Geology. — *The compacting pressure of sediments.* By J. VERSLUYS.

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Attention was drawn in 1908 by H. C. SORBY (1, pp. 227—32) to the phenomenon that most sediments, according as they attain a greater depth, become more compressed, so that the volume of pores decreases. This is especially of effect for sediments that arise from mud, i.e. clay, shale and slate, but also with quartzsand and sandstone at greater depths the porosity declines (6). As a rule this phenomenon is called compaction. This has recently been treated at length by H. D. HEDBERG (2) and W. W. RUBEY (6).

The compaction of clay at no great depth is a consequence of this, that the adsorbed water at the points of contact of the solid particles is driven out enabling these particles to approach each other. The voids — under these circumstances pores can not be yet spoken of — then become smaller. If the pressure be again diminished, the clay then expands, so long as no other manner of compaction has taking place (3). During this first compaction the particles will also become arranged otherwise and owing to this, on the cessation of the pressure, even though it may be able to absorb water, the clay will not again expand to the volume it had before compaction set in.

The second way of compaction, the other arrangement of the solid particles is still able to proceed, even after the mutual contact has been attained, but then friction of the solid particles over each other takes place. That process, upon further compression, however, will cease when at all the points of contact of the solid particles the angle the direction of the pressure forms with the normal of the common tangent plane of two particles is smaller than a certain angle of friction. The gliding of the particles, however, can even then take place, provided the pressure increases in one direction and the above mentioned angles change, a proportional increase of the pressure in all directions, however, cannot any longer effect any compaction in this sense. It would even be conceivable that under alround pressure, as long as this manner of compaction is of effect, the volume of the clay or rock will become smaller without any increase of pressure. With the first kind of compaction however, by driving out the adsorbed water, a certain connection must exist between volume, respectively porosity and pressure (2).

If the pressure increases, compaction can take place in two other ways, viz : by recrystallization and by the crushing of the particles. With the recrystallization it may be presumed that at the points of contact of the particles, solid matter becomes dissolved and that this again becomes crystallised elsewhere on the surface of these particles. A new texture then arises ; this phenomenon was described by W. H. RUSSELL (5) for sandstone. The sediment then undergoes a change from a granular to spongy mass.

The crushing of some particles, as is discussed above, must again be succeeded by compaction, owing to alteration of the arrangements, by which the smaller particles occur, due to the crumbling of the larger, and force a way into the former pores.

If it be taken that the deposit of the sediments and the sinking of those already deposited takes place so slowly that compaction at any depth may be always considered as having been completed, as regards the pressure operating there, the porosity, at any depth, might be considered as a function of the pressure and the porosity would then indirectly be a function of the depth. From the foregoing observation, however, it would then follow that this need not be a continuous function. At a slight depth this function is determined by adsorption, and at greater dephts by entirely other phenomena.

The pressure exercised by a sediment of a certain porosity can be easily calculated as long as the mass is still granular. We then have to do with the pressure exercised by submerged bodies. If the specific gravity of the grains be s, and if the porosity (i.e. the quotient of the volume of the pores and that of the entire mass) p, then the weight of the solid matter in the unit of volume of the mass is $(1-p)\,s$, and the pressure of the water upwards upon this quantity of matter $1-p$, hence the pressure per unit of volume is $(1-p)\,(s-1)$. The specific gravity of the water is presumed to be 1.

Therefore at a depth y the pressure is :

$$D = (1-p)(s-1)\,y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The hydrostatic pressure of a column of water of this height is y, hence the sediment exercises a pressure that is $(1-p)\,(s-1)$ times that of a column of water of the same height.

H. D. HEDBERG (2, p. 1038) states that according to C. K. LEITH and W. J. MEAD, the specific gravity of the solid particles of clay is 2.68, of shale 2.71 and 2.70—2.85 for slate. If 2.75 be taken as the average specific gravity :

$$(s-1)(1-p) = 1{,}75\,(1-p) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and for $p = 0.43$, the value of this form is 1. If the porosity of the sediment be 0.43, it then thus exercises a pressure, that is equally great as that of a column of water of the same height. If the porosity be smaller, the pressure becomes greater :

porosity	$(1-p)(s-1)$
0,4	1,05
0,3	1,22
0,2	1,40
0,1	1,57
0,01	1,73

The figures in the second column represent the pressure at the values of the porosity given in the first column, which a layer of granular sediment, having a thickness 1, exercises upon the grains of the sediment lying below there. This pressure multiplied by the depth, represents the pressure that aims at crushing, or bringing the grains closer together.

Formula (1) can also be deduced in another manner. If per unit of area at a depth y, the volume of the pores declines q, the whole mass of sediment with the water in its pores, with a thickness y, will sink a small depth q, on the other hand, however, a volume of water q will rise over a height y.

