Physics. — *The best method of measurement of a resistance thermometer.* (21^{st} Communication of results obtained by the aid of the "VAN DER WAALS-Fund"). By A. MICHELS and P. GEELS. (Communicated by Prof. J. D. VAN DER WAALS Jr.).

(Communicated at the meeting of December 17, 1927).

The present communication forms a continuation of the 19^{th} communication [1]) in which the replacement of the ice-point of the thermometer scale by another fixed point, reproducible to within $^{1}/_{4000}{}^{\circ}$, was proposed. In connection with the desired accuracy, it was found necessary to investigate the factors, which determine the accuracy of a resistance thermometer, and to find how the influence of these factors could be reduced to a minimum.

The following considerations are also partially applicable to other observations. The use of a resistance thermometer depends on the change of the resistance of a measuring wire with temperature, and other influences which result in an alteration to the resistance (for example the pressure effect) are amenable to similar treatment.

Besides the external factors, such as the choice of galvanometer, the accuracy of the resistance boxes used etc., which influence any resistance measurement, the most troublesome factor in the use of a resistance thermometer is the temperature rise of the measuring wire resulting from the measuring current.

In an absolute temperature measurement it is therefore desirable not to work with current which results in a temperature rise of the wire greater than the accuracy with which it is desired to establish the temperature. [2])

The temperature rise is determined by two factors, the amount of heat evolved by the Joule effect and the velocity with which this heat is dissipated to the surroundings. The latter is very greatly influenced by the construction of the thermometer and the best construction will be that with which the heat is dissipated as rapidly as possible, in other words a thermometer with as small a lag as possible, a factor also very desirable for other reasons.

The lower limit of this lag will be largely determined by other conditions such as insulation, stability etc. which will not be considered further.

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[1]) These Proceedings, **30**, p. 1017 (1927).

[2]) N.B. Actually it should be permissible to work with a constant temperature rise, but it would then be necessary to be certain of the constancy.

Even though it is understood that the best arrangement has been chosen so far as these factors are concerned, a large variation may still be made as to the length and diameter of the wire and the measuring current.

The temperature of the wire is given by the expression:

$$\delta t = \beta \frac{i^2}{d^3}$$

where β is a constant, i the measuring current and d the diameter of the wire. This relation between δt and i has already been tested and established [1]). The temperature rise of the wire is therefore proportional to the square of the measuring current.

From the 20[th] communication (to be published in the following number of these Proceedings, vol. **31**) it follows that, if i is the current strength in the measuring wire and dR an arbitrary alteration to the resistance R, then the galvanometer deflection is given by

$$a = C \frac{i \dfrac{dR}{R} \times R}{\sqrt{g + R_0 + R}},$$

when a moving coil galvanometer within its aperiodic limits is used.

If a moving coil galvanometer in a constant field is used, this expression only differs in the numerator, the root being replaced by the first power.

As a moving coil galvanometer is usually used, the derivation will be given for this instrument in its aperiodic limit and only the result given for the other case, which may be obtained in exactly the same way.

It is hardly necessary to mention that the formulae used hold for almost any circuit, whether a potentiometer, a differential or a bridge method is used. In the last circuit it is understood that $n \gg 1$ (loc. cit. for the notation used). These conditions are not sufficient in the case of the THOMSON Bridge, but this bridge is of little importance for the present purpose. The only alteration that can occur is in the value of R_0, which disappears in some cases, and which is always small compared to the galvanometer resistance g (loc. cit.).

In order to simplify the present calculations $g + R_0$ has been replaced by G, so that the above expression becomes

$$a = C \frac{i \dfrac{dR}{R} \times R}{\sqrt{G + R}}.$$

It is at once apparent from this expression that the accuracy of the measurement is directly proportional to i, whilst, as already observed, δt is proportional to i^2. These are therefore two opposed influences.

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[1]) 17[th] Communication of the VAN DER WAALS Fund. These Proceedings **30**, p. 47.

The limit of the measurable temperature interval is that interval which is just equal to the temperature rise of the wire itself.

It is thus necessary to know the length and diameter of the wire for a given resistance R, with which a minimum temperature rise is obtained.

As already indicated

$$\delta t = \beta \frac{i^2}{d^3}$$

whilst the resistance R is given by

$$R = \gamma \frac{l}{d^2}$$

where γ is a constant and 1 the length of the wire.

From these expressions it follows that the smallest temperature rise with a given current is obtained when d is as large as possible, and, therefore, for a given R, when the wire is as long as possible.

