y+L_c & \text{fig. 1. V} & Y+D+L_c & \text{fig. 3, 4. V} \\ X+D+L_d & \text{fig. 3, 4. V} & & & & \\ \end{array}$$
(33^a)

then, on addition of water, the liquid does not change its composition, so that a phases-reaction occurs. If we represent the composition of the liquid L_c in the first of the equilibria mentioned above, by:

$$x Mol X + y Mol Y + (1 - x - y) Mol W$$
. . . . (34)

then on addition of water the reaction:

$$(1-x-y)$$
 Mol $W + x$ Mol $X + y$ Mol $Y \rightarrow 1$ Mol L_c

occurs. Hence now follows:

so that the change of the O.W.A. is defined. Corresponding considerations as for (10^a) are valid for this value of $\triangle V_E$.

If we take the system:

$$E = (Y + H + L_d)_P$$
 fig. 2. V (36)

then, on addition of water, also herein a phases-reaction occurs in which, however, the liquid L_d does not participate. If we represent the composition of the hydrate H by: β Mol $Y + (1-\beta)$ Mol W, then this reaction is:

$$(1-\beta) \operatorname{Mol} W + \beta \operatorname{Mol} Y \rightarrow 1 \operatorname{Mol} H$$
. . . . (37)

Hence follows:

$$\Delta V_E = \frac{V_H - \beta V_Y}{1 - \beta} \qquad d\xi = -\frac{V_H - \beta V_Y}{1 - \beta} \cdot dP \quad . \quad . \quad (38)$$

by which the influence of a change in pressure on the O.W.A. of system (36) is defined.

We also can deduce this again in the following way. If we bring the pressure of (36) to P + dP then arises the equilibrium:

$$E' = (Y + H + L_d)_{P+dP}$$
 (39)

in which L'_d is represented by a point d', which we must imagine in fig. 2 V in the vicinity of point d. If we represent the composition of the liquid L_d by (34), then its O.W.A. is defined by:

$$\xi = -\zeta + x \frac{\partial \zeta}{\partial x} + y \frac{\partial \zeta}{\partial y} \quad \dots \quad \dots \quad \dots \quad (40)$$

As the liquid L_d is saturated with Y + H, are valid the equations:

$$\zeta - x \frac{\partial \zeta}{\partial x} + (1 - y) \frac{\partial \zeta}{\partial y} = \zeta_Y \qquad \zeta - x \frac{\partial \zeta}{\partial x} + (\beta - y) \frac{\partial \zeta}{\partial y} = \zeta_H \quad (41)$$

which we write, with the aid of (40) in the form:

$$\xi = \frac{\partial \zeta}{\partial y} - \zeta_Y \qquad \qquad \xi = \beta \frac{\partial \zeta}{\partial y} - \zeta_H \dots \dots \dots (42)$$

For the liquid L'_d of equilibrium E' under the pressure P + dP is valid then:

$$d\xi = \left(\frac{\partial V}{\partial y} - V_Y\right) dP + s \, dx + t \, dy$$
$$d\xi = \left(\beta \frac{\partial V}{\partial y} - V_H\right) dP + \beta \left(s \, dx + t \, dy\right)$$

from which follows by elimination of s dx + t dy:

which is in accordance with (38).

As the systems (33^a) and (39) define also the O.W.A. of the solid complexes X + Y, Y + D, X + D and Y + H, we also know, therefore, the change of the O.W.A. of those solid complexes, if we bring their pressure from P to P + dP.

We now take under the pressure P an equilibrium:

with a vapour G and a liquid L. In order to make the calculation not too extensive, we assume that E is a binary system. We represent the composition of the vapour by $x \operatorname{Mol} X + (1-x) \operatorname{Mol} W$ and that of the liquid by $x_1 \operatorname{Mol} X + (1-x_1) \operatorname{Mol} W$.

As the phases of this binary equilibrium have a constant composition at constant T and P, on addition of water, according as $x_1 - x$ or $x - x_1$ is positive, occurs one of the reactions:

$$1 Mol W + \frac{x}{x_1 - x} Mol L \rightarrow \frac{x_1}{x_1 - x} Mol G$$
$$1 Mol W + \frac{x_1}{x - x_1} Mol G \rightarrow \frac{x}{x - x_1} Mol L$$

If we represent the volume of the vapour by V and that of the liquid by V_1 then follows:

$$\Delta V_E = \frac{x_1 V - x V_1}{x_1 - x} \dots \dots \dots \dots \dots \dots \dots \dots (45)$$

by which is defined the change of the O.W.A. of system E on a change of pressure.

