

Applied Mechanics. — *Temperature differences occurring in gaslift.*

By J. VERSLUYS.

(Communicated at the meeting of October 27, 1928).

Recently a theoretical analysis was given of the phenomena occurring in gaslift. (I). As gaslift is to be considered a vertical tube in which a mixture of gas and liquid rises. The pressure of the gas at the bottom of the gaslift is the main source of energy. In the above mentioned analysis it was stated that the temperature in the gaslift is not invariable, for the temperature must drop as the mixture rises. The formulae, however, were deduced as if the process were isothermal. As appears from what follows, the differences in temperature are so small, that they may be disregarded, such in view of the inevitable inaccuracies in the determination of the coefficients.

In this paper a deduction of the theory of the gaslift is given, whereby the temperature is taken as variable. It will be shown that the decrease in temperature during the rise of the gas and liquid mixture is so small that the decrease of the work exerted by the expansion of the gas in consequence of this drop in temperature may be disregarded.

The following notation will be used:

volume of liquid flowing through per unit of time . . .	q
absolute temperature	T
absolute pressure	p
volume of gas flowing through per unit of time . . .	$\frac{RT}{p}$
specific gravity of the liquid	γ_l
specific gravity of the gas at pressure p and temperature T	$\frac{\gamma_g p}{T}$
total volume of liquid and gas passing through the section	
per unit of time	v
weight of gas and liquid passing through the section per	
unit of time	G
ditto for gas alone	G_g
ditto for oil alone	G_l
average rate of flow of the mixture in a cross section .	u
a coefficient of the dimension of rate of flow	b
specific gravity of the mixture	s
height above horizontal level of origin	y
radius of the cross section of the tube	r
amounts of energy per unit of time	$W_1, W_2, \text{ etc.}$

mechanical heat equivalent	,	A
specific heat of the liquid	c_l
specific heat of the gas	c_g
absorption coefficient	α
coefficient of absorption heat	β .

One may write:

$$G = q \gamma_l + R \gamma_g \quad (1)$$

and

$$v = q + \frac{RT}{p} = \frac{qp + RT}{p} \quad (2)$$

From which follows:

$$s = \frac{G}{v} = p \frac{q \gamma_l + R \gamma_g}{qp + RT} \quad (3)$$

and

$$u = \frac{v}{\pi r^2} = \frac{qp + RT}{\pi r^2 p} \quad (4)$$

If as elementary cylinder is to be understood that part of the contents of the gaslift lying between the horizontal levels at the heights y and $y + dy$, then the work of the pressure on the bottom end of this cylinder per unit of time is:

$$W_1 = pv = pq + RT \quad (5)$$

from which follows:

$$dW_1 = q dp + R dT \quad (6)$$

and this is the work done in the unit of time by the pressure upon the elementary cylinder.

The work performed by the force of gravity in the elementary cylinder per unit of time is:

$$dW_2 = -G dy = -(q \gamma_l + R \gamma_g) dy \quad (7)$$

The mixture entering the elementary cylinder per unit of time through the bottom end would, provided the rate of flow is the same at every point in the cross section and the liquid and the gas flow at the same rate, have a kinetic energy of:

$$-\frac{G}{2g} u^2 = -\frac{q \gamma_l + R \gamma_g}{2g} \left(\frac{qp + RT}{\pi r^2 p} \right)^2 \quad (8)$$

Assuming that the variation of this energy, caused by the speed of flow not being the same at all points of the cross section and the two substances (oil and gas) not having the same speed is expressed by a

constant coefficient ψ , then the energy applied in this manner to the elementary cylinder per unit of time is:

$$dW_3 = -\psi \frac{q\gamma_l + R\gamma_g}{g} \frac{qp + RT}{\pi^2 r^4 p} \left(\frac{R}{p} dT - \frac{RT}{p^2} dp \right) = \\ = -\psi \frac{(q\gamma_l + R\gamma_g)}{\pi^2 r^4 g} \left(\frac{qp + RT}{p^2} \right) \left(R dT - RT \frac{dp}{p} \right) \quad (9)$$