This is also clearly shown by eliminating d from the above two expressions to give

$$\delta t = A i^2 \left(\frac{R}{l}\right)^{3/2}$$

(A is a constant).

Thus, from either point of view, it is desirable to make the wire as long as possible. Other external factors, such as the winding space, necessary distance for insulation etc., will determine the value of l. If it is assumed that l is made as large as possible in relation to the method of measurement, l may be considered as a constant and will disappear as a variable from the equations.

The problem may then be solved as follows.

Let $\triangle t$ be the temperature alteration which it is desired to measure and δt the temperature increase which may be tolerated (this may be left undecided, if a relation is afterwards established between $\triangle t$ and δt).

As δt has been chosen, it may be treated as a constant.

The deflection of the galvanometer is given by

$$a = C \frac{i \frac{dR}{R} R}{\sqrt{G+R}}$$

in which $\frac{dR}{R}$ is proportional to $\triangle t$.

Put $C \frac{dR}{R} = D \triangle t$

$$a = D \triangle t \frac{iR}{\sqrt{G+R}}$$

It is now necessary to find the minimum value of $\triangle t$ under the given condition that the temperature rise is not greater than δt. This is a limiting condition capable of mathematical determination

$$\delta t = \beta \frac{i^2}{d^3} \quad \text{whilst} \quad R = \gamma \frac{l}{d^2}.$$

Eliminating d and bringing all the constants (including δt) under one letter, the limiting condition may be expressed as

$$i^4 R^3 = E.$$

There is an experimental value of α, which is the smallest value observable. Let this be μ, then the smallest value of $\triangle t$ is given by

$$\mu = D \triangle t \frac{iR}{\sqrt{G+R}}$$

when $\frac{iR}{\sqrt{G+R}}$, which may be represented by z, is made as large as possible within the limiting conditions.

If z is plotted in a space diagram as a function of i and R (the x- and y-axis respectively), the question is reduced to the determination of the maximum of a surface with a border condition. This condition defines a space curve on the surface. From the expression $z = \frac{iR}{\sqrt{G+R}}$ it appears that the surface is regular and that the boundary lines go through the R-axis and run parallel to the $(z - i)$ surface. The surface therefore possesses no *absolute* maximum, although it reaches a maximum value on the boundary, and the question is therefore reduced to the determination of the maximum of the space curve defined by the two equations

$$z = \frac{iR}{\sqrt{G+R}}$$

$$i^4 R^3 = E.$$

This determination is made as follows:

$$F = K \frac{iR}{\sqrt{G+R}} + \lambda (i^4 R^3 - E) \qquad i^4 R^3 - E = 0$$

$$F'_{(i)} = K \frac{R}{\sqrt{G+R}} + 4 \lambda i^3 R^3 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$F'_{(R)} = K \frac{i}{\sqrt{G+R}} - \frac{1}{2} K \frac{iR}{\sqrt{G+R}^3} + 3 \lambda i^4 R^2 = 0 \quad . \quad . \quad (2)$$

(1) and (2) give:

$$\frac{K}{\sqrt{G+R}} + 4\lambda i^3 R^2 = 0. \qquad \ldots \ldots \ldots \quad (3)$$

$$\frac{K}{\sqrt{G+R}} - \frac{1}{2}\frac{KR}{\sqrt{G+R}^3} + 3\lambda i^3 R^2 = 0.$$

$$\frac{1}{2}\frac{KR}{\sqrt{G+R}^3} + \lambda i^3 R^2 = 0$$

substituting in 3

$$\frac{K}{\sqrt{G+R}} - 2\frac{KR}{\sqrt{G+R}^3} = 0$$

$$R = G.$$

The result $R = G$ is independent of the value of E and therefore of δt and hence holds for the case chosen $\delta t = \triangle t$.

In the latter case it is possible to obtain a simpler solution, using the same proof that the maximum lies on the border curve, as the expression $\triangle t = \beta \frac{i^3}{a^3}$ is not then a condition for the maximum, but is an absolute equation.

Eliminating d between this equation and $R = \gamma \frac{l}{d^2}$ gives

$$\triangle t^2 = H i^4 R^3.$$

Solving for i and substituting the value in

$$a = D \triangle t \frac{iR}{\sqrt{G+R}}.$$

$$= L (\triangle t)^{3/2} \frac{R^{1/4}}{\sqrt{G+R}} \qquad (H \text{ and } L \text{ constant})$$

or for minimum $a = \mu$

$$\mu = L (\triangle t)^{3/2} . \frac{1}{\sqrt[4]{\frac{(R+G)^2}{R}}}.$$

The smallest value of $\triangle t$ is obtained when $\frac{(G+R)^2}{R}$ is a minimum.

