**Crystallography.** — *On* M. BEREK's *Methode der Charakteristischen Gangunterschiedsverhältnisse.* By W. NIEUWENKAMP. (Communicated by Prof. L. RUTTEN).

(Communicated at the meeting of January 29, 1927).

In his excellent work „Mikroscopische Mineralbestimmung mit Hilfe der Universaldrehtischmethoden" M. BEREK points out a new method for determining the optic axial angle, the optical character and the position of the optic axes of symmetry of a mineral, requiring only the measurement of some six or seven values of wave differences by means of the compensator after BEREK. These measurements are simple and convenient, but BEREK derives from them the axial angle by means of rather long and tedious calculations. I hope I have shown in this paper that these calculations can be simplified without diminishing the accuracy, and that in this way I have made this really fine method more efficient.

For a clear understanding it is desirable to describe first shortly the instrumental part of the method: The slide is clamped on a Federow-stage, between two hemispheres of glass, whose refractive index is as much as possible like that of the mineral under test, and we place in the ordinary way an optic plane of symmetry perpendicular to the axis $A_4$, i.e. the second (always) horizontal axis of rotation. Then the whole stage is rotated about the axis $A_5$, (the axis of the microscope) for 45°, so that the direction of the vibration of the fast ray in the mineral and in the compensator are at right angles to one another, and we can start our measurements. They are taken in a series of different directions (all of them in the plane of symmetry) by placing the mineral by rotation about $A_4$ in different positions, e.g. successively on the readings: $a_4 = -50°, -40°, -20°, 0°, 20°, 40°, 50°$. In this way we get a series of readings at the compensator, and in the table Kompensatorfunktion 10 000 $f(i)$, (p. 164 Drehtischmethoden) we can find a series of values proportional with the wave differences. They need not be multiplied with the constant of the compensator, for their rationes will be the same as those of the actual wave differences, and, as is suggested by its name, this method makes only use of these rationes. When we wish to intercompare the values of $f(i)$, they must first be reduced to the same length of way of the light in the mineral.

BEREK reduces them to the real thickness of the slide. It is unnecessary to be so scrupulous; the only condition is that the length of the way of the light has to be the same for every observation. The reduction becomes very simple when we reduce the observation to the interval the ray passes when $A_4$ stands in its initial position ($\alpha_4 = 0^\circ$). This is shown in fig. 1.

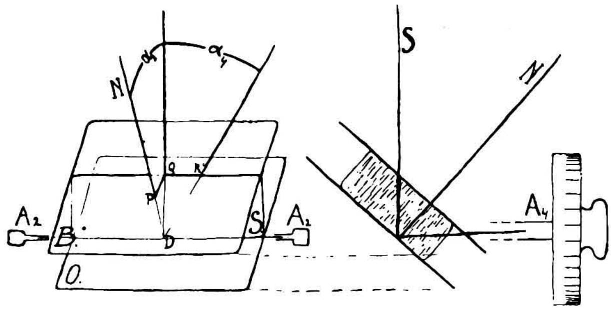


Fig. 1.

Here $O$, and $B$ represent the under and upper surface of the slide, $N$ is drown perpendicular to it. $S$ is the plane of symmetry in which we measure the wave differences; so the ray passes always through the mineral along one of the lines of $S$; when $A_4$ stands in its initial position ($\alpha_4 = 0^\circ$) along the line $DQ$, when the stage is inclined, along a line $DR$.

In the initial position the axis $A_2$ is horizontal. Consequently in that position also the line $QR$, i.e. the trace of the plane of symmetry runs horizontal, for our first manupulation was placing this plane parallel to $A_2$. The ray, in this case $DQ$, is vertical, so $DQ$ is at right angles with $QR$, and $QD : RD = \cos \alpha_4$; therefore in order to reduce the way travelled over by the light to $DQ$ we have only to multiply by $\cos \alpha_4$.

In BEREK's procedure the reduction runs as follows: he begins by plotting a curve with the measured wave differences as ordinates, and $\alpha_4$ as abscissae. On the same abscissae he also plots the ordinates that represent the angle $RDP$, corresponding with every $\alpha_4$, i.e. the angle between the ray and the normal on the slide. In a stereographic projection in which the plane of symmetry and the ray for every $\alpha_4$ are represented, these angles are measured by means of a WULLF's net, and so we can multiply the ordinates of every point of the first curve by the *cos* of the corresponding angle $RDP$ thus reducing indeed, all the wave differences to the real thickness of the slide.

