Physics. — *Experimental Verification of the theory of the paramagnetic rotatory polarisation in the crystals of xenotime.* By JEAN BECQUEREL, W. J. DE HAAS and H. A. KRAMERS. (Comm. Phys. Lab. Leiden 204b).

(Communicated at the meeting of November 30, 1929).

The investigations on the paramagnetic rotation of the plane of polarisation in a plate of xenotime (perpendicular to the optical axis) formed the subject of a recent paper [1]). In another one [2]) the following theoretical law has been given:

$$\varrho = \varrho_\infty \frac{\mu H}{\sqrt{\mu^2 H^2 + K^2}} \, tgh \frac{\sqrt{\mu^2 H^2 + K^2}}{\varkappa T}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where ϱ_∞ = saturation rotation,
μ = magnetic moment (= mg μ_B),
H = intensity of the magnetic field,
T = absolute temperature,
$\varkappa$ = constant of BOLTZMANN,
K = constant, the double value of which gives the distance between two energy levels extremely close to each other, resulting from a duplication of the fundamental state.

We intend to show, that this formula is in agreement with the experiments and to determine the most probable value for the magnetic moment.

Though the rotation is not a function of a single variable $\frac{H}{T}$, it is still easy to plot the curves by taking the values of $\frac{H}{T}$ as abscissae: this is allowable, as the temperatures at which the different series of measurements have been made are either absolutely constant or extremely little variable. In the figures 1 and 2 two graphs, representing reduced curves, have been given; they have the same scale for the ordinates but different scales for the abscissae.

Fig. 1 gives the ensemble of the measurements at 1.38°K. and at 4.23°K. The inferior curve I, which extends up to the abscissa 19330 corresponds to the temperature 1.38°: the superior curves II (the ex-

[1]) Comm. Leiden 204*a*.

[2]) Suppl. Comm. Leiden 68*b*.

plication of which will be given later on) do not extend further than the abscissa 6320 and refer to the measurements at 4.23°.

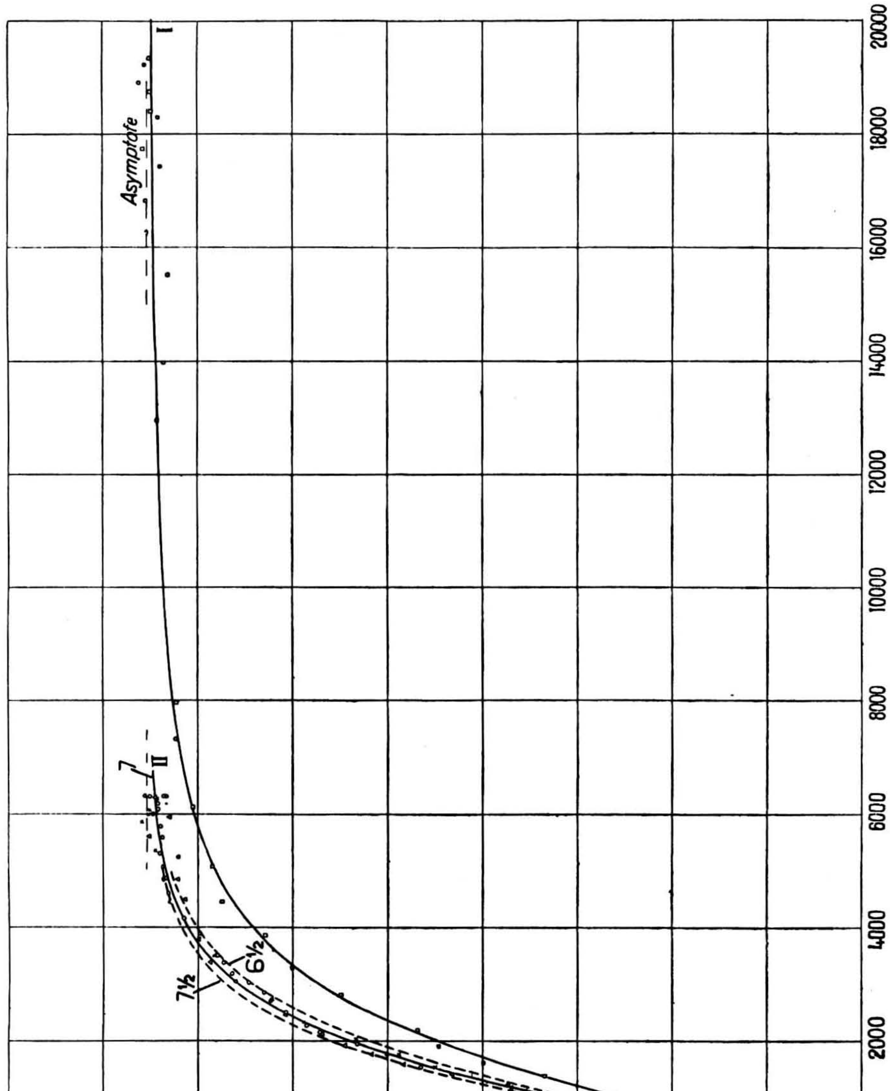

Fig. 1.

