Physics. — *Experiments on the velocity distribution in the boundary layer along a rough surface; determination of the resistance experienced by this surface.* By B. G. VAN DER HEGGE ZIJNEN. (Mededeeling N^0. 10 uit het Laboratorium voor Aerodynamica en Hydrodynamica der Technische Hoogeschool te Delft.) (Communicated by Prof. J. D. VAN DER WAALS Jr.)

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§ 1. *Introduction.*

In a discussion of the experimental data concerning the resistance experienced by the flow through pipes and channels with rough walls, HOPF [1]) deduced that this resistance is proportional to the square of the mean velocity, at least if the state of motion is entirely turbulent. According to FROMM [2]) the same law holds for the pressure drop in a rectangular channel, the walls of which are composed of surfaces having a well defined roughness. Evidently in these cases the resistance coefficient is independent of REYNOLDS' number and is determined entirely by the dimensions of the channel and of the projections on its surface.

However, the numerous experiments on the resistance of flow still need completion and extension; researches on the distribution of the velocity in the boundary layer, on the development of the boundary layer along the surface and more detailed experiments concerning the resistance of rough surfaces are of importance for a better understanding of the phenomena observed by several experimenters.

The opportunity for carrying out such researches in the Laboratory for Aerodynamics and Hydrodynamics of the Technical University at Delft presented itself in the beginning of 1925, when by the courtesy of Prof. VON KÁRMÁN a sheat of "waffle-plate", of about the same aspect as used by FROMM in his researches on rectangular channels, was put at our disposal. The experiments were finished in 1927, when the measurements on the velocity distribution could be completed and checked by weighing the resistance experienced by a board, covered on both sides with waffle-plate, directly on a balance.

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1) L. HOPF. Abhandl. a. d. Aerodynamischen Institut der Techn. Hochschule Aachen III, 1924, p. 1; Zeitschr. f. angewandte Math. u. Mech. 3, 1923, p. 329.

2) K. FROMM. Abhandl. a. d. Aerodynamischen Institut der Techn. Hochschule Aachen III, 1924; Zeitschr. f. angewandte Math. u. Mech., 3, 1923, p. 339.

§ 2. *Summary of the principal theoretical data about the motion in the boundary layer.*

Before describing the experiments performed and discussing their results, some formulae concerning the flow in the boundary layer along a rough surface may be deduced.

The following considerations are based upon the hypothesis of Prof. VON KÁRMÁN [3]) that the resistance of a rough surface is entirely due to the head resistance of the projections. It might be expected that as this head resistance, at least above a certain value of REYNOLDS' number, follows the quadratic law, the same will be the case with the resistance experienced by the entire surface; this supposition is supported by the work of HOPF and FROMM mentioned above.

The velocity distribution in a section of the boundary layer will now be supposed to satisfy a relation of the form:

$$u = V\left(\frac{y}{\delta}\right)^n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where u = velocity (parallel to the surface) in an arbitrary point of the section considered; V = velocity of the undisturbed flow just outside of the boundary layer; y = distance of the point in question from the surface; and δ = thickness of this section of the boundary layer.

This relation has to be verified by experiment. If the surface is not too rough, and if REYNOLDS' number for the boundary layer $\left(R_\delta = \frac{V\delta}{\nu}\right)$ is sufficiently high, it may be expected from the results of other researches that form. (1) will hold.

Although about the flow in the vicinity of the projections nothing can be predicted with certainty, following VON KÁRMÁN the velocity at the top of a projection of height h may be expressed by:

$$u_t = aV\left(\frac{h}{\delta}\right)^n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where a is an unknown numerical constant.

The resistance of a projection will now be proportional to u_t^2, and the resistance experienced by the surface per unit area may be written:

$$\tau_0 = c\varrho V^2\left(\frac{h}{\delta}\right)^{2n} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

In order to deduce a relation between δ and the distance x of the section

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[3]) TH. VON KÁRMÁN. Ueber die Oberflächenreibung von Flüssigkeiten, Vorträge aus dem Gebiete der Hydro- und Aerodynamik, Innsbruck 1922 (Berlin 1924), p. 146.

considered from the leading edge of the surface, we may use the formula given by VON KÁRMÁN [4]) for the loss of momentum. It has to be born in mind, however, that the experiments described below relate to a surface mounted in a wind tunnel, in which case the velocity V outside of the boundary layer is not constant, but increases down stream in consequence of the narrowing of the passage of the flow, caused by the growth of the boundary layers along the tunnel walls and along the surface.

The increase of V is rather small; putting:

$$\frac{dV}{dx} = \beta V \quad \ldots \ldots \ldots \ldots \quad (4)$$

we may neglect terms as $\beta^2 x^2$, $\beta^3 x^3, \ldots$, in the following formulae, even when x is equal to the length l of the plate.

In this case the integral of (4) is:

$$V = V_0 (1 + \beta x) \quad \ldots \ldots \ldots \quad (4^a)$$

The loss of momentum in the boundary layer per unit length will be:

$$\frac{d}{dx}\int_0^\delta u^2\, dy - V \frac{d}{dx}\int_0^\delta u\, dy = -\frac{\tau_0}{\varrho} + \delta V \frac{dV}{dx} \quad \ldots \quad (5)$$

Inserting form. (1), (3) and (4) and dividing by V^2, equation (5) is reduced to [5]):

$$\frac{n}{(2n+1)(n+1)}\frac{d\delta}{dx} + \frac{2n^2+3n}{(2n+1)(n+1)}\beta\delta = c\left(\frac{h}{\delta}\right)^{2n},$$

the integral of which is, when $\delta = 0$ for $x = 0$:

$$\delta = \left\{\frac{(2n+1)^2(n+1)c}{n}\right\}^{\frac{1}{2n+1}} x^{\frac{1}{2n+1}} h^{\frac{2n}{2n+1}} \left\{1 - \frac{2n+3}{2}\beta x\right\} \quad . \quad . \quad (6)$$

Hence from form. (3), inserting the value of V from (4a) and that of δ from (6), we get:

$$\tau_0 = c\varrho V_0^2 \left\{\frac{n}{(2n+1)^2(n+1)c}\right\}^{\frac{2n}{2n+1}} \left(\frac{h}{x}\right)^{\frac{2n}{2n+1}} \{1 + (2n^2+3n+2)\beta x\} \quad . \quad (7)$$

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[4]) TH. VON KÁRMÁN. Ueber laminare und turbulente Reibung, Zeitschr. f. angewandte Math. u. Mech., 1, 1921, p. 235 form. (5).

[5]) The relations for δ, τ_0 and c_i which are given here for the case of an accelerated flow, have been deduced by Prof. BURGERS.