Hence the following energy is exerted :

1⁰. Positive, by the sediment : $(1-p)\ syq$.

2⁰. Positive, by the water : pyq.

3⁰. Negative, by the water : yq.

The total energy that is exerted by the sinking of the sediments is therefore :

$$A = \{(1-p)\,s + p - 1\}\,y\,q = (1-p)(s-1)\,y\,q \quad . \quad . \quad . \quad (3)$$

The distance is q, hence the force of the pressure exercised in this case, is

$$D = \frac{A}{q} = (1-p)(s-1)\,y, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

as deduced in formula (1).

The last deduction, that of formula (4) is unaltered of application for a porous rock, for a spongy mass, as is referred to above.

If the pressure will be able to result in the further compaction of the sediments, it must then exercise energy :

1⁰. to cause the solid particles to approach each other, to cause matter to recrystallise, to effect a plastic change of form, to crush particles and to cause particles to glide over each other ;

2⁰. To get the water to move from the diminished pore space through the pores of the sediments.

The work referred under 1⁰, may be spoken of as the compaction work, that referred to under 2⁰, is a loss, owing to friction in a fluid. The question would be whether that loss is great or small.

According to DARCY's law (see 4) the volume of water that flows per unit of time under a rate of decline $\frac{dh}{dl}$, the unit of area of the cross section in a sediment is :

$$u = k\frac{dh}{dl}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which k is a factor dependent upon the nature of the sediment, if h is the pressure head and l the distance.

With an unvariable transversal section is :

$$u = k\frac{h}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or

$$h = \frac{ul}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

The energy exerted with this per unit of time is per unit of area :

$$a = uh = \frac{u^2 l}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If the sediments over the height y are homogeneous one can write :

$$a = \frac{u^2 y}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$u = \frac{q}{t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

if t is the time in which the volume of pores, as is above discussed has decreased q. From this follows :

$$a = \frac{q^2 y}{kt^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and the total energy per unit of area is :

$$A = at = \frac{q^2 y}{kt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Hence the energy absorbed by the friction of the water is inversely proportional to the time of the compaction. This energy would effect a great counter pressure, provided A had a great value. If the compaction, however, takes place slowly, t becomes great and the value of A small, so that the friction can become so small as to be negligible.

The value of the factor k is very small for the finer sediments, for the larger it is much greater. If fine sediments become intermingled with some coarser strata, the water will then be carried off laterally by the last mentioned, and, owing to this, the friction becomes much smaller.

In formula (1), as has already been observed, the value of s changes slightly with the depth, but that of p is dependent upon the pressure and consequently also upon the depth, hence may be written :

$$p = f(y) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and the pressure through a thin layer of sediment having a thickness dy is thus for every depth :

$$D = \{1 - f(y)\}(s - 1)\,dy, \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

which follows from formula (1). Hence the total pressure at a depth y is :

$$D = (s - 1)\left\{y - \int_0^y f(y)\,dy\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

As is mentioned above, for great values of y the function $f(y)$ is not continuous within the limits 0 and y, so the value of D will be better represented by :

$$D = (s - 1)\left\{y - \int_0^{y_1} f_1(y)\,dy - \int_{y_1}^{y_2} f_2(y)\,dy \ldots - \int_{y_{n-1}}^{y_n} f_n(y)\,dy\right\}. \quad . \quad (15)$$

The functions $f_1, f_2, \ldots\ldots, f_n$ are not known otherwise than from observation, if this be possible. If the value of p at various depths be known, instead of formula (15) we may be able to be satisfied with :

$$D = (s - 1)(y - \Sigma\, p \triangle y), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

in which $\triangle y$ are the various heights, concerning which the porosity is considered as having the certain value p. Hence $\Sigma \triangle y = y$.

H. D. HEDBERG (2, pp. 1052 and 1053) gives various figures for the porosity of clay, shale and slate, taken partially from H. C. SORBY (1). For slate these may decline to $\frac{1}{400}$.

To prevent misunderstanding it should further be pointed out that the pressure of the sediments which tries to cause those open spaces such as excavations at great depth, which are not entirely filled with water nor with air under great pressure, to collapse, must not be calculated in this manner. In this case at the deduction of formula (3) referred to sub. 3° the work qy does not intervene and in the place of formula (4) we may write :

$$\{(1 - p)s + p\}\,y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Hitherto it has been presumed that the specific gravity of the water is 1. If the water be salt, this is not so and thus the specific gravity of the water is s_1, the formula then becomes :

$$D = (1 - p)(s - s_1)\,y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Should the specific gravity of the water not be everywhere the same, in stead of s_1 a function of the depth y can be put.

LITERATURE.

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