In fig. 2 the scale of the abscissae has been tripled, in order to represent the experiments at 4.2° more distinctly. Now the inferior curve I represents only part of the measurements at 1.38°. The curves drawn on the right hand side of the graph with a displacement of the origin of coordinates, relate to the experiments at 14.36°K. (hydrogen under a pressure of 6 cm mercury). The marked points indicate the measured values. The five series of measurements mentioned in the preceding paper, cited above, are indicated in the following way:

1st series T constant 14.34°K. Points o, figure 2 curve III, right hand side of the graph.

2nd series $\left.\right\}$ 4.22$_2$° K const. Points o

T_{mean}

3rd series $\left.\right\}$ 4.25$_4$° K const. Points ×

4.23$_5$°

4th series $\left.\right\}$ 4.23$_0$° K. const. Points △

5th series T oscillating between 1.39$_3$° and 1.37$_1$° points ⊏, T_{mean} 1.38$_2$°.

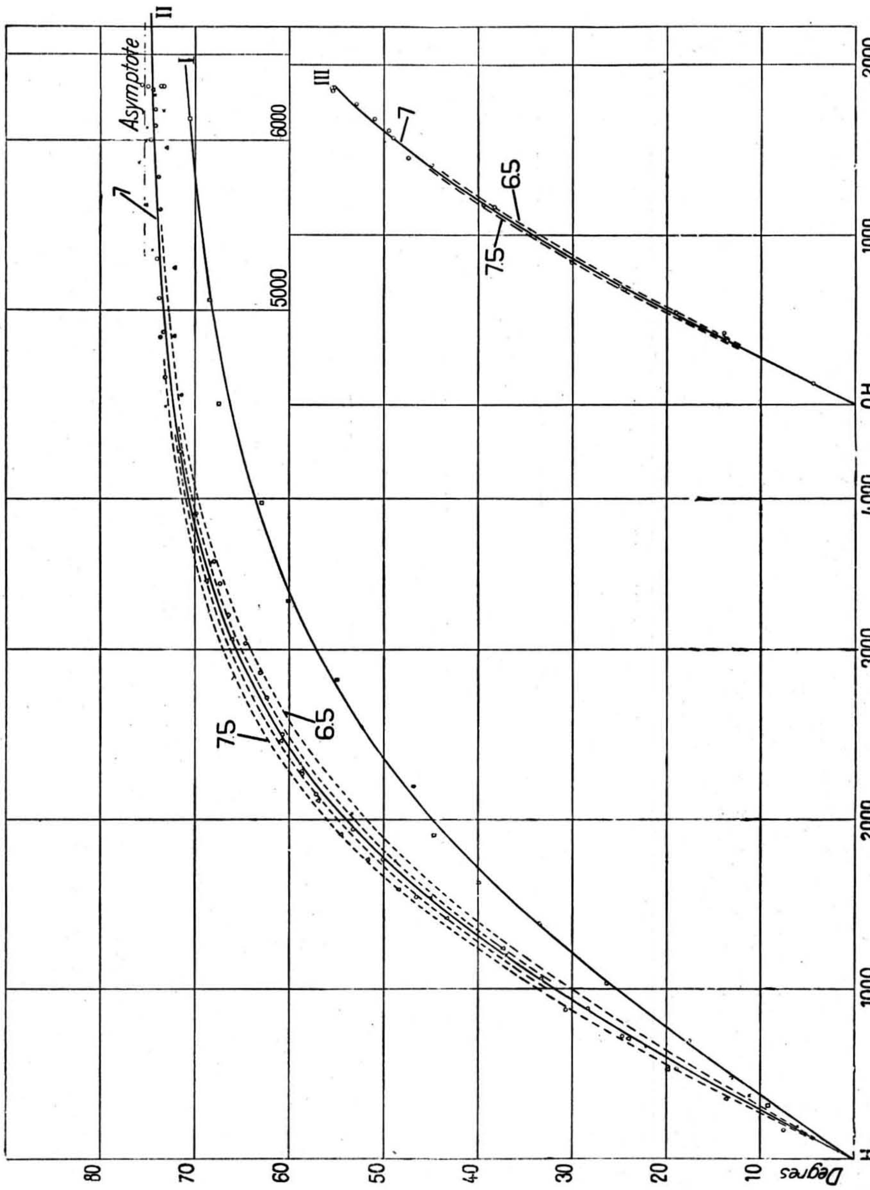

Fig. 2.

For the 5th series the temperatures have been determined from minute to minute; for each measurement $\frac{H}{T}$ has been calculated with the temperature at the moment of observation; for the theoretical curves the mean temperature 1.38_2° has been used.

A preliminary study of the results (first paper) has shown that passing from 1.38° to 4.23° and to 14.34° we find a tendency of the rotation to follow the simple law:

$$\varrho = \varrho_\infty \, tgh \frac{\mu H}{\varkappa T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

with μ nearly equal to $7\,\mu_B$.