The integral of (7) with respect to x, multiplied by the breadth b of the plate, gives the total resistance :

$$\left.\begin{aligned} W &= b\int_0^x \tau_0\, dx \\ &= c\varrho V_0^2 b x^{\frac{1}{2n+1}} h^{\frac{2n}{2n+1}} \left(\frac{1}{2n+1}\right)^{\frac{2n-1}{2n+1}} \left(\frac{n}{(n+1)c}\right)^{\frac{2n}{2n+1}} \left\{1+\frac{2n^2+3n+2}{2n+2}\beta x\right\} \end{aligned}\right\} \quad (8)$$

This expression is simplified by introducing :

$$I = \varrho \int_0^\delta u\,(V-u)\, dy = \frac{n}{(2n+1)(n+1)}\varrho\, V^2\,\delta \quad . \quad . \quad . \quad . \quad (9)$$

The quantity I will be called the defect of momentum in the boundary layer. It has to be noted that in (9) occurs the local value of the velocity, V, and not V_0.

By means of (4a), (6) and (9) form. (8) leads to :

$$W = bI\left\{1+\frac{(2n+1)^2}{2\,(n+1)}\beta x\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

If n and c are found from experimental results for a known value of β, the behaviour of δ, τ_0 and the value of W in the case of an unlimited flow (where V is constant), are easily deduced by putting everywhere $\beta = 0$ in the formulae.

§ 3. *Experimental arrangements.*

In order to collect data about the distribution of the velocity in the neighbourhood of rough surfaces and to check the formulae deduced for δ and for the resistance, experiments were performed with two sheats of "waffle plate" of dural, as used f.i. for covering treadles. The first plate served only for the measurements in the boundary layer ; later on, when a greater sheat of the metal was put at our disposal, a second plate with projections differing but slightly from those of the first, was used for measuring directly on a balance the total resistance experienced by a board covered on both sides with it and for determining the loss of momentum in the wake down stream. In order to compare the results of these experiments with those performed on the first plate, a few measurements of the velocity distribution in the boundary layer along the second plate were performed.

Waffle plate I. The surface may be characterized as follows : The projections had the shape of quadrilateral pyramids, arranged in regular horizontal and vertical rows without change ; their mean height was

TABLE I. VELOCITY IN THE BOUNDARY LAYER (IN CM/SEC).

WAFFLE PLATE I

y cm.	V = 811 cm/sec. x = 25 t	v	50 t	v	75 t	v	100 t	v	125 t	v	150 t	v	175 t	v
0.025	380	39	354	36	230	35	310	35	324	19	300	18	317	22
0.050	428	90	384	81	325	90	350	74	377	58	361	31	375	66
0.075	462	152	425	140	384	121	371	125	400	106	396	64	396	122
0.100	476	236	439	214	409	227	400	196	425	176	416	116	425	185
0.125	485	331	450	304	434	304	409	270	441	256	441	184	436	262
0.150	520	396	466	361	445	361	425	313	452	341	450	270	445	327
0.175	529	434	485	400	459	389	445	342	461	379	459	334	461	374
0.200	540	450	490	412	485	416	450	374	471	409	471	379	470	404
0.250	573	493	515	450	490	450	476	412	489	432	484	415	485	434
0.300	592	525	529	471	511	459	485	445	501	459	494	441	501	450
0.400	621	573	557	498	529	493	511	484	520	489	525	476	520	490
0.500	666	618	579	539	549	529	539	493	541	510	539	494	532	520
0.600	709	653	621	573	573	557	561	529	557	529	551	515	551	549
0.700	729	689	640	601	592	573	573	557	573	552	568	532	568	568
0.800	759	713	665	621	621	590	592	573	586	568	581	557	573	573
0.900	770	749	681	651	638	620	618	581	600	576	592	573	581	581
1.00	789	762	709	666	658	627	621	610	620	592	621	592	592	592
1.25	811	784	740	715	689	665	658	630	640	620	640	621	621	621
1.50	811	806	762	762	715	709	681	666	666	645	665	651	640	640
1.75	811	811	789	780	749	715	713	702	688	668	688	673	665	666
2.00	811	811	806	806	780	759	729	729	715	709	713	698	689	689
2.50	811	811	811	811	806	784	770	762	761	749	750	722	715	722
3.00	811	811	811	811	811	811	806	784	790	780	770	761	762	762
4.00	—	—	—	—	811	811	811	811	811	811	811	811	811	811
5.00	—	—	—	—	811	811	811	811	811	811	811	811	811	811
X =	200	200	175	175	175	175	175	175	200	200	200	200	200	200
Bar.	768	767	768	766	770	768	773	773	769	769	765	766	766	765
Temp.	18.4	17.3	17.8	17.8	16.1	17.4	18.6	17.5	18.0	19.6	18.0	18.4	18.4	17.1

y cm.	V = 1600 cm/sec. 25 v	50 v	75 v	100 v	125 v	150 v	175 v	V = 2400 cm/sec. 25 v	50 v	75 v	100 v	125 v	150 v	175 v
0.025	139	117	74	62	56	27	112	261	228	114	139	114	324	217
0.050	256	208	176	132	92	109	201	420	370	285	262	188	458	370
0.075	369	316	272	236	184	202	282	584	515	429	393	316	615	520
0.100	484	405	384	345	295	292	421	745	655	566	532	455	772	676
0.125	638	529	549	466	426	412	548	888	790	790	701	616	924	848
0.150	734	635	645	579	530	538	615	1056	912	950	851	771	1045	980
0.175	790	692	721	656	630	625	681	1176	1025	1060	950	911	1130	1050
0.200	853	751	780	691	691	697	761	1253	1135	1160	1020	1050	1185	1120
0.250	900	818	800	725	780	773	810	1421	1232	1240	1090	1220	1240	1225
0.300	999	870	830	809	821	818	841	1513	1310	1310	1175	1285	1332	1320
0.400	1082	942	900	890	898	906	911	1681	1460	1440	1360	1430	1430	1415
0.500	1163	1010	961	955	950	949	961	1823	1545	1550	1460	1520	1513	1500
0.600	1246	1082	1024	989	988	992	1010	1936	1640	1620	1555	1580	1570	1582
0.700	1332	1150	1056	1024	1015	1015	1040	2061	1722	1660	1620	1640	1632	1620
0.800	1436	1176	1096	1050	1035	1045	1060	2190	1840	1760	1660	1680	1675	1665
0.900	1460	1246	1130	1120	1070	1070	1080	2304	1918	1820	1740	1740	1725	1710
1.00	1513	1325	1183	1175	1100	1090	1095	2333	1955	1900	1760	1770	1765	1770
1.25	1600	1421	1262	1262	1200	1155	1175	2400	2162	2005	1900	1880	1835	1840
1.50	1600	1475	1355	1320	1262	1230	1262	2400	2245	2200	1990	1980	1935	1930
1.75	1600	1521	1450	1380	1320	1310	1320	2400	2315	2240	2120	2050	1995	2010
2.00	1600	1560	1500	1450	1360	1330	1380	2400	2345	2300	2240	2110	2060	2080
2.50	1600	1600	1550	1500	1515	1430	1450	2400	2372	2380	2300	2240	2190	2180
3.00	1600	1600	1585	1570	1560	1515	1520	2400	2400	2390	2380	2360	2275	2290
4.00	1600	1600	1600	1600	1600	1600	1570	2400	2400	2400	2400	2400	2360	2375
5.00	1600	1600	1600	1600	1600	1600	1600	2400	2400	2400	2400	2400	2400	2400
X =	200	200	175	175	200	200	200	200	200	175	175	200	200	200
Bar.	758	756	764	765	771	761	765	758	757	758	765	770	760	768
Temp.	19.7	19.1	18.8	15.4	18.6	19.1	17.0	18.8	17.7	18.0	19.2	16.8	18.6	18.6

