

Physics. — *On the anomalous g -values in the spectrum of ionized Argon ($Ar II$).* By C. J. BAKKER. (Communicated by Prof. P. ZEEMAN).

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1. *Introduction.*

In a former paper in these Proceedings ¹⁾ the investigation of the ZEEMAN-effect of 110 lines of the spectrum of ionized Argon ($Ar II$) has been communicated. It has appeared that many terms, which are known by the extensive analysis of the spectrum by DE BRUIN ²⁾, possess g -values deviating from the well known splitting factors of LANDÉ. For the terms arising from the $4s$, $4p$ and $5s$ electron the g -sum rule could be tested.

The g -formula of LANDÉ is derived on the assumption that the coupling scheme of RUSSELL-SAUNDERS ³⁾, which occurs mostly in the spectra, holds. Assumption of other coupling schemes gives rise by calculation to g -values deviating from those of LANDÉ. This communication contains the calculation of the g -values belonging to 4 different coupling schemes and the comparison of the g -values calculated in that way with those found experimentally in the spectrum of ionized Argon ($Ar II$) for the coupling $4s$, $4p$, $5s$ (3P).

2. *Method of calculation of the g -values.*

According to the theory of HEISENBERG, PAULI and HUND the energy levels of the spectrum of the simply ionized Argon atom ($Ar II$) can be deduced by joining to the double ionized Argon atom ($Ar III$), that is the atomic core of Argon II , in succession a $3p$, $3d$, $4s$, $4p$, $4d$, $5s$ electron and by coupling the quantum vectors of core and electron ⁴⁾.

We denote by

s_1 = rotation quantum number of the atomic core,

l_1 = azimuthal quantum number of the atomic core.

and

s_2 = rotation quantum number of the electron that is coupled.

l_2 = azimuthal quantum number of the electron that is coupled.

¹⁾ C. J. BAKKER, T. L. DE BRUIN and P. ZEEMAN: These Proceedings **31**, 780, 1928, Zeitschr. für Phys. **51**, 114, 1928. **52**, 299, 1928.

²⁾ T. L. DE BRUIN: These Proceedings **31**, 591, 1928 and **31**, 771, 1928. Zeitschr. für Phys. **48**, 62, 1928 and **51**, 108, 1928.

³⁾ H. N. RUSSELL and F. A. SAUNDERS: Astrophys. Journal **61**, 38, 1925. Also F. HUND: Zeitschr. für Phys. **33**, 345, 1925 and S. GOUDSMIT: Zeitschr. für Phys. **32**, 794, 1925.

⁴⁾ Resumed description by F. HUND, Linienspektren, Springer, Berlin, 1927.

From GOUDSMIT and UHLENBECK¹⁾ I take some coupling schemes and the method of calculation of the g -values for these schemes. To draw up the g -formulas one makes use consequently of a condition given by LANDÉ²⁾, which thus runs: When a vector z is composed of two vectors x and y whereby

$$|x - y| \leq z \leq x + y$$

then the splitting factor $g(z)$ is determined by:

$$g(z) = \frac{z(z+1) + x(x+1) - y(y+1)}{2z(z+1)} g(x) + \frac{z(z+1) + y(y+1) - x(x+1)}{2z(z+1)} g(y).$$

As given is supposed, that

$$g(l_1) = g(l_2) = 1.$$

and

$$g(s_1) = g(s_2) = 2$$

Scheme I. $\{(s_1 l_1)(s_2 l_2)\} = (j_1 j_2) = j.$

The reciprocal influence of core and electron is so small, that the couplings of s_1 with l_1 (both of the core) and of s_2 with l_2 (both of the electron) is not disturbed. The external magnetic field is not so strong that the coupling of j_1 and j_2 is disturbed.

The g -formula becomes

$$g(j) = \frac{j(j+1) + j_1(j_1+1) - j_2(j_2+1)}{2j(j+1)} g(j_1) + \frac{j(j+1) + j_2(j_2+1) - j_1(j_1+1)}{2j(j+1)} g(j_2).$$

in which is

$$g(j_1) = \frac{j_1(j_1+1) + s_1(s_1+1) - l_1(l_1+1)}{2j_1(j_1+1)} \cdot 2 + \frac{j_1(j_1+1) + l_1(l_1+1) - s_1(s_1+1)}{2j_1(j_1+1)} \cdot 1$$

and

$$g(j_2) = \frac{j_2(j_2+1) + s_2(s_2+1) - l_2(l_2+1)}{2j_2(j_2+1)} \cdot 2 + \frac{j_2(j_2+1) + l_2(l_2+1) - s_2(s_2+1)}{2j_2(j_2+1)} \cdot 1.$$

Scheme II. $[\{(s_1 l_1) s_2\} l_2] = \{(j_1 s_2) l_2\} = (s' l_2) = j.$

By means of the reciprocal influence of core and electron the coupling between s_2 and l_2 is now broken. The s_2 of the electron is coupled with the j_1 (resultant of s_1 and l_1) of the core, which gives a resultant s' . s' is coupled with l_2 , this gives the total resultant j .