In this way then BEREK obtains the reduced wave difference-curve. In this curve a maximum or a minimum has to be looked for; it is however not easy to find where it lies exactly. According to BEREK one begins by assuming a presumptive value for the abscissa of the maximum or minimum, and by marking out distances from it of 30°, 45° and if possible 60°, 75° and 90°, then the corresponding ordinates are divided by the ordinates of the maximum or minimum, and, we try to find in 5 diagrams, constructed by BEREK, and added to his work, by means of these 5 quotients 5 values for the axial angle. If these values are not equal, if their curve runs up or falls then the choice of the abscissa of the minimum is wrong and we have to do it over again. If now their curve moves in the other direction, we can find by means of interpolations and by other trials an abscissa for which the axial angle is the same for all quotients; this then is the real axial angle and the value of the abscissa of the maximum or minimum indicates the position of an optical axis of symmetry.

I think I can substitute the latter operation of trial and error by moving a curve projected on transparent paper, and obtained from the observations, over a set of curves on a diagram until the best possible agreement is attained.

That this can really be done is borne out by the following consideration: in our computation of the axial angle we availed ourselves of rationes of the wave differences; it is these rationes that determine the axial angle. When plotting the logarithms of the wave differences the shape of the curve will depend on the axial angle. The lengths of the ordinates representing the logarithms of the wave differences indeed also depend on the thickness of the slide and the compensator constant, but the shape of the curve is determined by the differences of the ordinates, and these differences represent the logarithms of the quotients of the wave differences and consequently depend only on the axial angle. So with each axial angle there is a definite shape of the curve of the log. wave differences. Now the later have been drawn on the diagrams for axial angles varying from 0°—90°. The absolute lengths of the ordinates are chosen so as to be most convenient for the construction and the selection of a definite curve.

From our theoretical discussions at the close of this paper it will appear that 2 diagrams are required, one for the optic axial plane and one for the 2 planes of symmetry perpendicular to it, but first we shall illustrate the use of the diagrams by a few examples.

| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $\alpha_4$ | Wave differences | $i = \frac{a-b}{2}$ | *log* wave differences | *log cos* $\alpha_4$ | Reduced *log* wave differences |
| 50 | 193.5 | 25.55 | 9.287 | 0.809—1 | 9.096 |
| 40 | 183.0 | 24.83 | 9.263 | 0.884—1 | 9.147 |
| 20 | 176.1 | 24.35 | 9.246 | 0.973—1 | 9.219 |
| 0 | 173.0 | 24.12 | 9.238 | | 9.238 |
| —20 | 170.6 | 23.95 | 9.232 | 0.973—1 | 9.205 |
| —40 | 168.9 | 23.83 | 9.228 | 0.884—1 | 9.112 |
| —50 | 175.2 | 24.28 | 9.243 | 0.809—1 | 9.052 |