y cm.	V = 3200 cm/sec. 25 v	50 v	75 v	100 v	125 v	150 v	175 v	WAFFLE PLATE II V_p = 2400 cm/sec. 198 cm. t	v	y cm.
0.025	368	388	210	221	185	335	243	1018	158	0.025
0.050	594	590	416	385	346	511	392	1142	295	0.050
0.075	761	761	625	510	538	686	548	1211	445	0.075
0.100	955	949	820	701	778	773	730	1260	615	0.100
0.125	1185	1130	1060	911	961	973	956	1296	824	0.125
0.150	1386	1295	1320	1120	1204	1290	1160	1340	1011	0.150
0.175	1586	1415	1460	1225	1384	1475	1340	1384	1116	0.175
0.200	1650	1585	1595	1410	1544	1545	1450	1421	1190	0.200
0.250	1864	1705	1740	1500	1665	1700	1630	1513	1310	0.250
0.300	1975	1805	1850	1620	1798	1780	1730	1568	1384	0.300
0.400	2210	1935	1990	1820	1850	1936	1884	1648	1513	0.400
0.500	2395	2095	2110	1950	1971	2061	1990	1714	1600	0.500
0.600	2580	2200	2245	2015	2100	2135	2060	1772	1714	0.600
0.700	2721	2315	2315	2100	2172	2190	2125	1815	1772	0.700
0.800	2860	2430	2400	2150	2256	2245	2200	1849	1823	0.800
0.900	2962	2520	2460	2237	2315	2305	2237	1884	1884	0.900
1.00	3062	2600	2525	2375	2375	2375	2294	1936	1936	1.00
1.25	3200	2860	2745	2500	2525	2440	2440	2043	1989	1.25
1.50	3200	3025	2845	2610	2610	2580	2540	2116	2061	1.50
1.75	3200	3115	2950	2765	2740	2725	2663	2190	2116	1.75
2.00	3200	3180	3100	2875	2845	2800	2767	2247	2190	2.00
2.50	3200	3200	3140	3080	2950	2960	2920	2381	2285	2.50
3.00	3200	3200	3160	3140	3114	3105	3030	2430	2333	3.00
4.00	3200	3200	3200	3200	3200	3180	3160	2510	2500	4.00
5.00	3200	3200	3200	3200	3200	3200	3200	2560	2560	5.00
X =	200	200	175	175	200	200	200	129	129 cm.	
Bar.	755	758	758	766	773	762	763	767	767 mm.	
Temp.	19.2	19.5	19.9	18.9	19.7	20.1	19.7	13.1	14.3 °C.	

1.7 mm ; their distance in longitudinal direction 6.5 mm, in transverse direction 6.33 mm. The tops of the projections, as well as the valleys at their root, were slightly rounded off.

Comparing this with the rough surfaces used by FROMM, our "waffle plate I" seems to approach FROMM's "Waffelblech II", although the height of the projections of the latter plate was less (height 0.858 mm, distance of the tops 6.62 mm).

This surface was nailed on a rectangular wooden board of 189.5 $\times$ 50 cm, the leading edge of which was sharpened over 15 cm. The dimensions of the metal sheat allowed to cover one side only ; however, the metal was bent around the leading edge of the board and covered the back over 10 cm. Two wooden clamps were provided at the uncovered side to give the board a greater stiffness.

This board was mounted vertically in the wind tunnel at a distance of 175 to 225 cm from the honey comb at the entrance (this distance is called X in table I).

The arrangement for the determination of the velocity distribution in the boundary layer was the same as that used in the experiments on a glass plate, described before [6]). The measurements were carried out with a hot wire anemometer (length of wire 37 mm, diameter 0.05 mm, heated to about 600° C above the surrounding atmosphere), which was mounted in a screw micrometer with displacement perpendicular to the plate. The distance between the hot wire and the surface could be regulated to 0.01 mm. The screw micrometer was mounted at the outside of the tunnel on a strong iron frame, which carried at the same time four round bars, two of which supported the upper edge of the plate and the two others the lower edge.

The displacement of the hot wire in the direction of the flow was performed by shifting the micrometer along the iron frame, without altering the position of the plate.

The mean velocity of the flow in the tunnel, V, was determined with a Pitot-tube, connected to an alcohol micromanometer ; the value of V was kept constant by the experimenter. This Pitot-tube was mounted 268 cm behind the honey comb at midheight of the tunnel and 60 cm from the vertical tunnel wall facing the experimenter ; the velocity indicated by this Pitot-tube will be called V_p.

Waffle plate II. The surface of the second metal sheat, which is called further on "waffle plate II", consisted of quadrilateral pyramids, 1.5 mm high, at distances of 6.65 mm in the longitudinal direction and of 6.4 mm in the transverse direction; hence it differed slightly from the surface of plate I.

Two sheats of this metal were fixed to the sides of a wooden board, which was sharpened at the leading edge over 15 cm, while the upper- and

6) B. G. VAN DER HEGGE ZIJNEN. Measurements of the velocity distribution in the boundary layer along a plane surface (thesis Delft, 1924).

the lower edges were sloped over 5 cm in order to get a sharp edge in the direction of the plate. The sheats were fixed as tightly as possible to the board by means of small nails; at the leading edge, where the wood was thin, the sheats were riveted. The dimensions of the covered board are: length of the metal sheat: 199.4 cm, height (measured along the curved sheat): 50 cm; thickness of the board: 2.4 cm.

The arrangement for the measurement of the resistance is represented in fig. 1. The board C is supported by two thin steel wires D, E (diameter 0.33 mm, length 290 cm) in the vertical plane of symmetry of the tunnel; they passed the top of the tunnel with sufficient clearance. At the lower edge of the board two pairs of steel wires F and G were provided in order to avoid transverse oscillations; they were stretched slightly, so that the plate was free to swing longitudinally.