We are able to find this in the following way also.

The O.W.A. of the vapour (and consequently also that of the liquid and that of the total system E) is defined by:

$$\xi = -\zeta + x \frac{\partial \zeta}{\partial x}$$
 (46)

If we bring this system under the pressure P + dP, while its total composition remains constant, then we get the equilibrium:

of which the O.W.A. is defined by

and in which dx has a value defined by dP. For the equilibrium E is true viz.:

$$\zeta - x \frac{\partial \zeta}{\partial x} = \zeta_1 - x_1 \frac{\partial \zeta_1}{\partial x_1} \qquad \frac{\partial \zeta}{\partial x} = \frac{\partial \zeta_1}{\partial x_1} \quad . \quad . \quad . \quad (49)$$

from which follow for equilibrium E' the equations:

$$\left(V - x\frac{\partial V}{\partial x}\right)dP - xr\,dx = \left(V_1 - x_1\frac{\partial V_1}{\partial x_1}\right)dP - x_1\,r_1\,dx_1$$
$$\frac{\partial V}{\partial x}\,dP + r\,dx = \frac{\partial V_1}{\partial x_1}\,dP + r_1\,dx_1$$

If we substitute, after having eliminated dx_1 , in (48) dx then we find:

which is corresponding with the value of $d\xi$ which follows from (45).

In order to define the sign of the coefficient of dP we distinguish two cases.

1. The vapour G has a smaller amount of water than the liquid L; consequently we have $1 - x < 1 - x_1$ or $x > x_1$. As in general the volume V of the vapour is many hundred times as large as the volume V_1 of the liquid, $x_1V - xV_1$ will be, therefore, generally positive, unless in the special case that x (viz. the concentration of the substance X in the vapour) is many hundred times as large as x_1 (the concentration of the substance X in the substance X in the liquid).

If we exclude this case, then the coefficient of dP in (50) is positive, therefore.

2. The liquid L has a smaller amount of water than the vapour G; consequently we have $1 - x_1 < 1 - x$ or $x_1 > x$. The numerator of (50) now always is positive, the denominator negative, so that the coefficient of dP is negative.

Consequently we can say:

the O.W.A. of the binary system G + L becomes larger on increase of pressure, when the vapour has the smallest amount of water, and smaller if this is the case with the liquid.

We now take the osmotic equilibrium:

in which on both sides of the membrane the system (9) discussed above, under the pressure P. We distinguish two cases.

1. The solubility of the solid substance Y increases on increase of pressure.

If we assume that the total concentration of the substance Y is smaller at the left side of the membrane than on the right side, then, as on increase of pressure the quantity of solid Y decreases on both sides of the membrane, under a definite pressure $P_1(P_1 > P)$ the solid Y at the left of the membrane will disappear, while it is still present at the right side; then (51) passes into the osmotic equilibrium:

in which the liquid has the same composition on both sides. If we increase the pressure still with $\triangle P$, then the liquid at the left side of the membrane remains unchanged; at the right side of the membrane, however, it gets a somewhat other composition L'_1 . The O.W.A. at the left and at the right side of the membrane then increases with:

$$(d\xi)_l = - \bigtriangleup V_W \cdot dP$$
 $(d\xi)_r = -\bigtriangleup V_E \cdot dP \cdot \cdot \cdot \cdot (53)$

in which $\triangle V_W$ and $\triangle V_E$ are defined by (10^a), (15) and (16). It now follows from (53):

$$(d\xi)_l - (d\xi)_r = \frac{y}{1-y} \cdot \bigtriangleup V_y \cdot dP.$$
 (54)

As the solubility of the substance Y increases on increase of pressure, $\triangle V_Y$ must be negative, as is apparent from (14) and (15); consequently follows from (54):

$$(d\xi)_l < (d\xi)_r$$
 (55)

On an increase of pressure $\triangle P(\triangle P > 0)$ consequently (52) passes into the osmotic system:

in which the water diffuses in the direction of the arrow; this diffusion continues till the liquid L_1 gets the composition L'_1 .

2. The solubility of the solid substance Y decreases on increase of pressure. Then we must decrease the pressure, in order to make disappear the solid substance Y from the system Y + L; the osmotic equilibrium (52) then arises under a pressure P_1 smaller than P. As $\triangle V_y$ in (54) is positive on a decreased of pressure $\triangle P$ [in (54) then is $dP = -\triangle P$] (52) passes into the osmotic system:

in which the water diffuses in the same manner from left to right; the water-diffusion continues also here till the liquid L_1 gets the composition L'_1 .

(To be continued.)

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