Physics. — *The deviation from* LAMBERT's *law for incandescent tungsten and molybdenum.* By C. ZWIKKER. (Communicated by Dr. G. HOLST.)

(Communicated at the meeting of September 24, 1927)

According to LAMBERT's law the amount of light, emitted by an incandescent surface in a direction which makes an angle α with the normal to the surface, is proportional to $\cos \alpha$. Therefore the apparent brightness of a luminous surface is independent of the angle at which we look at it.

WORTHING [1]) has called attention to the fact that the intensity of the light emitted by incandescent tungsten varies considerably from LAMBERT's cosine law. At an angle $\alpha = 75°$ the brightness reaches a maximum, which, according to this author, exceeds that for normal emission by about 16 %.

In a previous paper [2]) I had some reason to doubt the numerical value of this excess. Now new measurements have been performed all of which indeed give a smaller effect than was mentioned by WORTHING.

A tungsten ribbon filament was pyrometered optically at different angles with respect to the normal. The effective wavelength of the light used was in most cases 0.652 μ.

Figure 1 shows the ratio of the brightness at various angles to the normal brightness as a function of the angle α. The three curves refer to three intervals during the seasoning of the ribbon-filament, viz.

curve 1 : the tungsten ribbon filament is not yet recrystallised, it has still the fibrous structure caused by the rolling ;

curve 2 : the ribbon has glowed for some time at a temperature of 2300°K, as a consequence of which it has become much more polished ;

curve 3 : the ribbon has once more glowed during 40 hours at a temperature of 2400° ; it has become ideally polished and for the main part it consists of a single crystal.

In all these three intervals we have performed our measurements at more than one temperature. The effect appeared to be independent of the temperature. In fig. 1 WORTHING's results are indicated by dots, marked

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1) Astr. Journ. **36**, 1912, p. 345.
Phys. Rev. **35**, 1912, p. 76.
Report Intern. Comm. of the Illum. 1924.
Astr. Journ. **61**, 1925, p. 146.
Rev. Gén. de l'Électr. **20**, 1926, p. 310.
Jl. Opt. Soc. Am. **13**, 1926, p. 635.

2) Diss. Amsterdam 1925, p. 32.
Arch. Néerland, IX, 1925, p. 237

W. The effect, which I find is the more pronounced the better the ribbon is polished, but it remains considerably smaller than WORTHING's values.

For the same unicrystalline tungsten ribbon filament, we have tried to

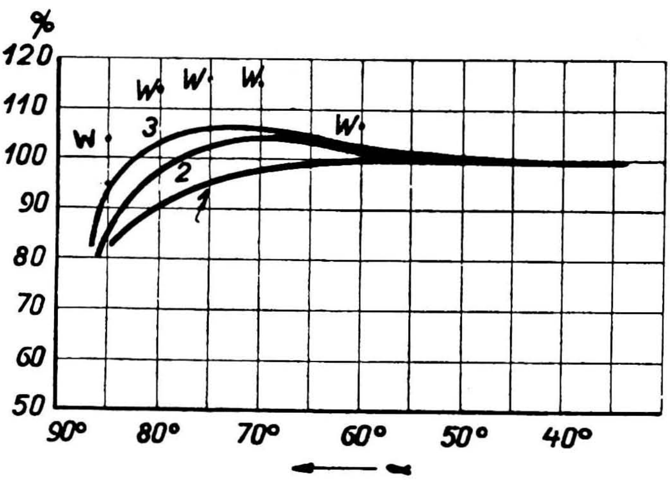

Fig. 1.

find out, whether there is any influence of the direction of emission with regard to the direction of rolling. For three of the four principal directions the effect was the same (see fig. 2 curve *b*), but for the fourth direction the effect was reproduceably found to be smaller (curve *a*). The writer