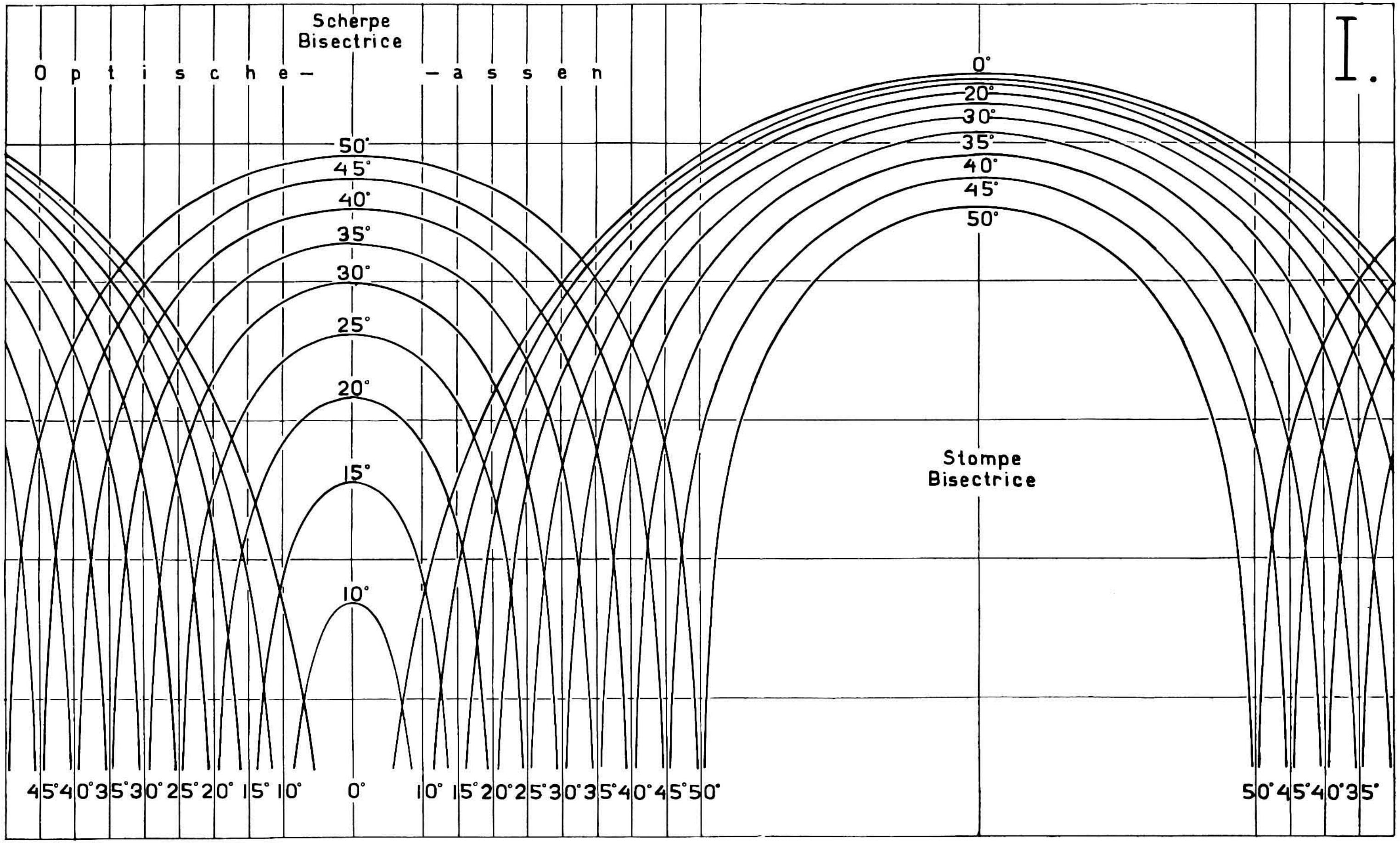
In BEREK's work we read on p. 121 that when measuring a plagioklase he found corresponding with the $\alpha_4$ from column 1 the wave differences as indicated in column 2. These are not the real wave differences but the values (proportional with the wave differences) of table: "Kompensatorfunktion 10 000 $f(i)$", and we can find the $i = \frac{a-b}{2}$ (not given in his work) with which BEREK started his calculations, and upon which we will base our example. So we take the $i$ of column 3 for measured with the compensator, and in the table "Für logarithmisches rechnen, Kompensatorfunktion *log* $f(i)$", we look up the *log* $f(i)$. They are given in column 4, but have still to be reduced to the same length of the way of the light. Therefore we have only to add the *log cos* $a_4$. *Log cos* $a_1$ is given in column 5, by addition we get the values of column 6.

Now the calculations are already finished, and we can plot the curve. To get it on the same scale as the accompanying diagrams we have to mark off on the axis of abscissae 1 cm for every 10° of $a_4$; on the axis of ordinates 5 cm for every unit of the logarithm. On a sheet of millimeter paper this scale for abscissa and ordinate should be marked; if we cover it with a sheet of transparent paper the dotting of the curve on the right scale is an easy operation. Care should be taken to indicate at the same time the direction of the axis of abscissae, which may be done by drawing a horizontal line across the transparent paper. Care should then be taken that while moving the paper over the diagram, we always keep this line parallel to the horizontal lines of the diagram.

