Mathematics. — *Two Representations of the Congruence of* REYE.
By Prof. JAN DE VRIES.

(Communicated at the meeting of April 28, 1928).

§ 1. By the aid of a well known cubic transformation [1]) the twisted cubics k^3 through four given points are transformed into the rays of space. In this case the congruence Γ of the k^3 through five given points (congruence of REYE) is represented on the rays m of a sheaf, hence indirectly on the points of a plane. In my paper "Congruences of twisted curves in connection with a cubic transformation" (These Proceedings, Vol. **11**, p. 84) I have shown how the properties of Γ appear through the transformation in question.

Any ray m of the sheaf about a point M is a bisecant of one k^3 of Γ; the point of intersection of m and an image plane may be considered as the image of k^3. I have treated this representation in a paper in Vol. **30**, p. 850, of these Proceedings [2]).

If we consider Γ as the product of two pencils of quadratic cones that have a base generatrix in common, there arises a representation of Γ on the points of a field if we bring both pencils into projective correspondence with two pencils of the image plane; in this case the point of intersection of two rays of these plane pencils may be considered as the image of the k^3 that is produced by the corresponding cones [3]).

In what follows we shall treat two other representations.

§ 2. The congruence Γ with base points B_k $(k=1,2,3,4,5)$ contains ten systems of composite figures k^3. For the line $B_1 B_2$ forms a k^3 with any conic k^2 in the plane β_{345} $(B_3 B_4 B_5)$ that passes through B_3, B_4, B_5 and the point of intersection C_{12} of b_{12} $(B_1 B_2)$ with β_{345}; let us indicate this system of curves k^3 by Σ_{12}.

Let r be a straight line through B_5, R its point of intersection with the plane ϱ. I shall consider R as the image of the k^3 that touches r (at B_5).

The transits S_1, S_2, S_3, S_4 of the lines $b_{15}, b_{25}, b_{35}, b_{45}$ are evidently *singular* image points; each of them represents a system Σ.

The other six systems Σ are represented on point ranges; the k^3 of

[1]) We mean the transformation $x_1 x'_1 = x_2 x'_2 = x_3 x'_3 = x_4 x'_4$..

[2]) A Representation of the Congruence of REYE.

[3]) This representation has been investigated by H. P. HOESTRA ("Onderzoek van eenige bilineaire congruenties van kubische ruimtekrommen door een afbeelding op het puntenveld", thesis for the doctorate, Utrecht 1928, p. 45).

Σ_{12} have their images on the line s_{12} that joins $S_3 S_4$. The point of intersection of s_{12} and s_{34} is the image of the k^3 that consists of b_{12}, b_{34} and the line through B_5 that rests on b_{12} and b_{34}.

§ 3. Let us now consider the system Λ of the k^3 that cut the line l. As l rests on one k^2 of any system Σ, the image curve λ of Λ has only the points S_3, S_4 and the image R of a k^3 belonging to Σ_{12} in common with s_{12}; it is, therefore, a cubic.

Between the points of λ^3 and the points of l there exists a correspondence (1,1); accordingly λ^3 is *rational*. Its double point is the image of the k^3 that has l as bisecant.

Two curves λ^3 have 5 points R in common; consequently on two arbitrary lines there rest *five* k^3 and the curves of Λ form a surface of the *fifth* degree.

The intersection of Λ^5 and β_{123} consists of the lines b_{12}, b_{13}, b_{23} and a conic.

§ 4. Let us also consider the system Φ of the k^3 that touch a plane φ. β_{123} contains two conics k^2 that touch φ; the locus of the points where φ is touched by curves of Γ is, therefore, a conic φ^2.

As any system Σ contains two k^3 of Φ, the image curve of Φ has double points in S_3 and S_4 and s_{12} contains two more points R of that curve. Because of the (1,1) correspondence between φ^6 and φ^2 the image curve of Φ is a *rational* φ^6; the six double points which it contains outside the points S, are images of the k^3 that osculate φ.

Accordingly an arbitrary plane *osculates six* curves of Γ. Two curves φ^6 have 20 points R in common; there are, therefore, *twenty* k^3 that touch two planes.

A curve λ^3 has ten points R in common with φ^6; accordingly there are *ten* k^3 that touch a plane and cut a line. And the curves of Φ form a surface of the *tenth* degree.

A point range (R) is the image of the system of the k^3 that touch a plane pencil (r); these k^3 form a cubic surface, for (R) has three points in common with a λ^3.

§ 5. In order to arrive at another representation of Γ we pass a plane ϱ through b_{45} and we consider the third point of intersection R of a k^3 with ϱ as the image of that k^3.

The transits S_{12}, S_{13}, S_{23} of b_{12}, b_{13}, b_{23} are singular points; S_{12} is e.g. the image of all k^3 of the system Σ_{12}.

A k^3 of Σ_{45} is represented by the points of b_{45} and by the point R which its k^2 has besides in common with ϱ.

The image curve of the system Λ passes evidently through the points S; with the line through these points it has besides only the point R in common that is the intersection of a k^2 of Σ_{45} and l; it is, therefore, a *rational* curve λ^4. It has double points in B_4 and B_5; the third double point is the image of the k^3 that has l as chord.

Two curves λ^4 have five points R in common outside the five singular points; accordingly there are *five* k^3 that rest on two lines.

§ 6. The singular point B_4 is the image of all k^3 that touch ϱ at B_4; these curves evidently form a quadratic surface $\beta^2{}_4$. The k^3 on $\beta^2{}_4$ define an involution I^3 on the conic in which a plane φ' cuts $\beta^2{}_4$. Accordingly on $\beta^2{}_4$ there lie four k^3 that touch φ and B_4 is a quadruple point of the image curve of the system Φ; this is consequently a *rational* curve φ^8 with quadruple points B_4 and B_5. It has double points in the points S_{kl}, for the corresponding system Σ_{kl} contains two conics that touch φ. The six other double points of φ^8 are the images of the *six* k^3 that osculate φ.