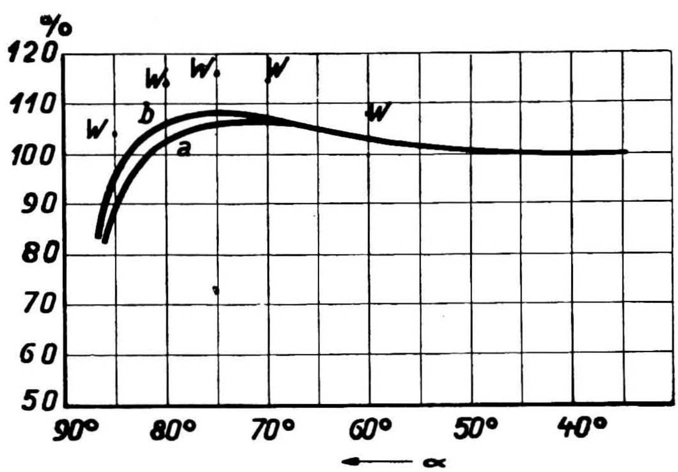

Fig. 2.

believes that here the position of the crystal-axes with respect to the direction and plane of rolling plays a roll. Tungsten has a body-centred cubic lattice. In the rolled ribbon the crystals tend to take up a position so that a (100) plane lies in the rolling plane and a (110) plane lies perpendicularly to the rolling direction. This orientation is however only approximate, so that the surface consists of steps. Therefore it will make a difference for the effect of the deviation from LAMBERT's law in which direction we look over the surface.

The mean effect for the four principal directions of observation is given in the following table :

α	e/e_0
40°	1.01
50°	1.02
60°	1.03
70°	1.07
75°	1.08
80°	1.04^5
85°	0.93

The knowledge of the quantity e/e_0 makes it possible to calculate the normal brightness of an incandescent tungsten filament from the total candle power (e.g. the "horizontal" candle power, that is the candle power in a direction perpendicular to the filament axis). Dividing the horizontal candle power by the visible surface, as we should do in case LAMBERT's law would hold strictly, we find too great a value for the normal brightness. The correction is 3 % according to WORTHING, and only 1.6 % according to me. Of even more importance is the correction for the calculation of the normal brightness from the total spherical lightflux of the filament. In this case WORTHING requires a correction of 5 %, the present writer only 2.5 %.

Moreover it is not certain, whether the deviation from LAMBERT's law is constant throughout the whole visible spectrum. The maximum of the light impression lies in the green region and in some cases the deviation from LAMBERT's law is much smaller for green light than it is for red light. This has already been assumed by WORTHING [1]) and was also found by us. We shall have to deal with this question further on. Summarising we can say that, according to our own measurements, the ratio of the mean spherical candle power to the horizontal candle power of a tungsten filament varies less than 1 % from the theoretical value $\frac{\pi}{4} = 0.785$. Indeed, direct measurements of the factor gave a value, which coincided with the theoretical value within the experimental errors [2]).

We have measured the deviation from LAMBERT's law for molybdenum

[1]) Astr. Journ. **36**, 350 (1912).

[2]) Arch. Néerland IX, 239 (1925).

with a ribbon filament which was well-seasoned and well-polished. The filament consisted in this case of a great many little crystals. We could find no difference between the values for the effect in different directions with respect to the rolling direction. Evidently we measure directly the mean effect, because all crystal orientations occur simultaneously. The mean of all our measurements can be seen from fig. 3 and from the there following table ($\lambda = 0.652\ \mu$).

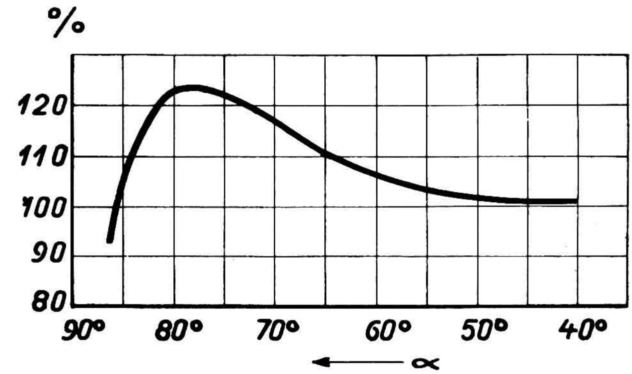

Fig. 3.

angle α	e/e_0
40°	1.01
50°	1.02
60°	1.06^5
70°	1.17
75°	1.22
80°	1.23
85°	1.06

For green light ($\lambda = 0.541\ \mu$) we found practically the same effect as above for red light. Assuming therefore that the deviation from LAMBERT's law for molybdenum is constant throughout the whole visible spectrum, we compute from these figures that the correction for the normal brightness, when calculated from the horizontal candle power, comes to 3.4 %. On the other hand when calculated from the mean spherical candle power, this correction amounts to 6 %. This last number is in agreement with a note of WORTHING [1]).