The shape of the curve of our example shows directly that it must fit to one of the curves of diagram 2, and is intermediate between the curve for $V$ 40°, and that for 45°. If the curve bends so little as in the present case we may for convenience' sake magnify the ordinates 10 times and then use diagram $2^A$, a magnification of the top part of diagram 2. With both diagrams the result is of course the same. The correspondence to the curve for $42^1/_2$° is not yet quite sufficient. We see that our points are lying in a curve slighty less arched, we have therefore to go up a little, and shall than find that the best correspondence is obtained with a curve between 43° and 44°. So the total axial angle is established at 87°.

When in plotting the curve we have also indicated the scale of the abscissa we are able to indicate directly the position of an optic axis of symmetry, (in this case the optical normal, with $a_4 = 3°$). The bisectrix lying in the contemplated plan of symmetry, is that which has the angle $V$, indicated in the diagram, on either side, so it is acute bisectrix for the curves 0°—45°, and obtuse bisectrix for those from 45°—90° (in our case acute bisectrix). We now also know the third axis of symmetry and consequently we can also indicate the optical character of the mineral, when we know from our compensator the directions of the vibration of the slow and fast ray.

An example of the use of diagram 1 is to be found on page 116 of

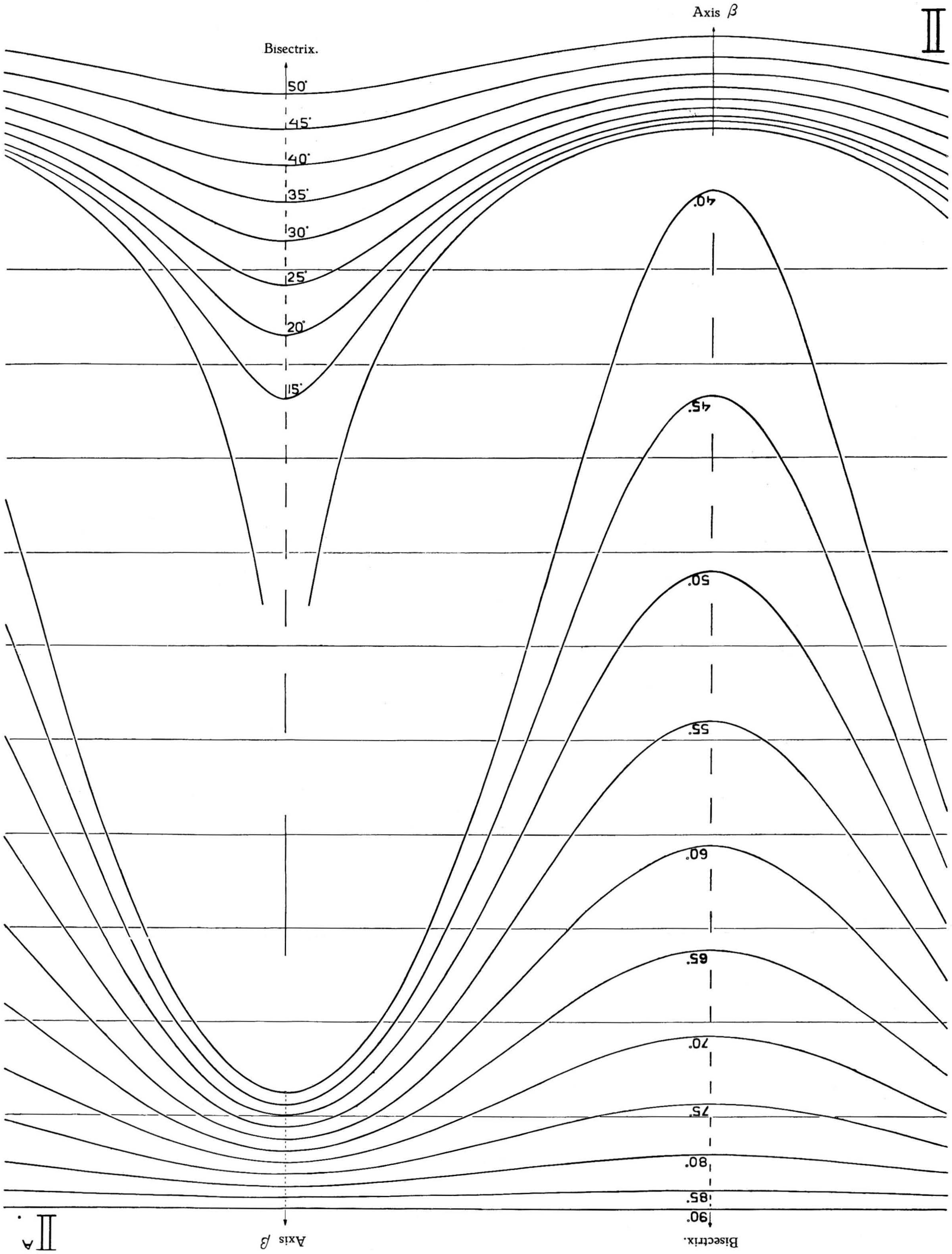



Scherpe Bisectrice = Acute bisectrix.
Optische assen = Optic axes.