From the leading edge a steel wire H led to a sensitive balance B outside of the tunnel, the slope of which was $30°.30'$; its end was fixed to the tunnel wall at K and a certain tension was given to this wire. The measurement of the forces acting on the model was performed in such a way that the other arm of the balance B was loaded with a determined weight, after which the wind speed in the tunnel was regulated until B was in balance. The resistance experienced by the plate was afterwards deduced from the weighed forces by resolving them graphically. During the measurements the distance of the leading edge of the board from the honey comb (X) was 158 cm; the lower edge of the board was 17.1 cm above the bottom of the tunnel, while the distance between board and the front wall of the tunnel was 41.6 cm. The Pitot-tube A was fixed at 143 cm behind the honey comb, 20 cm above the bottom and 52.4 cm from the front wall. The velocity V_p indicated by it will practically be equal to V_0 at the beginning of the board ($X = 158$ cm).

The resistance measured on the balance will be affected by the resistance of the steel wires carrying the board, and on the other hand by the suction at the trailing edge. In order to arrive at the true surface friction, corrections have to be applied for both influences.

The wire resistances are of minor importance; it is sufficient to deduce them from their length and diameter by means of the diagram for the resistance coefficients of cylinders given by PRANDTL [7]).

The suction at the trailing edge of the model is determined according a method also given by PRANDTL [8]) by mounting a brass tube of 9 mm diameter, provided with 9 holes of 0.8 mm in the space left between both metal sheats at the rear. This tube was closed at the bottom, while the upper

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[7]) L. PRANDTL. Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen II (München 1923), p. 24.

[8]) L. PRANDTL. Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen I (München 1921), p. 120.

end, projecting above the board, was connected with an alcohol micromanometer; the holes were directed down stream. For various values of V the suction in this space was compared to the static pressure on Pitot-tube A, which was supposed to be equal with sufficient accuracy to the pressure at

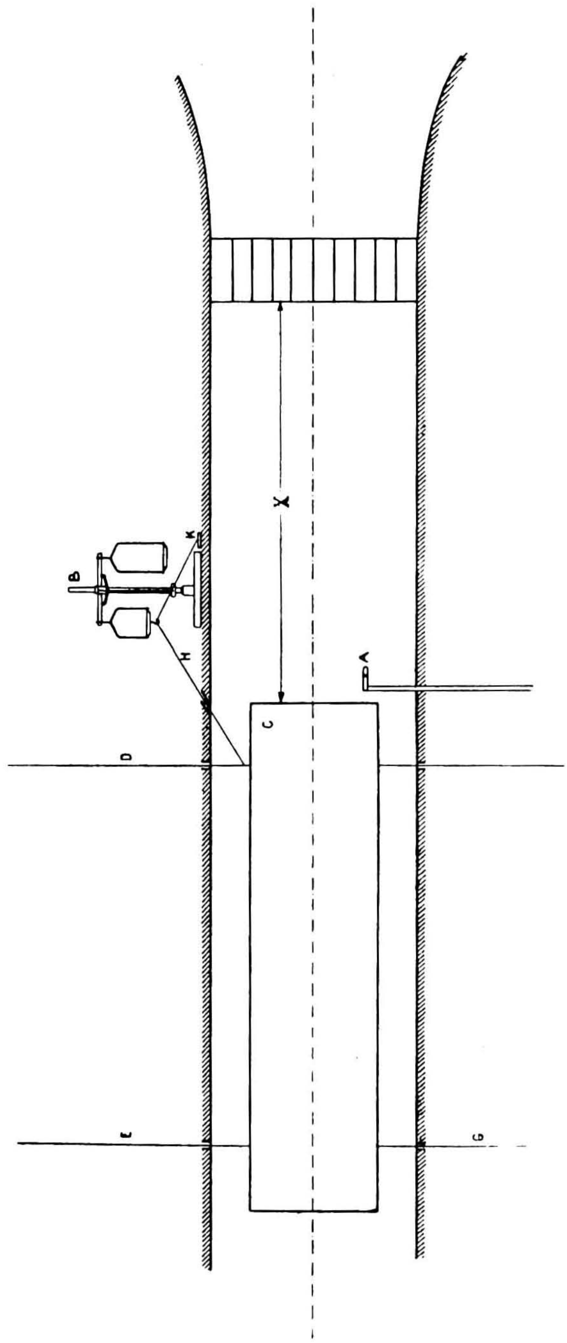

Fig. 1. Scheme of the arrangement for the direct determination of the resistance.

the leading edge of the plate. A second manometer was used for the determination of the air speed.

The pressure differences at the leading and at the trailing edge of the board, multiplied by the area of the trailing edge gives the suction experienced by the model.

This brass tube was absent during the resistance measurements with the balance.

The few measurements of the velocity distribution in the boundary layer along plate II were performed in the same way as those in the case of plate I, with the exception that here the first pair of iron bars, which held the board, was taken away and replaced by steel wires in order to avoid any disturbance of the flow as far as possible. The results of the velocity measurements with plate I had proved that these bars might cause a disturbance of the flow in the boundary layer in down stream sections. The pair of bars in the neighbourhood of the hot wire anemometer, however, could not be dispensed with, so that only the foremost pair, which gave the greater trouble, was removed.

§ 4. *Measurements in the boundary layer along plate I.*

The measurements on the velocity distribution along plate I were performed at a great number of distances from the leading edge and at various values of V.

As stated in § 2, the velocity outside of the boundary layer will increase down stream. However, during the experiments the wind speed was regulated in such a way that V had the same value at every value of x, as this made a check and a comparison of the results more easy.

As it proved to be impossible to determine the velocity gradient in the immediate neighbourhood of the surface in order to evaluate the shearing stresses, the velocity in the boundary layer was measured only at values of $y \geqslant 0.025$ cm. Some of the results have been collected in table I (the last columns of this table give the results of the measurements on plate II). Every velocity mentioned here is obtained as the mean of 6 readings. The indices t and v respectively relate to the measurements in which the distance y is estimated from a "top" or from a "valley" between the pyramids. The series t were observed immediately behind the series v and therefore the distance x had in this case to be increased by about 3 mm.

The experiments have been performed with $V = 811$ cm/sec in the sections $x = 5$, 10, 15, 20, 25, 37.5, 50, 62.5, 75, 87.5, 100, 125, 150 and 175 cm, y being reckoned in this case from a top and from a valley respectively; with $V = 1200$, 1600, 2000, 2400, 2800 and 3200 cm/sec the velocity was observed in the sections: $x = 25$, 50, 75, 100, 125, 150 and 175 cm for the series v only.

As might be expected, the series t and v show appreciable differences at the smaller values of y; they disappear for the greater part, however, when in the series t the value of y is increased by the height h of the projections, $h = 1.7$ mm.

