Physics. — *Determination of the ratio of the specific heats (c_p/c_v) of helium gas at the boiling point of oxygen, by means of the velocity of sound.* By W. H. KEESOM and A. VAN ITTERBEEK. (Communication N⁰. 209a from the Physical Laboratory at Leiden.)

§ 1. *Introduction.* With the intention of performing measurements, which will furnish data of the specific heats of gases at very low temperatures and at pressures less than one atmosphere, we have worked out a method, which enables us to determine the velocity of sound in a gas at low temperatures with an accuracy of 0.3 %. We have tested this method for helium gas at the boiling point of oxygen and a pressure of one atmosphere. We could compare our result with a direct determination of the specific heat by SCHEEL and HEUSE [1]).

For later measurements at the temperatures of liquid hydrogen and liquid helium, we have used a modified method. Yet we think a communication of this measurement at the temperature of the boiling point of oxygen is desirable because of the degree of accuracy obtained with it.

§ 2. *Method and apparatus.* We follow a resonance method, which is between the method of QUINCKE [2]) (open resonator, changeable length, constant frequency) and that of THIESEN [3]) (closed resonator, constant length, changeable frequency). That is to say, we make use of a closed resonator with changeable length and constant frequency. The last gives the advantage, that one is not disturbed by the occurrence of the proper tones of the supply tubes.

The apparatus is represented in Fig. 1. As a source of sound we used a telephone T, soldered in a copper box D, that is tightly closed. The telephone is separated from the wall of the box D by a thin layer of cotton-wool W. The box D is provided with a little cone K, so that the sound in passing from the box D to the leading-tube B_1 is damped as little as possible. The apparatus was filled with helium gas through the tube H. A case Ka filled up with cotton-wool and saw-dust is placed around the box D, to prevent disturbance by the sound originating from the telephone.

1) K. SCHEEL and W. HEUSE. Ann. d. Phys. (4) **40**, 473, 1913.
2) G. QUINCKE. Pogg. Ann. **128**, 177, 1866. Wied. Ann. **63**, 66, 1897.
3) M. THIESEN, Ann. d. Phys. (4) **25**, 506, 1908.

T_1 represent the supply-wires of the telephone, connected to the secundary coil of the transformer.

The sound reaches the copper resonator tube R through the tube B_1, which is soldered to the cryostatcap C. The length and the diameter of R
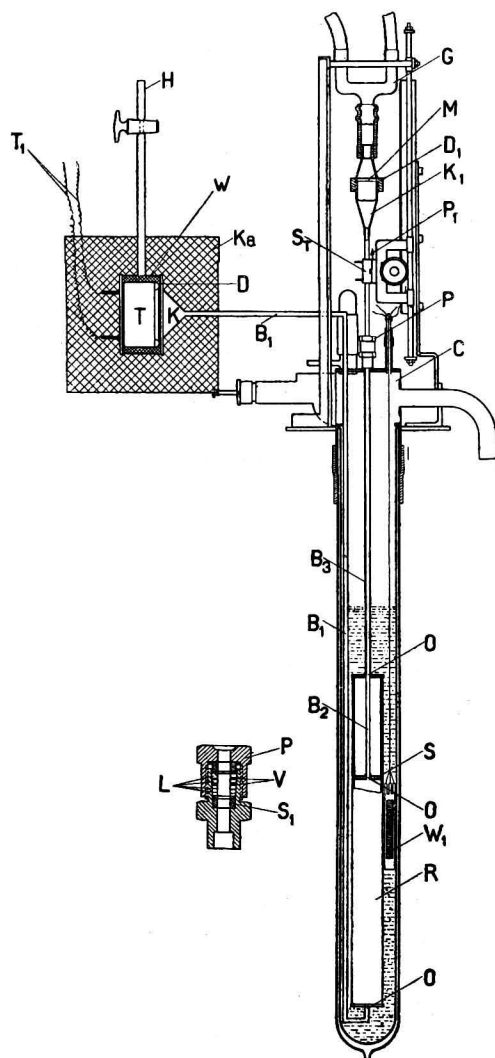

Fig. 1.

are respectively 30 cm and 2.6 cm, the openings O are 4 mm. The inside of this resonator tube was very accurately turned and polished on the lathe.

In this resonator tube was a moveable disk S, provided with a little opening of 3 mm. On the upper part of the disk a piece of felt was pasted, in order to avoid the sound caused by friction of the disk against the wall of the resonator.