Diagram I.

Stompe Bisectrice = Obtuse bisectrix.

W. NIEUWENKAMP: ON M. BEREK'S "METHODE DER CHARAKTERISTICHEN GANGUNTERSCHIEDSVERHÄLTNISSE".



the "Drehtischmethoden". This is a measurement made in the optic axial plane. We have not taken measures close to the optical axis, because there the small wave differences render them too inaccurate; on the other hand we have a determination of the abscissa of the optical axis; on the transparent sheet it is marked by a vertical line with $\alpha_4 = 21{,}2^\circ$, that is the $\alpha_4$ with which BEREK has found the optical axis. For the other points of the curve we find in the same way as in the first example from BEREK's wave differences:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\alpha_4 =$ | 50° | 40° | 10° | 0° | —20° | —40° | —50° |
| *log* wave diff. = | 8,706 | 8,618 | 8,529 | 8,811 | 9,057 | 9,105 | 9,077 |

It follows from the fact that the measurements have been made in the plane of the axes, that diagram I is to be used, which moreover is shown clearly by the shape of the curve. When fitting the curve on the diagram care should be taken that the same angle $V$ is found on either branch of the curve and also that the same $V$ stands at the line marking the position of the optical axis. We can place the right part of our curve first on the curve for 35°, but then the left branch lies at 32° (anyhow if we have laid the line for the optical axis along that for 35°). So we have to shove the paper upward slantingly to the left. With 33° there is a fair concordance, with 32° it decreases already. So we find $V = 32^1/_2{}^\circ$ ($2\ V = 65^\circ$), in complete accordance with BEREK's result.

Here also we can easily mark where in our mineral an optic axis of symmetry is to be found, and which of the axes it is, as well as the remaining 2 axes and the optical character.

One more remark about this graphical arithmetical method:

The points dotted on the transparent paper have not been connected by a continuous line; this is quite irrelevant as we do something like it when shoving the paper over the diagrams. Nay, it is better even to let the points alone, because otherwise we should most likely draw a curve of a shape that cannot occur with perfectly correct measurements, and is impossible with any axial angle, whereas in the shoving process the points are connected by a curve that is theoretically possible. In this case therefore we make a selection from curves that of themselves may occur, though perhaps with another axial angle, wereas in the first case we can hit upon a good curve only with perfectly correct measurements. Generally however we shall find one that will give trouble rather than pleasure.

The data for the construction of the diagram are found in the following way: According to BEREK (p. 154 sqq. of his Drehtischmethoden), when our measurements are taken in an optical plane of symmetry which is not the axial plane:

$$\frac{\sin\,(r,\,n_x)}{\sqrt{\frac{\Delta_r}{\Delta_x} - 1}} = tg\,V \text{ or } \frac{\sin\,(r,\,n_x)}{\sqrt{1 - \frac{\Delta_r}{\Delta_x}}} = \sec\,V.$$

Here $r$ is the direction of the ray, $n_x$ an optical axis of symmetry, $\triangle_r$, $\triangle_x$ the wave differences in the direction $r$ resp. $n_x$. If furthermore $V$ is counted from 0°—90°, one trigonometrical function will do (it is then needless to mention: or cotg $V$, resp. cosec $V$).