The work of the turbulence resistance may be expressed as follows, assuming that the mixture behaves as a single liquid or a single gas:

$$dW_4 = -\pi r^2 f u dy = -\pi r^2 f \frac{qp + RT}{\pi r^2 p} dy = -f \frac{qp + RT}{p} dy \quad (10)$$

in which f (compare I p. 67) is a function:

$$f = \frac{\varphi v^2 s}{r^5} = \frac{\varphi}{r^5} \frac{(qp + RT)(q\gamma_l + R\gamma_g)}{p} \quad (11)$$

φ being a constant so that:

$$dW_4 = -\frac{\varphi}{r^5} \left(\frac{qp + RT}{p} \right)^2 (q\gamma_l + R\gamma_g) dy \quad (12)$$

The work per unit of time performed by the expansion of the gas in the elementary cylinder is:

$$dW_5 = p dv = p \left(\frac{R}{p} dT - \frac{RT}{p^2} dp \right) = R dT - \frac{RT}{p} dp \quad (13)$$

Finally, the energy per unit of time in the elementary cylinder converted into heat due to the difference in speed (see I page 67) may be expressed as the product of the weight of the liquid in the cylinder and a coefficient b , which, if the volume of the liquid is much greater than that of the gas, is about as great as the difference in the rate of flow, thus:

$$dW_6 = -b\gamma_l \pi r^2 \frac{pq}{pq + RT} dy \quad (14)$$

An equation of equilibrium can be written as follows:

$$dW_1 + dW_2 + dW_3 + dW_4 + dW_5 + dW_6 = 0 \quad (15)$$

The external energy applied per unit of time to the elementary cylinder is the sum of dW_1 , dW_2 and dW_3 . Since the temperature of the substance flowing out of the cylinder is different from that of the

inflowing substance, a certain amount of heat is applied per unit of time. This is equivalent to the external energy, thus:

$$(q \gamma_l c_l + R \gamma_g c_g) dT - A (dW_1 + dW_2 + dW_3) = 0 \quad . \quad . \quad (16)$$

For the present no consideration is given to the value of the coefficient of the specific heat of the gas in connection with the change in pressure and volume. Later it will appear, that the terms in which this coefficient occur, may be disregarded for the present purpose.

Substituting in (15) and (16) the expressions of dW_1, \dots, dW_6 then two simultaneous differential equations are obtained, including p, dp, T, dT and dy .

The equations are considerably simplified if, as has been found admissible in many practical cases, one takes:

$$dW_1 = R dT \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and:

$$dW_3 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Then (16) is written:

$$\frac{(q \gamma_l c_l + R \gamma_g c_g) dT}{A} - R dT + (q \gamma_l + R \gamma_g) dy = 0, \quad . \quad . \quad (19)$$

from which follows:

$$T = C - \frac{(q \gamma_l + R \gamma_g) A}{(q \gamma_l c_l + R \gamma_g c_g) - A R} y, \quad . \quad . \quad . \quad . \quad (20)$$

in which C is the integration constant.

For (15) may now be written:

$$\left. \begin{aligned} R dT - (q \gamma_l + R \gamma_g) dy - \frac{\varphi}{r^5} \left(\frac{qp + RT}{p} \right)^2 (q \gamma_l + R \gamma_g) dy + \\ + R dT - \frac{RT}{p} dp - b \gamma_l \pi r^2 \frac{pq}{pq + RT} dy = 0 \end{aligned} \right\} \quad (21)$$

From this dy can be eliminated by substituting for the first two terms the form deduced from (19) and for the third term:

$$(q \gamma_l + R \gamma_g) dy = R dT - \frac{(q \gamma_l c_l + R \gamma_g c_g) dT}{A}, \quad . \quad . \quad . \quad (22)$$

which relation follows from (19). We then get the following equation:

$$\left. \begin{aligned} \frac{(q \gamma_l c_l + R \gamma_g c_g) dT}{A} - \frac{\varphi}{r^5} \left(\frac{qp + RT}{p} \right)^2 \left\{ R - \frac{q \gamma_l c_l + R \gamma_g c_g}{A} \right\} dT + \\ + R dT - \frac{RT}{p} dp - b \gamma_l \pi r^2 \frac{pq}{(pq + RT)} \left\{ R - \frac{A(q \gamma_l c_l + R \gamma_g c_g)}{(q \gamma_l + R \gamma_g)} \right\} dT = 0 \end{aligned} \right\} \quad (23)$$