The results were plotted on a logarithmic scale (log u against log y); one of these diagrams, relating to the measurements at $x = 175$ cm, has been reproduced in fig. 2. By means of the diagrams it was inquired

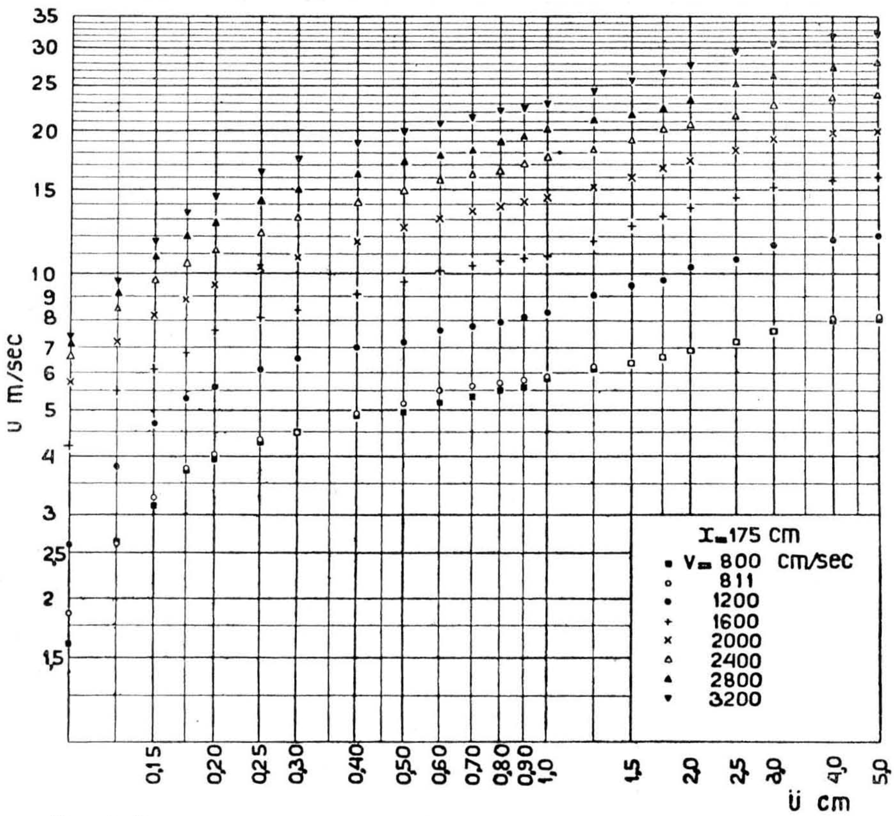

Fig. 2. Logarithmic scale diagram of the velocity in the boundary layer with $x = 175$ cm.

whether the velocity in the boundary layer could be represented by the formula $u = V\left(\frac{y}{\delta}\right)^n$ with a reasonable degree of accuracy. This proved to be the case; however, for all series the observed values of u seem to be arranged rather wavingly along the mean straight line [9]). This circumstance renders it difficult to determine the value of n with sufficient accuracy. The mean values, deduced from these diagrams, have been collected in table II.

They show a clearly marked decrease when x increases; probably this is due to the fact that the turbulent state of motion is not yet fully developed when x is small. On the contrary, n seems to be independent

[9]) The same phenomenon, although less pronounced, has been observed in the researches on a smooth surface, mentioned in note [6]) (p. 28, fig. 8).

TABLE II. Values of n.

x	V cm/sec 800 (v)	811 (v)	811 (t)	1200 (v)	1600 (v)	2000 (v)	2400 (t)	2800 (v)	3200 (v)	Mean
15	—	0.33	0.38	—	—	—	—	—	—	0.35
20	—	0.34	0.37	—	—	—	—	—	—	0.35
25	—	0.32	0.36	0.34	0.37	0.36	0.36	0.38	0.37	0.36
37.5	—	0.30	0.32	—	—	—	—	—	—	0.31
50	—	0.30	0.31	0.33	0.34	0.33	0.34	0.33	0.33	0.33
62.5	—	0.285	0.31	—	—	—	—	—	—	0.30
75	0.27	0.275	0.29	0.27	0.27	0.27	0.27	0.27	0.27	0.27
87.5	—	0.275	0.28	—	—	—	—	—	—	0.28
100	—	0.27	0.28	0.29	0.30	0.29	0.28	0.29	0.29	0.29
125	—	0.24	0.25	0.24	0.27	0.24	0.24	0.24	0.26	0.25
150	—	0.25	0.24	0.25	0.26	0.24	0.25	0.245	0.25	0.25
175	0.24	0.24	0.24	0.25	0.24	0.25	0.25	0.25	0.25	0.25

TABLE III. Values of δ (in cm).

x cm	V cm/sec 800	811	1200	1600	2000	2400	2800	3200	Mean
5	—	0.4	—	—	—	—	—	—	0.4
10	—	0.63	—	—	—	—	—	—	0.63
15	—	0.90	—	—	—	—	—	—	0.90
20	—	1.00	—	—	—	—	—	—	1.00
25	—	1.20	1.16	1.14	1.10	1.06	1.02	1.08	1.11
37.5	—	1.50	—	—	—	—	—	—	1.50
50	—	2.00	1.90	1.80	1.96	1.80	1.64	1.76	1.84
62.5	—	2.18	—	—	—	—	—	—	2.18
75	2.55	2.55	2.55	3.1	2.7	2.5	2.55	2.45	2.62
87.5	—	3.00	—	—	—	—	—	—	3.00
100	—	3.20	3.20	3.0	2.85	2.9	2.9	3.0	3.01
125	—	3.75	3.80	4.0	4.0	3.5	3.7	3.1	3.69
150	—	3.70	4.0	4.0	4.1	4.0	3.5	3.3	3.80
175	3.80	3.80	4.0	3.8	3.6	3.7	3.65	3.5	3.74

of V. With $x \geqslant 100$ cm, n becomes about 0.25. Our researches are not sufficient to prove whether this is the limiting value, or whether n will decrease still further when x increases to the value $^1/_7$ which holds for a smooth surface [10]).

The logarithmic diagrams give at the same time the thickness δ of the boundary layer; the results have been collected in table III (all numbers mentioned here relate to the series v).

The value of δ decreases, though not much, when V increases, contrary to what has been supposed in § 2. However, δ never can be determined with great accuracy and the sensibility of the hot wire decreases at higher values of V.