The disk is fastened to a tube B_2 of German silver, of inner diameter

3 mm, which moves in the tube B_3. The latter is also soldered to the cryostat C.

The moveable tube B_2 is clamped in a copper piece S_T, that is moved by a rack-and-pinion system.

The end of the tube B_2 is connected by means of a second cone K_1 to a box D_1, diameter 2.5 cm. The box is closed from the air by means of a membrane M of German silver, 0.02 mm. thick.

The tube B_2 moves, through the packing box P, while remaining hermetically closed (see Fig. 1). By using special thin pieces of leather it was possible to reach a vacuum of 10^{-5} mm mercury-pressure in the resonator. S_1 is a conical massive copper piece, L represents the thin pieces of leather soaked in oil, V are layers of grease.

The maxima were observed by means of a stethoscope G. The positions of the tube for two succeeding resonance maxima were determined by a cathetometer, adjusted upon the sharp point P_1. Thus it was possible to determine each time the length of a half wave-length up to 0.01 mm.

By adjusting on two succeeding maxima one is independent of the correction of the opening O.

The temperature was determined by means of a resistance thermometer W_1, of a rather small type, in view of the restricted dimensions of the cryostat-vessel. The platinum resistance wire was wound upon a porcelain tube, suspended in a glass tube.

After having exhausted the resonator to 10^{-5} mm mercury pressure, it was filled with helium gas to 1 atmosphere. The pressure was read on an open mercury manometer by means of the same cathetometer, with which the position of the moveable disk was determined.

§ 3. *Generator. Determination of the Frequency.* As generator we used the classical oscillating arrangement in which the anode circuit is coupled inductively with the grid circuit, using three triode-valves connected in parallel ; as valves we used Philips $T A\,04/5$, heating current 1.6 Amp., anode potential 400 Volts.

The frequency was changed by changing the capacity of the oscillating circuit between 2 μF and 0.5 μF.

In this way and also by connecting other selfinductions in circuit, it was possible to reach the frequency range between 700 and 2800 vibrations per second.

The accuracy of the method is defined by the accuracy, with which the frequency is known.

For the determination of the frequency we made use of a string galvanometer, recording the movement of the string, which was a 4 μ Wollaston wire. Fig. 2 represents this apparatus.

The image of the string of the galvanometer S_g is projected vertically by a system of lenses on the horizontal slit of a registration drum R. B represents an arc lamp, which illuminates the string. The little water-

cuvette W serves for taking up the heat of the arc lamp. TR represents a low frequency transformer, the primary coil of which is connected up in the anode circuit of the tonegenerator.

The registration drum is moved by means of a motor M, 1/16 HP. The current for the motor was delivered by a 32 Volt battery
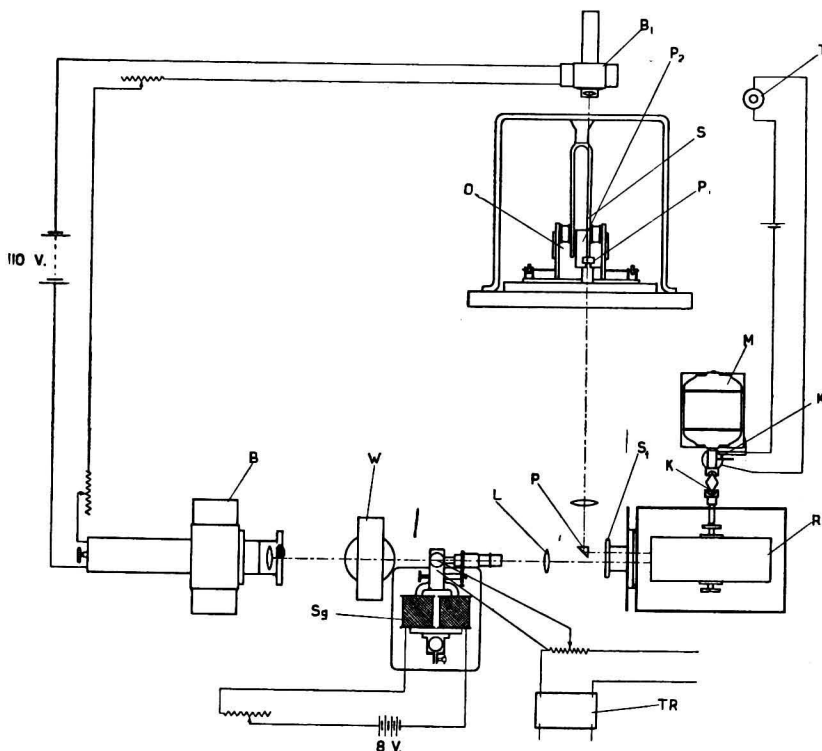