Our measurements have been completed by a study of the deviation from LAMBERT's law for the two poralised components of the light emitted. In fig. 4 curve p represents the results, obtained with tungsten and with

[1]) Phys. Rev. **28**, 1926, 195.

light, which has its magnetic vector in the plane of emission. Curve *s* shows the results for light with the magnetic vector normal to the plane of emission. Denoting the normal emission coefficient by *e*, the emissivities in the direction α are *pe*, resp. *se*. Then, according to KIRCHHOFF's law, the

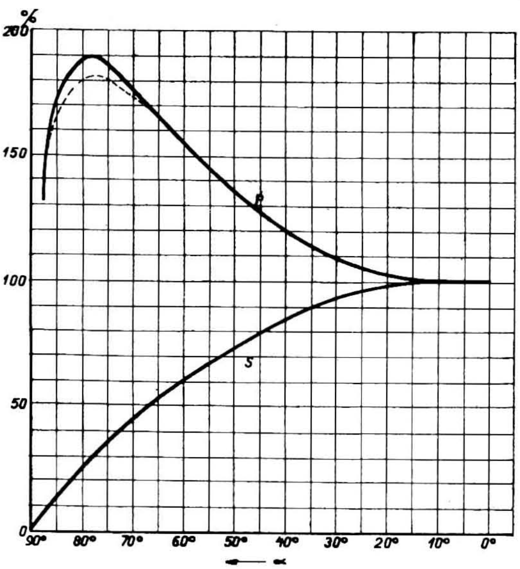

Fig. 4. Tungsten.

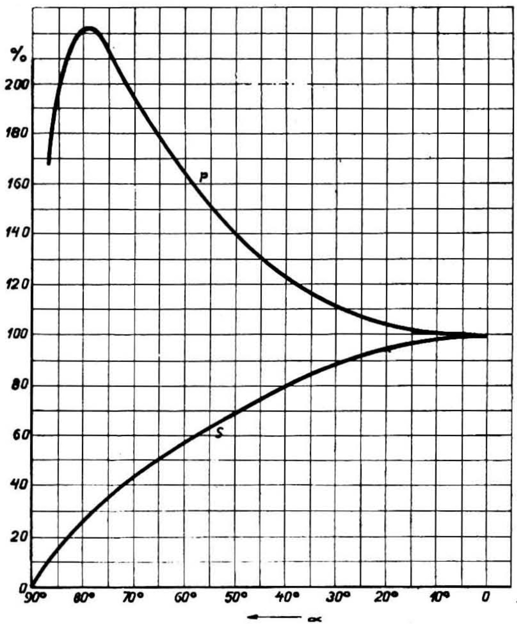

Fig. 5. Molybden.

corresponding reflectivities are 1—*pe* and 1—*se*.

Now in first approximation [1])

$$\frac{1-pe}{1-se}=\frac{(m-tg\,\alpha\,sin\,\alpha)^2+\varkappa^2}{(m+tg\,\alpha\,sin\,\alpha)^2+\varkappa^2}$$

where $m=$ refractive index.

$\varkappa=$ coefficient of extinction.

If, for convenience, we put $tg\ \alpha\ sin\ \alpha = a$, then the above equation becomes :

$$(p-s)\,e\,(\varkappa^2+m^2)=\{2-(p+s)\,e\}\,.\,2\,ma-(p-s)\,ea^2.$$

which can be considered as a linear equation with two unknown quantities, viz. $(\varkappa^2+m^2)$ and *m*. Substituting the measured values of *p* and *s* and the well-known value for *e*, we get an equation for each value of α. I solved this set of equations graphically by plotting them as straight lines on an $m-(x^2+m^2)$ diagram and by determining the point of intersection which is common to all these lines. The position of these straight lines is very sensitive for small errors in α. I only could make the lines pass through one point, if I shifted the scale of the angle α with half a degree. This is a correction which is very probably smaller than the experimental