From a numerical integration of the value of $u\,(V-u)$ over the thickness of the boundary layer the defect of momentum I has been determined, according to form. (9). Writing:

$$c_i = \frac{I}{^1/_2\,\varrho V^2 x} = \frac{2I}{\varrho V^2 x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

the coefficient c_i has the values given in table IV (again for the series v):

TABLE IV. Values of $c_i \times 1000 = \dfrac{2I}{\varrho V^2 x} \times 1000.$

x	V cm/sec								Mean
	800	811	1200	1600	2000	2400	2800	3200	
5	—	21.6	—	—	—	—	—	—	21.6
10	—	15.8	—	—	—	—	—	—	15.8
15	—	15.8	—	—	—	—	—	—	15.8
20	—	14.0	—	—	—	—	—	—	14.0
25	—	12.7	13.2	13.76	13.4	12.56	12.60	13.62	13.12
37.5	—	10.1	—	—	—	—	—	—	10.1
50	—	10.6	11.1	11.48	12.18	11.30	10.48	10.80	11.13
62.5	—	8.4	—	—	—	—	—	—	8.4
75	9.08	9.46	9.30	10.12	9.50	8.64	8.86	8.66	9.22
87.5	—	8.90	—	—	—	—	—	—	8.90
100	—	8.60	8.60	8.48	7.84	7.80	8.40	8.32	8.29
125	—	7.39	7.10	7.40	7.40	6.80	7.94	7.00	7.29
150	—	6.52	6.72	6.84	6.98	6.68	6.52	6.08	6.62
175	5.68	5.68	5.84	5.92	5.48	5.56	6.04	5.66	5.73

These values do not vary systematically with V.

[10]) Compare the papers mentioned in notes [4]), (p. 238) and [6]) (fig. 7 and 8).

§ 5. *Discussion of the results.*

It was of importance to compare the experimental results collected in tables II to IV with the formulae deduced in § 2.

In the first place this has been done with the exponent n. Writing form. (11) by means of (9) :

$$c_i = \frac{2n}{(2n+1)(n+1)} \frac{\delta}{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and taking the mean values of δ and c_i for every value of x, we arrive at the following results for the expression $\frac{n}{(2n+1)(n+1)}$:

TABLE V.

x	δ	δ/x	c_i	$\frac{(2n+1)(n+1)}{n}$
25	1.11	0.0444	0.01312	7.77
50	1.84	0.0368	0.01140	6.63
75	2.63	0.0351	0.00922	7.61
100	3.01	0.0301	0.00830	7.25
125	3.69	0.0295	0.00748	7.89
150	3.80	0.0253	0.00662	7.64
175	3.72	0.0213	0.00574	7.42
			Mean...	7.44

This gives for n about 0.25 (exactly 0.255), which is in agreement with the value deduced from the measurements on the velocity distribution for the higher values of x.

If this value of n is valid over the whole surface, form. (6) will become :

$$\delta = 5{,}02\, c^{2/5} \left(\frac{x}{h}\right)^{3/5} (1 - 1{,}75\, \beta x)$$

Now it was found in the experiments on waffle plate II that the velocity outside of the boundary layer increased by 160 cm/sec over the full length (198 cm) of the model, when V_0 was equal to 2400 cm/sec. This leads to $\beta = 0.000337$. If we accept that the same value of β holds for all series the factor $(1—1.75\, \beta\, x)$ will become equal to $1—0.00059\, x$.

Now $h = 0.17$ cm ; therefore we get for the constant the values mentioned in table VI :

TABLE VI.

x	δ	$c^{2/3}$	δ calculated from form. (13)
25	1.11	0.0473	1.21
50	1.84	0.0493	1.89
75	2.63	0.0555	2.44
100	3.01	0.0531	2.92
125	3.69	0.0571	3.32
150	3.80	0.0527	3.70
175	3.72	0.0459	4.03
	Mean...	0.0516	

With the mean value 0.0516, the expression for δ becomes

$$\delta = 0{,}259 \left(\frac{x}{h}\right)^{2/3} h\,(1 - 0{,}00059\,x) \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The values of δ derived from this formula have also been mentioned in table VI. From form. (12) we now find for c_i :

$$c_i = 0{,}0691 \left(\frac{x}{h}\right)^{-1/3} (1 - 0{,}00059\,x) \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Table VII gives the comparison between the values of c_i calculated from this expression and those derived from the observed values of I.

TABLE VII.

x	(c_i) calculated	(c_i) experimental
25	0.0130	0.01312
50	0.0111	0.01140
75	0.00871	0.00922
100	0.00779	0.00829
125	0.00709	0.00729
150	0.00660	0.00662
175	0.00617	0.00573

Although the differences between both sets of numbers are important, it may be said that in general the results found for n (as deduced from the logarithmic diagrams for the velocity distribution), for the thickness of the boundary layer and for the defect of momentum in the boundary layer. confirm the relations deduced in § 2.

With the value of the constant mentioned before, form. (3) gives for τ_0:

$$\tau_0 = 0{,}01'7\,\varrho V^2 \sqrt{\frac{h}{\delta}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The resistance experienced by a single wall of length l and breadth b, in a flow with the constant velocity V, now becomes according to (8) (with $\beta = 0$) :

$$W = 0{,}0690 \left(\frac{h}{x}\right)^{1/8} \tfrac{1}{2}\varrho V^2\, bx \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

§ 6. *Measurements on the velocity distribution along plate II.*

As stated in § 3, two series of measurements on the velocity distribution in the boundary layer along plate II were performed in order to check the results found with plate I.

The plate was mounted in the same way as mentioned at the end of § 3, with the leading edge at $X = 129$ cm behind the honey comb of the wind tunnel. The measurements were carried out at a distance of $x = 198$ cm from the leading edge, starting from a valley; a second series was performed 3 mm down-stream starting from a top. The hot wire anemometer was put at 40 cm above the bottom of the tunnel. Measurements have been performed only for the value $V_p = 2400$ cm/sec (Pitot-tube 42 cm behind honey comb; 52 cm from the front wall and 20 cm above the tunnel bottom), in which case V_x proved to be 2560 cm/sec according to the readings of the hot wire, which instrument had been calibrated carefully before. This leads to the velocity increase of 160 cm/sec along the surface, as stated before.

The results have been collected in the last columns of table I. They prove, as was the case with plate I at the higher values of x, that the velocity in the boundary layer increases with the 0.25-power of y; here too in a logarithmic diagram the observed values of the velocity are grouped more or less wavingly along the mean straight line. The exact evaluation of n and δ becomes rather difficult on account of this phenomenon.

The value of δ proved to be about 4.8 cm.