From the first formula follows:

$$\frac{\triangle_r}{\triangle_x} = \frac{sin^2\,(r,\,n_x)}{tg^2\,V} + 1 = \frac{sin^2\,(r,\,n_x) + tg^2\,V}{tg^2\,V}.$$

$$lg\,\triangle_r = lg\,\{sin^2\,(r,\,n_x) + tg^2 V\} - lg\,tg^2\,V + lg\,\triangle_x\,.$$

FOR DIAGRAM I, $lg\,|\,sin^2\,(r,n_x) - sin^2\,V\,|$

| $(r,n_x)$ \ $V$ | 0° | 5° | 10° | 15° | 20° | 25° | 30° | 35° | 40° | 45° |
|---|---|---|---|---|---|---|---|---|---|---|
| 0° | | 7.881 | 8.479 | 8.826 | 9.068 | 9.252 | 9.398 | 9.517 | 9.616 | 9.699 |
| 5° | 7.881 | | 8.352 | 8.774 | 9.039 | 9.233 | 9.384 | 9.507 | 9.608 | 9.692 |
| 10° | 8.479 | 8.352 | | 8.567 | 8.939 | 9.172 | 9.342 | 9.476 | 9.583 | 9.672 |
| 15° | 8.826 | 8.774 | 8.567 | | 8.699 | 9.048 | 9.262 | 9.418 | 9.539 | 9.636 |
| 20° | 9.068 | 9.039 | 8.939 | 8.699 | | 8.790 | 9.123 | 9.326 | 9.472 | 9.583 |
| 25° | 9.252 | 9.233 | 9.172 | 9.048 | 8.790 | | 8.854 | 9.177 | 9.370 | 9.507 |
| 30° | 9.398 | 9.384 | 9.342 | 9.262 | 9.123 | 8.854 | | 8.898 | 9.213 | 9.398 |
| 35° | 9.517 | 9.507 | 9.476 | 9.418 | 9.326 | 9.177 | 8.898 | | 8.925 | 9.233 |
| 40° | 9.616 | 9.608 | 9.583 | 9.539 | 9.472 | 9.370 | 9.213 | 8.925 | | 8.938 |
| 45° | 9.699 | 9.692 | 9.672 | 9.636 | 9.583 | 9.507 | 9.398 | 9.233 | 8.938 | |
| 50° | 9.768 | 9.763 | 9.746 | 9.716 | 9.672 | 9.611 | 9.527 | 9.411 | 9.239 | 8.938 |
| 55° | 9.827 | 9.822 | 9.807 | 9.781 | 9.744 | 9.692 | 9.624 | 9.534 | 9.411 | 9.233 |
| 60° | 9.875 | 9.871 | 9.857 | 9.834 | 9.801 | 9.757 | 9.699 | 9.624 | 9.527 | 9.398 |
| 65° | 9.915 | 9.910 | 9.898 | 9.878 | 9.848 | 9.808 | 9.757 | 9.692 | 9.611 | 9.507 |
| 70° | 9.946 | 9.942 | 9.931 | 9.912 | 9.884 | 9.848 | 9.801 | 9.743 | 9.672 | 9.583 |
| 75° | 9.970 | 9.966 | 9.956 | 9.937 | 9.912 | 9.878 | 9.834 | 9.781 | 9.716 | 9.636 |
| 80° | 9.987 | 9.983 | 9.973 | 9.956 | 9.931 | 9.898 | 9.857 | 9.807 | 9.746 | 9.672 |
| 85° | 9.997 | 9.993 | 9.983 | 9.966 | 9.942 | 9.911 | 9.871 | 9.822 | 9.763 | 9.692 |
| 90° | 10 | 9.997 | 9.987 | 9.970 | 9.946 | 9.915 | 9.875 | 9.827 | 9.769 | 9.699 |