(In fact, if K is small compared with $\varkappa T$, the formulae (1) and (2) give only slightly differing results as is evident from formula (1)).

It is therefore reasonable to try this value of μ to represent the experiments at the temperatures 4.23° and 1.38° by formula (1).

Having plotted the *experimental* curve representing the measurements at 1.38° as exactly as possible, we have passed the theoretical curve (formula 1) through the point of the experimental curve with the abscissa 5000 under assumption of the value of ϱ_∞ (75.3° for $\lambda = 5615.7$ and for a thickness of 0.80 mm.) from the first paper. The values adopted for the magneton and for the constant of BOLTZMANN are the following:

$$\mu_B = 0.9216.10^{-20} \text{ (corresponding to } \frac{e}{m} = 1.769.10^{7})$$

$$\varkappa = 1.3708.10^{-16} \text{ (after BIRGE).}$$

In this way we find:

$$\text{for } \mu = 7\mu_B,\ K = 1.96_9.10^{-16}.$$

The full-drawn curves on the two graphs are the theoretical curves calculated with these values of μ and of K. For 1.38° the theoretical curve is found to coïncide nearly with that one drawn initially without intervention of any formula. For the temperatures of 4.23° and of 14.34° the theoretical curves are equally satisfactory.

It is however interesting to examine with what precision the values of μ and of K can be determined from the experiments.

Let us consider first the measurements at 1.38°. If μ is kept fixed, formula (1) is very sensitive to a variation of the value assumed for K: e.g. if we always take $\mu = 7\,\mu_B$, varying K^2 over $\pm$ 10%, we obtain two curves (not drawn in the graphs) which are not so satisfactory for small or medium values of $\frac{H}{T}$ (5000 f.i.) as the curve obtained with the constant K calculated above.

If now we choose a value of μ considerably different from $7\,\mu_B$, e.g.

$6^1/_2\,\mu_B$ or $7^1/_2\,\mu_B$ and if we pass the theoretical curve through the same point as above, we obtain new values for K, with which the representation of the measurements remains good (these curves very close to the drawn curve are omitted in the figure).

If thus we only consider the experiments at 1.38° a wide margin remains for the choice of the value of μ. If on the contrary μ is given, the constant K is sufficiently well defined.

Fortunately the situation is reversed with respect to the curves for 4.23° and 14.34°. For a given value of μ they are nearly insensitive to a variation of K (within the limits compatible with the experiments at 1.38°), but they are strongly influenced by a variation of μ.

The results are evident as soon as we remark that, as has been said already, the values of ϱ corresponding to formula (1) approach those given by formula (2) more and more as the temperature rises.

From this follows, that the determination of μ and of K will be improved by the comparison of the results obtained at the three temperatures used. In Fig. 1 for $T = 4.23°$ the full-drawn curve ($\mu = 7\,\mu_B$) is surrounded by the curves (dotted) calculated with $6^1/_2\,\mu_B$ (inferior curve) and $7^1/_2\,\mu_B$ (superior curve); in each case the value taken for K is that which is best suited to represent the measurements at 1.38°.

In Fig. 2 these same curves for $6^1/_2\,\mu_B$ and for $7^1/_2\,\mu_B$ are found again, moreover we have inserted the curves calculated with 6.8 μ_B and with 7.2 μ_B.

Except for the immediate neighbourhood of the origine and of the asymptote (regions where the different curves are very close together) the points representing the measurements lie between the curves corresponding to $6^1/_2\,\mu_B$ and to $7^1/_2\,\mu_B$; it is even likely that these limits can be further reduced and that μ must lie between 6.8 μ_B and 7.2 μ_B.

For the temperature 14.34° the ordinate of the asymptote can be calculated with the least squares, assuming as a first approximation the form of a hyperbolical tangent: the value found depends a little on the choice of μ. The full-drawn curve, as has been said above, has been calculated for 7 μ_B: we find then $\varrho_\infty = 78.4°$ [1]). The dotted curves have been calculated for $6^1/_2\,\mu_B$ and for $7^1/_2\,\mu_B$ in such a way that they cut the curve for 7 μ_B in the same point with abscissa the highest value of $\frac{H}{T}$ used [2]). It is evident, that the curve corresponding to 7 μ_B represents measurements best.

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1) The asymptote found in this way is higher than at 4.23° (difference 3.1°) and this needs not surprise us: the same is found with the tysonite, the parisite and the rare earth glasses. The absorption by the active bands diminishes when the temperature decreases; it becomes constant only at extremely low temperatures.

2) These curves are closer together than those calculated for the temperature 4.2°, because we have passed the curves through the same point near the origin instead of giving them the same asymptote.

The result of this discussion is that formula (1) has been well verified and that the most probable value of the active magnetic moment is 7 magnetons. The constant K has then the value $1.97 . 10^{-16}$. In the second paper cited above, it is said that this magnetic moment is probably that of the gadolinium ion.

Finally we wish to express our thanks to Mrs CAPEL and BLOM for their valuabie assistance in the experiments.