§ 7. *Determination of the resistance of plate II.*

The total resistance of the model, as found by means of the balance, has been represented in fig. 3 as a function of V_0 ($+ \ldots . +$); in the same diagram the suction at the trailing edge ($\times \ldots . \times$) and the resistance of the suspension wires (d) have been given. When the total resistance is diminished by the suction and by the resistance of the

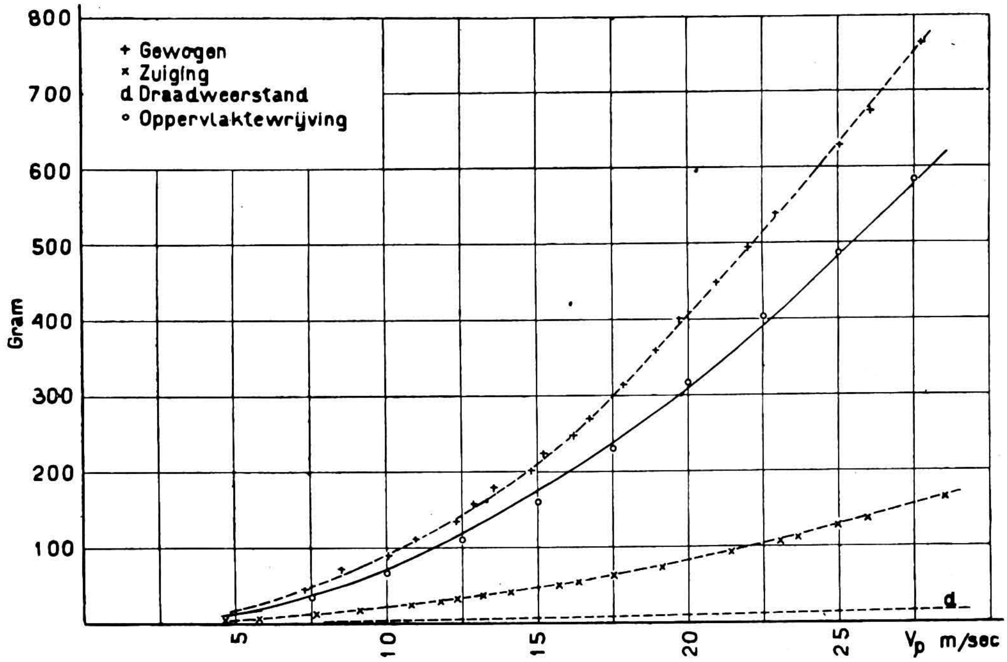

Fig. 3. Total resistance and surface friction as function of the velocity.

suspension wires we get the surface friction. From the diagram the following values have been deduced for this surface friction (∘ . . . . ∘).

TABLE VIII.

V_0 m/sec	Surface friction (both sides), in grammes	V_0 m/sec	Surface friction (both sides), in grammes
5	12	17.5	230
7.5	34	20	316
10	66	22.5	404
12.5	110	25	487
15	160	27.5	584

They may be represented by the interpolation formula :

$$W = 0{,}77\, V_0^2 \quad (V_0 \text{ in m/sec}),$$

which is in agreement with the supposition that the resistance is proportional to the square of the velocity. This formula is represented in fig. 3 by the full drawn line.

The results of the weighing have now been compared with the value of the resistance as deduced from the experiments on the velocity distribution in the boundary layer. To this end the value of

$$I = \varrho \int_0^\delta u\,(V - u)\,dy$$

has been calculated from the measurements mentioned in § 6.

It has to be born in mind that the velocity in the first part of the boundary layer cannot be found with sufficient accuracy. Starting from a valley f.i. we might expect that the hot wire anemometer, which is at first more or less screened by the pyramids, will be influenced at higher values of y by the v- and w-components of the velocity. On the other hand, when we start from a "top", and call the distance y between the hot wire and the surface zero when the wire just touches the pyramids, as has been done before, we still have to take into account the flow between top and valley, which region of course was not accessible with the apparatus used by us. We have allowed ourselves to extrapolate the u-y-curve from top to valley i.e. over the distance h until the value $u = 0$ is reached at the base of a pyramid. The region between top and valley accounts for about 4 % of the total defect of momentum, and therefore an estimation of the flow in this part of the boundary layer will not lead to important errors.

In this way we found for the expression $\frac{I}{\varrho} = \int_0^\delta u\,(V - u)\,dy$ when starting from a valley: 3469000 cm^3/sec^2, and when starting from a top, taking the flow between top and valley into account, 3242000 cm^3/sec^2 (as mentioned in § 6, V_p was 2400 cm/sec). The difference between the two values is due to the region of high velocities (i.e. high values of y), where the smaller sensibility of the hot wire anemometer leads to less accurate readings of the velocity than in the region of low air speed near the surface.

The mean value of $\frac{I}{\varrho} = 3346000$ cm^3/sec^2 gives for the surface friction on both sides of the plate together at $V_p = 2400$ cm/sec, according to form. (10) with $\varrho = \frac{1}{8000}$ and $b = 50$ cm:

$$W = \frac{3346000}{8000}\,(1 + 0{,}9\,\beta x) = 444 \text{ grammes}$$

when we suppose that the velocity distribution is the same for all heights.

The surface friction as determined by weighing is, according to fig. 3, at $V_p = 2400$ cm/sec : 454 grammes, which differs little from the value deduced from the measurements on the velocity distribution in the boundary layer.

It is of importance to compare this value of W with that given by the formula deduced from the measurements with plate I, taking x equal to 198 cm and $h = 0.15$ cm. Form. (13) leads in this case to:

$$\delta = 4{,}13 \text{ cm}$$

hence form. (9) gives:

$$\frac{I}{\varrho} = \frac{4{,}13}{7{,}5} (V_{198})^2 = \frac{4{,}13}{7{,}5} (1{,}066)^2 \, V_0^2 = 0{,}626 \, V_0^2,$$

and form. (10):

$$W = 0{,}83 \, V_0^2 \; (V_0 \text{ in m/sec}),$$

which leads to $W = 478$ gram at $V_0 = 24$ m/sec. The check must be considered fair, especially as it is not easy to determine h with sufficient accuracy.

§ 8. *Resistance deduced from total head loss in the wake.*

Moreover the resistance experienced by the model has been evaluated from the total head loss of the flow in the wake of the model. We suppose the flow to be stationary and will neglect the components of the velocity perpendicular to the direction of the undisturbed wind as the model is a long, thin board, mounted in the direction of the flow. Calling the static pressure in a section of the tunnel in front of the model p_A and the static pressure in a section behind the model p_B we get as the resultant of the pressure forces for a section of unit height:

$$W_p = \int (p_A - p_B) \, dy.$$

The change of momentum per unit time of the air passing the sections A and B is found from the total head loss. The mass of air entering A per unit time is $\int \varrho u_A \, dy$; its momentum: $\int \varrho u_A^2 \, dy$; the momentum of the air leaving B is: $\int \varrho u_B^2 \, dy$. Consequently:

$$W = \int (p_A + \varrho u_A^2) \, dy - \int (p_B + \varrho u_B^2) \, dy \quad . \quad . \quad . \quad . \quad (17)$$

By introducing the total head: $H = p + \frac{1}{2} \varrho u^2$, (17) is reduced to:

$$W = \int \triangle H \, dy + \int {}^1/_2 \, \varrho u_A^2 \, dy - \int {}^1/_2 \, \varrho u_B^2 \, dy \quad . \quad . \quad . \quad (18)$$

The second and third members at the right hand side of (18) are nearly equal; taken together they can be treated as a correction applied to the first term. They can be simplified when u_A and u_B are written as the sum of the undisturbed velocity V and a disturbing velocity u_a, u_b ; in this way the correction becomes :

$$\left.\begin{aligned}\int {}^1/_2\,\varrho\,(V^2 + 2u_a V + u_a^2)\,dy - \int {}^1/_2\,\varrho\,(V^2 + 2u_b V + u_b^2)\,dy = \\ = \varrho V \int (u_a - u_b)\,dy + {}^1/_2\,\varrho \int (u_a^2 - u_b^2)\,dy\end{aligned}\right\} . \quad (19)$$

The equation of continuity, applied to the region between A and B, proves that the first term at the right hand side of (19) is zero. Putting $u_A = V =$ constant which holds if the section A is taken far enough in front of the model and the flow is measured not too near to the honey comb, u_a will be zero, and (18) is reduced to:

$$W = \int \triangle H\,dy - {}^1/_2\,\varrho \int u_b^2\,dy \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where the integral is extended only over the region of turbulent motion in the wake.