## For Diagram II, $lg\,\{sin^2\,(r,\,n_x) + tg^2\,V\}$.

| $V$ / $(r,\ n_x)$ | 0° | 5° | 10° | 15° | 20° | 25° | 30° | 35° | 40° | 45° | 50° | 55° | 60° | 65° | 70° | 75° | 80° | 85° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0° | | 0.8839—3 | 0.4926—2 | 0.8561 −2 | 0.1221—1 | 0.3373—1 | 0.5230—1 | 0.6905—1 | 0.8476—1 | 0 | 0.1524 | 0.3095 | 0.4771 | 0.6627 | 0.8779 | 1.1439 | 1.5075 | 2.1159 |
| 5° | 0.8806—3 | 0.1833—2 | 0.5876—2 | 0.8998—2 | 0.1464—1 | 0.3523—1 | 0.5327—1 | 0.6971—1 | 0.8523—1 | 0.0033 | 0.1547 | 0.3111 | 0.4782 | 0.6635 | 0.8784 | 1.1442 | 1.5076 | 2.1160 |
| 10° | 0.4793—2 | 0.5776—2 | 0.7878—2 | 0.0084—1 | 0.2111—1 | 0.3937—1 | 0.5595—1 | 0.7164—1 | 0.8658—1 | 0.0129 | 0.1615 | 0.3159 | 0.4815 | 0.6656 | 0.8797 | 1.1449 | 1.5079 | 2.1161 |
| 15° | 0.8260—2 | 0.8730—2 | 0.9917—2 | 0.1423—1 | 0.2998—1 | 0.4540—1 | 0.6025—1 | 0.7461—1 | 0.8870—1 | 0.0282 | 0.1724 | 0.3235 | 0.4867 | 0.6690 | 0.8818 | 1.1460 | 1.5084 | 2.1162 |
| 20° | 0.0681—1 | 0.0956—1 | 0.1706—1 | 0.2759—1 | 0.3970—1 | 0.5243—1 | 0.6535—1 | 0.7834—1 | 0.9144—1 | 0.0481 | 0.1867 | 0.3337 | 0.4937 | 0.6737 | 0.8846 | 1.1476 | 1.5090 | 2.1163 |
| 25° | 0.2519—1 | 0.2701—1 | 0.3216—1 | 0.3986—1 | 0.4929—1 | 0.5977—1 | 0.7092—1 | 0.8254—1 | 0.9458—1 | 0.0714 | 0.2038 | 0.3460 | 0.5022 | 0.6793 | 0.8881 | 1.1495 | 1.5098 | 2.1165 |
| 30° | 0.3979—1 | 0.4110—1 | 0.4489—1 | 0.5076—1 | 0.5826—1 | 0.6697—1 | 0.7659—1 | 0.8694—1 | 0.9796—1 | 0.0969 | 0.2228 | 0.3597 | 0.5119 | 0.6857 | 0.8921 | 1.1517 | 1.5108 | 2.1167 |
| 35° | 0.5172—1 | 0.5272—1 | 0.5564—1 | 0.6029—1 | 0.6642—1 | 0.7375—1 | 0.8211—1 | 0.9134—1 | 0.0141 | 0.1235 | 0.2429 | 0.3745 | 0.5223 | 0.6928 | 0.8965 | 1.1541 | 1.5119 | 2.1170 |
| 40° | 0.6161—1 | 0.6241—1 | 0.6477—1 | 0.6857—1 | 0.7370—1 | 0.7997—1 | 0.8730—1 | 0.9559—1 | 0.0482 | 0.1502 | 0.2633 | 0.3896 | 0.5332 | 0.7001 | 0.9011 | 1.1566 | 1.5130 | 2.1173 |
| 45° | 0.6990—1 | 0.7056—1 | 0.7252—1 | 0.7572—1 | 0.8011—1 | 0.8558—1 | 0.9208—1 | 0.9958—1 | 0.0807 | 0.1761 | 0.2834 | 0.4047 | 0.5441 | 0.7076 | 0.9058 | 1.1593 | 1.5142 | 2.1176 |
| 50° | 0.7685—1 | 0.7741—1 | 0.7909—1 | 0.8186—1 | 0.8569—1 | 0.9054—1 | 0 9638—1 | 0.0323 | 0.1109 | 0.2005 | 0.3025 | 0.4193 | 0.5547 | 0.7149 | 0.9105 | 1.1619 | 1.5153 | 2.1179 |
| 55° | 0.8267—1 | 0.8317—1 | 0.8465—1 | 0.8709—1 | 0.9050—1 | 0.9487—1 | 0.0019 | 0.0650 | 0.1384 | 0.2230 | 0.3204 | 0.4330 | 0.5648 | 0.7219 | 0.9149 | 1.1644 | 1.5164 | 2.1182 |
| 60° | 0.8751—1 | 0.8794—1 | 0.8927—1 | 0.9148—1 | 0.9457—1 | 0.9856—1 | 0.0348 | 0.0935 | 0.1626 | 0.2430 | 0.3365 | 0.4455 | 0.5740 | 0.7284 | 0.9191 | 1.1667 | 1.5174 | 2.1184 |
| 65° | 0.9146—1 | 0.9186—1 | 0 9307—1 | 0.9509—1 | 0.9795—1 | 0.0165 | 0.0625 | 0.1178 | 0.1834 | 0.2604 | 0.3506 | 0.4565 | 0.5822 | 0.7341 | 0.9228 | 1.1688 | 1.5184 | 2.1187 |
| 70° | 0.9460—1 | 0.9497—1 | 0.9610—1 | 0.9800—1 | 0.0067 | 0.0416 | 0.0851 | 0.1378 | 0.2006 | 0.2749 | 0.3623 | 0.4658 | 0.5892 | 0.7390 | 0.9260 | 1.1706 | 1.5192 | 2.1189 |
| 75° | 0.9699—1 | 0.9734—1 | 0.9841—1 | 0.0021 | 0.0276 | 0.0609 | 0.1025 | 0.1533 | 0.2141 | 0.2862 | 0.3717 | 0.4731 | 0.5947 | 0.7430 | 0.9285 | 1.1721 | 1.5199 | 2.1190 |
| 80° | 0.9867—1 | 0.9901—1 | 0.0004 | 0.0177 | 0.0423 | 0.0745 | 0.1150 | 0.1644 | 0.2237 | 0.2944 | 0.3784 | 0.4785 | 0.5988 | 0.7458 | 0.9304 | 1.1732 | 1.5203 | 2.1191 |
| 85° | 0.9967—1 | 0.0000 | 0.0101 | 0.0270 | 0.0511 | 0.0827 | 0.1225 | 0.1711 | 0.2295 | 0.2994 | 0.3825 | 0.4817 | 0.6012 | 0.7476 | 0.9316 | 1.1738 | 1.5206 | 2.1192 |
| 90° | 0. | 0.0033 | 0.0133 | 0.0301 | 0.0540 | 0.0854 | 0.1249 | 0.1733 | 0.2315 | 0.3010 | 0.3839 | 0.4828 | 0.6021 | 0.7482 | 0.9320 | 1.1741 | 1.5207 | 2.1193 |