¹⁾ S. GOUDSMIT and G. E. UHLENBECK: Zeitschr. für Phys. **35**, 618, 1926.

S. GOUDSMIT and E. BACK: Zeitschr. für Phys. **40**, 530, 1927.

S. GOUDSMIT: Physica **5**, 419, 1925.

²⁾ E. BACK and A. LANDÉ: ZEEMAN-effekt und Multipletstruktur, Springer, Berlin, 1925.

The g -values are calculated from

$$g(j) = \frac{j(j+1) + s'(s'+1) - l_2(l_2+1)}{2j(j+1)} g(s') + \frac{j(j+1) + l_2(l_2+1) - s'(s'+1)}{2j(j+1)} \cdot 1,$$

in which

$$g(s') = \frac{s'(s'+1) + j_1(j_1+1) - s_2(s_2+1)}{2s'(s'+1)} g(j_1) + \frac{s'(s'+1) + s_2(s_2+1) - j_1(j_1+1)}{2s'(s'+1)} \cdot 2$$

in which $g(j_1)$ is calculated as in scheme I.

Scheme III. $[\{(s_1 l_1) l_2\} s_2] = \{(j_1 l_2) s_1\} = (l' s_2) = j.$

Now the l_2 of the electron is coupled with the j_1 of the core, which gives a resultant l' . l' coupled with s_2 gives the total resultant j .

The g - formula is

$$g(j) = \frac{j(j+1) + l'(l'+1) - s_2(s_2+1)}{2j(j+1)} g(l') + \frac{j(j+1) + s_2(s_2+1) - l'(l'+1)}{2j(j+1)} \cdot 2.$$

in which is

$$g(l') = \frac{l'(l'+1) + j_1(j_1+1) - l_2(l_2+1)}{2l'(l'+1)} g(j_1) + \frac{l'(l'+1) + l_2(l_2+1) - j_1(j_1+1)}{2l'(l'+1)} \cdot 1.$$

in which $g(j_1)$ is calculated as in scheme I.

Scheme IV. $\{(s_1 s_2) (l_1 l_2)\} = (s l) = j.$

This is the coupling scheme of RUSSELL and SAUNDERS, also the most frequently occurring. The reciprocal influence of core and electron is large, so that s_1 is coupled with s_2 and l_1 with l_2 . The external magnetic field is not so strong, that the coupling of s and l is disturbed.

The g - values are calculated from

$$g(j) = \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} g(s) + \frac{j(j+1) + l(l+1) - s(s+1)}{2j(j+1)} g(l)$$

in which

$$g(s) = \frac{s(s+1) + s_1(s_1+1) - s_2(s_2+1)}{2s(s+1)} \cdot 2 + \frac{s(s+1) + s_2(s_2+1) - s_1(s_1+1)}{2s(s+1)} \cdot 2 = 2$$

and analogical

$$g(l) = 1.$$

One thus finds here the g -formula of LANDÉ

$$g(j) = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}.$$

3. Application to Ar II.

The energy levels of the simply ionized Argon atom (*Ar II*), of which the g - values are determined experimentally, are levels built upon the stable ground state of the double ionized atom (*Ar III*). This is a 3P state.

The quantum numbers s_1 and l_1 are inherent to this 3P state. With the assumption, that for this lowest term in the energy scheme of double ionized Argon the coupling of the quantum vectors is the normal one, according to scheme IV, one must give the quantum numbers the values

$s_1 = 1$ (gives account for the triplet state) and

$l_1 = 1$ (gives account for the P state).

If a s - electron is coupled, then: $s_2 = 1/2$, $l_2 = 0$.

" " p - electron " " " $s_2 = 1/2$, $l_2 = 1$ and so on.