perature drops 3° Celsius, while the pressure from the bottom to the top drops from 40 to 1 atmosphere, then with isothermal expansion the gas would exert an energy per unit of volume of:

$$-\int_{p=1}^{p=40} p \, dv = \int_1^{40} p \frac{dp}{p^2} = \int_1^{40} \frac{dp}{p} = \log 40 = \pm 6. \quad \dots \quad (31)$$

If the temperature afterwards drops 3° at about $T=300$, then the decrease of volume is $\frac{1}{100}$ at a pressure 1. The work then performed is $\frac{1}{100}$. Thus with isothermal expansion the work per unit of volume would be 6, of which about $\frac{1}{6}\%$ would be lost through a difference in temperature of 0.01.

If, as also occurs, the pressure in the top of the gaslift is 4 atmospheres then the worth of isothermal expansion is $\log 10 = 2.3$, from which would be deduced the product of the pressure prevailing, the volume and the decrease of volume 0.01 or 0.4%.

In case of a gaslift where water is raised say 60 metres, while the pressure at the bottom is 4 atmospheres, then the temperature decrease will be $60 \times \frac{1}{427} = 0.14$. The energy exerted per unit of volume of gas at isothermal expansion would be $\log 4 = 1.39$, whilst through decrease of temperature this would be reduced by $\frac{0.14}{300} = 0.0005$, or about 0.03%.

In the treatise (I) referred to in the beginning, the problem of the action of the gaslift was dealt with, the solubility of the gas being taken into consideration. In order to understand the heat phenomena in the deduction account would have to be taken of the absorption heat of the gas in the liquid. In the case of water and air there is little absorption. With petroleum and the co-existing gases the absorption coefficient α in units of volume may be $\frac{1}{2}$, which is to say that with a pressure of 1 atmosphere half a litre of gas, measured at that pressure, would dissolve in 1 litre of oil. If the pressure at the bottom of the gaslift is p_1 and at the top p_2 , then the volume of gas set free during the rise, measured at 1 atmosphere is $\alpha(p_1 - p_2)$, and the weight of this volume of gas would be $\frac{\gamma_g \alpha (p_1 - p_2)}{T}$.

Assuming that the absorption heat β corresponds to the heat of evaporation, then this would presumably lie between 50 and 150. Then the absorbed heat per unit of volume of oil is:

$$\frac{\beta \gamma_g \alpha (p_1 - p_2)}{T}$$

and the resultant temperature decrease

$$\frac{\beta \gamma_g \alpha (p_1 - p_2)}{c_t T},$$

for which may be taken as average about

$$\frac{100 \times \frac{1}{1000} \times \frac{1}{2}}{\frac{1}{2}} (p_1 - p_2) = \frac{1}{10} (p_1 - p_2).$$

The heat absorption capacity of the gas is disregarded. In very deep wells it might be possible that $(p_1 - p_2) = 100$, in which case the difference in temperature might be 10° Celsius. In less deep wells $p_1 - p_2$ might be 36, and then the difference in temperature is 3.6° C.

In the latter case, where the difference of temperature is 3.6° C. the isothermal energy per unit of volume of gas would be 2.3, as already known, while the decrease of temperature would mean a loss of $\frac{3.6}{300} = 0.012$. This would be about $1/2\%$. In the former case, where the difference in temperature is 10° C., with isothermal expansion the energy per unit of volume of gas would be 6.9 and the loss through difference in temperature $\frac{10}{300} = 0.033$, which is also about $1/2\%$.

So in the theory of gaslift there is no reason for taking into account the differences in temperature.

LITERATURE.

1. J. VERSLUYS: De werking van de gaslift, *De Ingenieur*, 1928, N^o. 37, pp. M 65—M 70 (with a bibliography).
2. T. A. HALL: Fundamental principles of flowing wells, *The Oil and Gas Journal*, October 4, 1928, p. 30,