[1]) See e.g. SCHUSTER—NICHOLSON. The theory of optics. Third edition 1924, p. 276. Other authors (DRUDE, WOOD) put the extinction coefficient equal to $n\varkappa$.

errors. After this correction being applied the optical constants for the unicrystalline tungsten were found as follows :

$$m = 2.96$$
$$\varkappa = 3.36.$$

Fig. 5 shows our measurements performed in polarised light with molybdenum. These measurements are in perfect agreement with measurements of WORTHING [1]) on this same subject. In determining the values of $m^2 + \varkappa^2$ and m in the graphical way as described above, it came out that the straight lines

$$(p - s)\, e\, (\varkappa^2 + m^2) = \{2 - (p + s)\, e\}\, .\, 2\, ma - (p - s)\, ea^2,$$

did not pass through one point so well as this was the case for tungsten. As the most probable values for the optical constants for molybdenum we took :

$$m = 3.75$$
$$\varkappa = 4.25.$$

We have seen that our measurements with molybdenum were in perfect accordance with WORTHING's work. For tungsten, however, our results vary greatly from his. Therefore, I measured the deviation from LAMBERT's law for another tungsten ribbon, well seasoned, but consisting of small crystals, just as the molybdenum ribbon filament, mentioned above. Also the polishing was the same as that of the molybdenum, both these fine-crystallised ribbons being somewhat less polished than the unicrystalline tungsten ribbon filament. As can be seen from the following table, the results obtained with this fine-crystallised tungsten ribbon differed from those, which were obtained with the unicrystalline ribbon :

α	Fine-crystallised			Unicrystalline
	p	s	$^1/_2\,(p+s)$	$^1/_2\,(p+s)$
10°	1.01	0.99	1.00	1.00
20	1.03^5	0.96	1.00	1.00
30	1.10	0.91	1.00	1.00
40	1.19	0.83	1.01	1.01
50	1.35	0.73	1.04^5	1.02
60	1.52	0.61	1.06	1.03
70	1.73	0.47	1.10	1.07
75	1.85	0.38	1.11	1.08
80	1.85	0.28	1.06^5	1.04^5
85	1.68	0.15	0.91^5	0.93

[1]) Jl. of the Opt. Soc. of Am. **13**, 640 (1926).

The s-line for the fine-crystallised tungsten lies above the s-line for the unicrystalline tungsten and is responsible for the greater deviation from LAMBERT's law.

The two specimens differed still in another respect, viz. the deviation from LAMBERT's law for green light. For fine-crystalline tungsten, just as for molybdenum, the effect is the same for red light and for green light. The unicrystalline tungsten showed hardly any effect in green light. The emissivity exceeded its normal value slightly between $\alpha = 10^\circ$ and $\alpha = 80^\circ$ to a maximum excess of 3 %.

Although the deviation from LAMBERT's law for the second tungsten sample is greater than for the first, I cannot account for the still much greater effect, found by WORTHING. His p line reaches a maximum of 204 %, mine never exceeds 190 %.

For completeness I must mention measurements performed by Dr. DE GROOT in this laboratory on tungsten ribbon filaments in natural light (these measurements have not yet been published). His tungsten was not recrystallised, but exceedingly well-polished. The deviation from LAMBERT's law which he found was the same as was found by me for unicrystalline tungsten.

Summarising, we come to the conclusion that, at least for tungsten, the deviation from LAMBERT's law depends on the condition of the surface in a not quite controlable manner. It is evident that a better polishing tends to make the effect smaller.

In conclusion, I tabulate the values for the optical constants, which I have computed from the various measurements. For the normal emissivity has been taken the value 0.44 for tungsten and 0.37 for molybdenum.

Observer	Material	m	$\varkappa$
WORTHING	Tungsten	4.28	3.33
ZWIKKER	Tungsten, unicrystalline	2.96	3.36
ZWIKKER	Tungsten, multicrystalline	2.90	3.35
WORTHING + ZWIKKER	Molybdenum, multicrystalline	3.75	4.25

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Natuurkundig Laboratorium der
N. V. PHILIPS' *Gloeilampenfabrieken.*

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