Differentiation gives $G = R$.

It thus appears that the best value is found when R is made equal to G. R is therefore determined, and where l is fixed, d is also established. A simple numerical calculation shows that the maximum is not very pronounced and that a variation of 50 % in d does not make any appreciable alteration to the best conditions.

The value of $\triangle t$ corresponding to the value of R can only be calculated when the necessary experimental data relating to the radiation, galvanometer sensitivity etc. are known.

The above deduction is only practicable when the value $R = G$ lies within the limits in which the variable shunt resistance can be regulated.

A similar calculation for a moving coil galvanometer in a constant field gives $R = {}^1/_3\, G$.

In conclusion a few notes on a circuit with overlapping shunts will be given in connection with the above.

In this circuit it is only the difference between the two currents passing through the galvanometer circuits, that acts as a directing current on the galvanometer. This results in a large current being passed through each of the galvanometer coils and the limitations of the galvanometer current being reached before those of the current in the measuring wire. This inconvenience may be avoided in the following way:

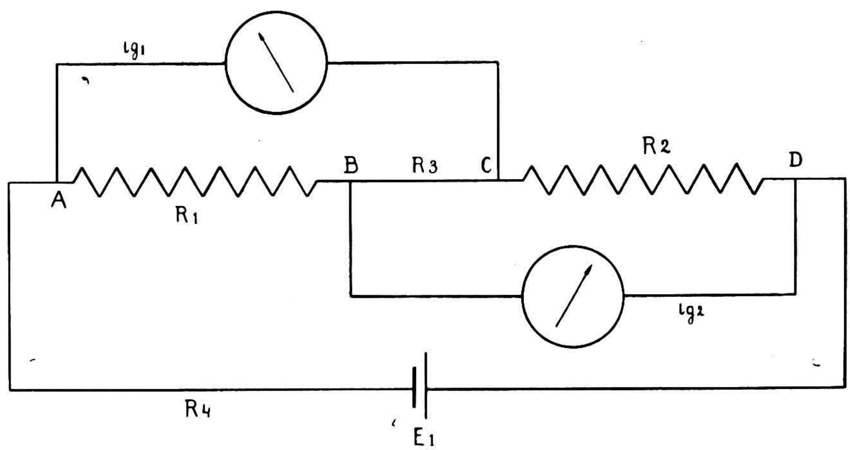

Fig. 1.

Fig. 1 is a schematic diagram of a Kohlrausch circuit, the current commutation being omitted.

Take, as is sufficient for the present derivation, the case when the two resistances R_1 and R_2 are equal. The galvanometer is adjusted to give no deflection. An alteration of the resistance from $R \rightarrow R + dR$ gives a deflection, which is determined by the algebraic sum of i^{g_1} and i_{g_2}.

In the calculation of the deflection dR may be replaced by an *E.M.F.* idR, E left out of consideration and R_4 broken.

In the equilibrium condition an *E.M.F.* in R_3 will also make no alteration to the algebraic sum of i_{g_1} and i_{g_2}.

According to the superposition law an *EMF* may be introduced into R_3 as well as into R_4. The two *EMF*'s together will not influence the algebraic sum $i_{g_1} + i_{g_2}$. The latter, and therefore the galvanometer deflection, will remain exclusively determined by idR. If E_1 and E_2 are chosen opposite in sign i_{g_1} and i_{g_2} may be both reduced to a very small value by the exact choice of E_1 and E_2.

The Steinwehr commutator must be modified to incorporate this addition.

It is desirable to place a shunt across both E_1 or E_2 in order to obtain an exact regulation.

Figure 2 gives the potential fall in the main circuit.

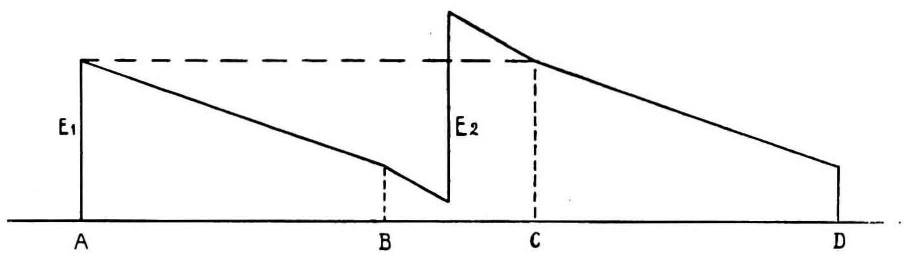

Fig. 2.