Mathematics. — *On the Limits of Holomorphic Functions.* By Prof. J. WOLFF. (Communicated by Prof. L. E. J. BROUWER).

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FATOU's theorem: "if $f(z)$ is holomorphic and limited for $|z| < 1$, $\lim_{\rho \to 1} f(\varrho e^{\varphi i})$ exists, except perhaps for values of φ the set of which has the measure zero", has been completed by the brothers RIESS in the following way (Stokholm Congress 1916): "If a is an arbitrary number this limit is equal to a only for a φ-set of the measure zero".

In the following note we shall derive this result without making use of integrals of LEBESGUE.

Without loss of generality we may assume $f(0) \neq a$. We suppose $|f| < M$. We have for $0 < \varrho < 1$

$$2\pi \lg |f(0) - a| \leqq \int_0^{2\pi} \lg |f(\varrho e^{\varphi i}) - a| \, d\varphi \quad . \quad . \quad . \quad . \quad (1)$$

Suppose $1 > \varepsilon > 0$ and let E_n be the set of intervals in which

$$\left| f \left\{ \left(1 - \frac{1}{n}\right) e^{\varphi i} \right\} - a \right| < \varepsilon.$$

If μE_n is the measure of E_n it follows from (1) and from $|f - a| < 2M$, that

$$2\pi \lg |f(0) - a| \leqq \mu E_n \, . \, \lg \varepsilon + 2\pi \lg (2M),$$

hence $\mu E_n \leqq \dfrac{C}{\lg \dfrac{1}{\varepsilon}}$, $n = 1, 2, \ldots;$ C constant.

Now the values of φ for which $\lim_{\rho \to 1} f(\varrho e^{\varphi i}) = a$, belong to the limes inferior of E_n for $n \to \infty$.

Hence the measure of the set of these values is at most $\dfrac{C}{\lg \dfrac{1}{\varepsilon}}$ and as ε may be chosen arbitrarily between 0 and 1, this measure is zero.

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