Mathematics. — *Representation of a Bilinear Congruence of Conics.* By Prof. JAN DE VRIES.

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1. A congruence $[k^2]$ of conics is called bilinear, if an arbitrary point of space defines one conic k^2 and an arbitrary straight line is a chord of one k^2. Evidently the planes of the conics pass through one fixed point O and form, therefore, a sheaf. Accordingly a line s through O is a chord of ∞^1 conics (*singular line*); these form a cubic surface Σ^3. This contains 5 pairs of lines that belong to $[k^2]$.

The surfaces Σ_1^3 and Σ_2^3 corresponding to the lines s_1 and s_2, have the k^2 in the plane $s_1 s_2$ in common. Any point S of the curve σ^7 which they have besides in common, carries a k^2 that cuts s_1 twice and a k^2 that cuts s_2 twice; accordingly through S (*singular point*) there pass ∞^1 conics.

The surface Σ^3 defined by the line OS, has a *double point* in S. This holds especially for the surface Ω^3 through the conics containing O.

The plane of any k^2 passes through O and cuts σ^7 in *six* more points; these lie on k^2.

2. In order to arrive at a *representation* of the congruence we shall consider two lines a_1 and a_2 chosen at random as directrices of a congruence of rays; this contains one chord b of a k^2; the transit B of b through a fixed plane β is considered as image of the k^2.

A point B of β usually carries one transversal b of a_1 and a_2 and is, therefore, the image of the k^2 lying in the plane Ob.

The conic k_0^2 that has the transversal b_0 in β as chord, is represented in the point range (B_0) of b_0.

The transit A_1 of a_1 is *singular* for the representation; the lines that join A_1 to the points of a_2, are chords of the conics that cut OA_1 twice and, consequently, form a surface Σ^3. Analogously A_2 (transit of a_2) is *singular*.

The transversal a of a_1 and a_2 through O is a chord of ∞^1 k^2; this is represented in the transit A of a; accordingly also A is *singular*.

3. The *conics* that have a straight line s as *chord*, are represented by the rays of the scroll with directrices s, a_1 and a_2. Their images form, therefore, a conic β^2 through A_1, A_2 and A (one of these k^2 passes through O).

Any β^2 through A_1, A_2 and A is the image of a system of conics that have a line s as chord. For if we choose two points B_1 and B_2 on β and if through these points we draw the transversals b_1 and b_2, the line of intersection s of the planes Ob_1 and Ob_2 defines the system of the k^2.

Two curves β^2 have one point B in common besides the singular points; it is the image of the k^2 lying in the plane $s_1 s_2$.

4. The *point range* (B) on a line c of β is the image of a system Γ of conics each of which has a transversal of c, a_1 and a_2 as chord. As these chords form a scroll, their planes *touch a quadratic cone* with vertex O.

Hence through a point S there pass two k^2 of Γ; on the surface of the conics of this system σ^7 is a *nodal curve*. The intersection of this surface with a surface Σ^3 consists, therefore, of the curve σ^7, to be counted twice, and of the two k^2 that are represented in the points of intersection of c and the β^2 defined by Σ^3. Consequently c is the image of a *surface* Γ^6.

5. Let Λ be the surface formed by the k^2 that cut a given line l; as *double curve* it has the k^2 that cuts l twice.

The surface Σ^3 corresponding to a point S contains three k^2 resting on l; hence σ^7 is a *triple curve* on Λ.

The image curve λ of the system has a triple point in A. Also A_1 and A_2 are triple points, for AO_1 and AO_2 are chords of three conics that rest on l. Besides the points A, A_1 and A_2, λ has three points B in common with a curve β^2, the images of the k^2 of Λ that have the line s as chord. Accordingly Λ has an image curve λ^6 with *three triple points*.

Two curves λ^6 have nine points B in common; there are, therefore, 9 k^2 that rest on two lines l, and the *surface* Λ has the degree 9.

6. A plane φ is cut by $[k^2]$ in the pairs of an involution. The pairs on the rays of a plane pencil with vertex M lie on a curve μ^3. Four rays of this plane pencil are tangents of conics k^2; hence OM is a chord of four k^2 that touch the plane φ. The conic β^2 corresponding to OM contains, therefore, the image points B of four k^2 touching φ.

The image curve of the system of the k^2 that touch φ, has *quadruple points* in A_1, A_2 and A; it is, therefore, a φ^8.

Two curves φ^8 have 16 non singular points in common; there are, accordingly, 16 k^2 that touch two given planes.

A φ^8 has 12 points B in common with a λ^6; consequently the conics that touch φ, form a *surface* Φ^{12} with *quadruple curve* σ^7.

7. The *pairs of lines* belonging to $[k^2]$ form a scroll $\triangle$. As any surface Σ contains five of these pairs, the image curve δ of $\triangle$ has quintuple points in A_1, A_2 and A and any β^2 contains five more points B of δ. Accordingly the image curve is a δ^{10}. It has 15 non singular points in common with a λ^6; hence $\triangle$ is a *scroll* of the degree *fifteen*. Indeed, a line s cuts five lines of $\triangle$ in O and ten lines outside O.