From the second formula follows:

$$\frac{\triangle_r}{\triangle_x} = \frac{-\sin^2(r, n_x)}{\sec^2 V} + 1 = \frac{\cos^2(r, n_x) - 1 + (1 + tg^2 V)}{1 + tg^2 V},$$

$$lg\, \triangle_r = lg\,[\sin^2\{(r, n_x) \pm 90^\circ\}\, tg^2 V] - lg\,(1 + tg^2 V) + lg\, \triangle_x\,.$$

The shape of the curve for a definite $V$ is now in the first case the same as in the second, so one curve suffices for both cases, if only in the second case we begin to measure the angle $(r, n_x)$ from a point that differs $90^\circ$ from the $n_x$ in the first case.

In the axial plane (see BEREK l. c.) is:

$$\frac{\sin^2(r, n_x)}{\sqrt{1 \pm \frac{\triangle_r}{\triangle_x}}} = \sin V,$$ ($V$ counted from $0^\circ$—$90^\circ$) from which follows:

$$\frac{\triangle_r}{\triangle_x} = \left| \frac{\sin^2(r, n_x)}{\sin^2 V} - 1 \right| = \frac{|\sin^2(r, n_x) - \sin^2 V|}{\sin^2 V}$$

$$lg\, \triangle_r = lg\, |\sin^2(r, n_x) - \sin^2 V| - lg\, \sin^2 V + lg\, \triangle_x\,.$$

For the construction of the diagrams, which have to indicate for each definite $V$, the differences between $log\, \triangle_r$ when the angle $(r, n_x)$ varies, only that part of the formula is of consequence in which $(r, n_x)$ occurs; it does not matter if a constant is added or not. So for the construction we require the formulae:

$lg\, \triangle_r = lg\,\{\sin^2(r, n_x) + tg^2 V\}$ resp. $lg\, \triangle_r = lg\, |\sin^2(r, n_x) - \sin^2 V|$

For those who should wish to attain a greater accuracy than is possible with the small diagrams reproduced here, and would construct them on a larger scale, the results have been tabulated here.

That as will be seen in formula 2, the $log\, \triangle_r$ becomes $-\infty$ for $(r, n_x) = V$, need not discompose us very much, because (on BEREK's suggestion) we do not take any measurements in the neighbourhood of an optical axis, so that there we have no points to be fitted on the curve, and consequently a part of it can be missing without vitiating the results.