In evaluating the experimental data, the correction proved to be of the order of 10 % of the uncorrected value of W. On the other hand it was found that the values of H varied in a rather important and unsystematic manner at various heights above the tunnel bottom. On account of this it was considered superfluous to calculate the correction in a more refined manner.

The section B was chosen 40 cm behind the trailing edge of the model, while the values of u_A and H_A were derived from a Pitot-tube put at 42 cm behind the honey comb, 20 cm above the bottom of the tunnel and 52 cm from the front wall. The values in the section B were determined by means of a Pitot-tube which could be shifted perpendicularly to the plate. In order to observe the variations of $\triangle H$ with the height, H_B was read at the following values of z: 12, 15, 17, 20, 30, 40, 44, 48, 52, 56, 60, 63, 65 and 68 cm (measured from the tunnel bottom). In a direction perpendicular to the plate the Pitot-tube was set at the following values of y: 20, 22, 23, 24, 25, 26, 28, 30, 31, 32, 33, 34, 34½, 35, 35½, 36, 36½, 37, 37½, 38, 38½, 39, 39½, 40, 40½, 41, 41½, 42, 42½, 43, 43½, 44, 44½, 45, 45½, 46, 47, 48, 49, 50, 52, 54, 55, 56, 57, 58, 60 cm (measured from the front wall).

During these experiments the model was fixed by means of thin steel wires, in the same way as during the measurements with the balance. On account of the dimensions of the tunnel and of the position of the section B, the distance X had to be reduced to 129 cm ; the distance from the model to the vertical front wall was 41.5 cm, measured from half way the thickness of the board.

While reading H_B the velocity of the air in the tunnel as indicated by Pitot-tube A, was kept constant at $V_p = 2400$ cm/sec. At every position of B 10 readings were taken of the differences of the static pressures and of those of the dynamic pressures $(p + \frac{1}{2} \varrho V^2)$ on A and B, at intervals of 5 seconds. A third manometer connected to A allowed the determination of the velocity V_p. When the pressures in a certain position had been observed, the Pitot-tube B was shifted to the next position and again 10 readings were taken; etc. During the readings at a constant value of z the fan of the wind tunnel was kept in action; it had to be stopped when z was modified.

The values of $\triangle H$ were plotted as function of y for every value of z; the areas of the curves obtained in this way were determined by means of a planimeter. In first approximation the area J of each of them represents the resistance of the model per cm height at the particular value of z at which $\triangle H$ has been determined. Then the correction

$$C = \int \tfrac{1}{2} \varrho u_b^2 \, dy$$

was deduced from $\triangle H$ and $\triangle p$ for the same values of z. Taking this correction into account, so that $J_{corr.} = J - C$, the area of the curve for $J_{corr.}$ as a function of z gives the resistance of the whole model. The results are collected in table IX (J being expressed in mm water $\times$ cm, i.e. in kg/m^2 $\times$ cm).

TABLE IX. Total head loss in the wake of waffle plate II.

z cm	12	15	17	20	30	40	44	48	52	56	60	63	65	68
J mm.cm	18.3	104.7	116.2	111.6	138.9	145.0	132.5	129.8	127.9	107.4	91.4	98.8	97.6	12.3
Correction	?	6.6	8.8	7.9	12.3	14.0	11.2	10.7	10.6	7.8	5.7	6.3	4.8	?
J corr.	18.3	98.1	107.4	103.7	126.6	131.0	121.3	119.1	117.3	99.6	85.7	91.5	92.8	12.3

Integrating the value of $J_{corr.}$ with respect to z, we find:

$$W = \int J_{corr.} \, dz = 561{,}5 \text{ grammes.}$$

Taking into account the suction at the trailing edge, the value of which is 120 grammes for $V_p = 2400$ cm/sec, the surface friction on both sides of the model will be 441.5 grammes.

Notwithstanding the uncertainty in the determination of the resistance from the total head loss, the agreement between the resistance found in this way and that deduced from other methods seems to be fair.

§ 9. *Summary.*

From measurements of the distribution of the velocity in the boundary layer the value of the exponent n in the relation $u = V\left(\frac{y}{\delta}\right)^n$ has been deduced. This exponent proves to be independent of the velocity V and seems to be determined entirely by geometrical relations. The results provisionally lead to the conclusion that n approaches to the limit of 0.25 for increasing values of δ.

The value of the thickness of the boundary layer δ deduced from the experiments has been compared with the relations found in § 2 for the fully developed turbulent state of motion along rough surfaces. As may be expected from the formulae, δ does not depend on V and is determined entirely by x, by the height h of the pyramids on the surface, and by a numerical factor. This factor, which also occurs in the formula for the resistance, is connected with the shape and the arrangement of the pyramids on the surface. It may be expected that the relations for δ, for the shearing stresses near the surface and for the resistance coefficient deduced from our measurements, will hold for a series of rough surfaces similar to those investigated. The experimental material, however, is not sufficient to decide whether this supposition is of general validity.

The surface friction of both sides of plate II, as weighed on the balance, is in agreement with the resistance as determined from the total head loss in the wake by means of a Pitot-tube ; the surface friction deduced from the loss of momentum in the boundary layer (determined with a hot wire anemometer) leads to the same value. The measurements with the balance lead to the formula :

$$W = 0{,}77\, V_0^2 \ (V_0 \text{ in m/sec}),$$

while form. (10) deduced from the measurements performed at plate I, inserting $\beta = 0.000337$, gives :

$$W \doteq 0{,}83\, V_0^2.$$

The results of the determinations of the surface friction of both sides together according to several methods may be summarized as follows (the length of the board being 198 cm, its breadth 50 cm and the velocity $V_0 = 2400$ cm/sec) :

Plate II, from measurements with the balance	454 gr.
from total head loss in the wake	441 gr.
from loss of momentum in the boundary layer	444 gr.
from form. (8), $x = 198$, $h = 0.15$ cm, $\beta = 0.000337$	478 gr.