It may be supposed well known that the different coupling schemes give with each choice of $s_1 l_1$ and $s_2 l_2$ the same number of terms with the same quantum numbers j .¹⁾

TABLE 1. Terms of the $4p$ -electron.

j	Scheme I		Scheme II		Scheme III		Scheme IV	g -values				
	j_1	j_2	s'	l_2	l'	s_2		Scheme I	Scheme II	Scheme III	Scheme IV	Observed
$5/2$	2	$3/2$	$5/2$	1	2	$1/2$	$4P_3$	1.44	1.53	1.53	1.60	1.60
$3/2$	2	$3/2$	$3/2$	1	1	$1/2$	$4P_2$	1.47	1.49	1.50	1.73	1.73
$1/2$	1	$1/2$	$3/2$	1	0	$1/2$	$4P_1$	1.78	2.11	2.00	2.67	2.67
$7/2$	2	$3/2$	$5/2$	1	3	$1/2$	$4D_4$	1.43	1.43	1.43	1.43	1.43
$5/2$	1	$3/2$	$3/2$	1	2	$1/2$	$4D_3$	1.40	1.40	1.40	1.37	1.33
$3/2$	0	$3/2$	$3/2$	1	2	$1/2$	$4D_2$	1.33	1.29	1.30	1.20	1.20
$1/2$	0	$1/2$	$1/2$	1	1	$1/2$	$4D_1$	0.67	0.67	0.67	0.00	0.00
$3/2$	2	$1/2$	$5/2$	1	1	$1/2$	$4S_2$	1.67	1.84	1.83	2.00	2.00
$5/2$	2	$1/2$	$3/2$	1	3	$1/2$	$2D_3$	1.33	1.24	1.24	1.20	1.24
$3/2$	1	$1/2$	$1/2$	1	2	$1/2$	$2D_2$	1.22	1.11	1.10	0.80	0.90
$1/2$	1	$3/2$	$1/2$	1	1	$1/2$	$2P_1$	1.22	0.89	1.00	0.67	0.99
$3/2$	1	$3/2$	$1/2$	1	1	$1/2$	$2P_2$	1.38	1.33	1.33	1.33	1.23
$1/2$	2	$3/2$	$3/2$	1	1	$1/2$	$2S_1$	1.67	1.67	1.67	2.00	1.68

¹⁾ S. GOUDSMIT and G. E. ÜHLENBECK: *Zeitschr. für Phys.* **35**, 618, 1926.
R. DE L. KRONIG: *Proc. Nat. Ac. of Sc.* **12**, 330, 1926.

TABLE 2. Terms of the 5s-electron.

j	Scheme I		Scheme II		Scheme III		Scheme IV	g -values				
	j_1	j_2	s'	l_2	l'	s_2		Scheme I	Scheme II	Scheme III	Scheme IV	Observed
$5/2$	2	$1/2$	$5/2$	0	2	$1/2$	$4P_3$	1.60	1.60	1.60	1.60	1.60
$3/2$	2	$1/2$	$3/2$	0	2	$1/2$	$4P_2$	1.40	1.40	1.40	1.73	1.63
$1/2$	0	$1/2$	$1/2$	0	0	$1/2$	$4P_1$	2.00	2.00	2.00	2.67	2.53
$3/2$	1	$1/2$	$3/2$	0	1	$1/2$	$2P_2$	1.67	1.67	1.67	1.33	1.43
$1/2$	1	$1/2$	$1/2$	0	1	$1/2$	$2P_1$	1.33	1.33	1.33	0.67	0.81

Table 1 contains the observed and calculated g -values according to the 4 schemes for the terms, arising from the coupling of the 4 p -electron. Table 2 gives the same for the terms of the 5 s -electron.¹⁾ For so far terms with the same inner quantum number are mentioned there is a difficulty as to the manner in which one must adjoin the terms of the schemes I, II and III to those of scheme IV. In the tables the g -values of the schemes I, II and III are as much as possible taken in accordance with those of scheme IV²⁾. In table 2 the g -values 1.40 and 1.67 of the terms with $j = 3/2$ are for example yet changeable. In connection with the observed g -values the given allotment seems the most probable.

As is to be expected on account of theoretical considerations, the tables show, that for the terms of the 4 p and 5 s -electron none of the coupling schemes holds exactly.

4. Summary.

In connection with the anomalous g -values, that appear in the spectrum of the simply ionized Argon atom (*Ar II*), g -values are calculated according to 4 different coupling schemes for the terms arising from the coupling of the 4 p and 5 s -electron.

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¹⁾ The g -values of the terms of the 4s-electron agree exactly with those of LANDÉ (scheme IV).

²⁾ S. GOUDSMIT: Thesis, Leiden, 1927.

F. HUND: Zeitschr. für Phys. **34**, 296, 1925.