Fig. 2.

with constant current. The axis of the registration drum is joined to the axis of the motor, by a special coupling K. The velocity of the motor was determined optically by means of a tuning fork S, which intercepts with its proper frequency the light of an arc lamp B_1. The tuning fork is driven electrically (in the figure the tuning fork has been drawn in projection). One prong of the tuning fork carried a light plate P_1 with a small opening O (0.1 mm). The opening O of the plate P_1 vibrates before the equally large opening of a second plate P_2, on which the light of the arc lamp B_1 is focussed. We thus obtain on the registration drum, with the aid of the total-reflecting prism P, a small point, which appears every other half period of the tuning fork. The points derived from the fork and the sine-curves of the string are registered on sensitive bromide paper. A blind-shutter S_1, adjusted to "time", is placed before the slit. K represents a mercury cap, by which the current in the telephone is closed each time after a complete revolution of the drum. In this way it was possible to open the shutter at a definite "tick" and to shut it at a following "tick"; thus

preventing a later part of the image on the sensitive paper falling over the first part.

Assume: l_m the length between a whole number of points of the tuning fork,

l_x the length of a whole number of sine-curves of the string,

n_m the number of the points,

n_x the number of the curves,

f_m the frequency of the tuning fork,

f_x the unknown frequency.

We have:

$$f_x = \frac{f_m \, n_x \, l_m}{n_m \, l_x} \, .$$

In this way it was possible to get a relative accuracy of 1 in 10.000. The absolute accuracy is fixed by the accuracy with which the frequency f_m of the electrically driven tuning fork is known. The frequency f_m was determined by an electrically driven tuning fork of the Cambridge Instrument Co. with frequency 300.0, calibrated by the Cambridge Instrument Co. with an accuracy of 0.1 %. The anode current of it is lead through the string galvanometer. We found for f_m:

$$\text{February } 10^{\text{th}} \; 1930 \quad f_m = 132.4$$
$$\text{,,} \quad\;\; 27 \quad \text{,,} \quad\;\; \text{,,} = 132.3$$
$$\text{March} \quad 27 \quad \text{,,} \quad\;\; \text{,,} = 132.5.$$

We took for the value of f_m the average $f_m = 132.4$.

§ 4. *Calibration of the Generator. Determination of the velocity of sound in helium gas at the boiling point of oxygen.* We have examined how far the frequency of the generator is dependent on the heating current i of the valves at constant anode potential.

Coils I	$1 \, \mu F$	$0.5 \, \mu F$
$i = 4.8$ amp.	$f_x = 1833$	$f_x = 2465$
4.9	1832	2460
4.7	1847	2518

4.8 amp. represents the maximal current intensity. We see, that, when this intensity is reached, the frequency changes but little.

For the measurements we used coils I and capacities $1 \, \mu F$ and $0.5 \, \mu F$; the heating current was maintained constant at 4.8 amp.

We found:

f_x	$\frac{\lambda}{2}$ average in cm.	velocity of sound in m/sec
1833	$15.25^1 \pm 4^0$	559.1
2465	$11.34^1 \pm 3^5$	559.1

The pressure in the resonator was 76.5 cm; the temperature of the oxygen bath was maintained constant at —182.90°.

§ 5. *Determination of the ratio of the specific heats.* Assume:

W = velocity of sound in cm/sec,
V = molecular volume in cm³/g,
P = pressure in dynes/cm²,
p = „ „ atmospheres,
M = molecular weight = 4.00 (HEUSE, TAYLOR),
c_p = specific heat at constant pressure,
c_v = „ „ „ „ volume,
R_M = molecular gasconstant (erg/° C. Mol.) = 8.316×10^7,
R = gasconstant in atm. theoretical normal volume/° C. = 1/273.1.

From

$$W = \sqrt{-\frac{c_p}{c_v}\left(\frac{\partial P}{\partial V}\right)_T \frac{V^2}{M}}, \quad \cdots \cdots \quad (1)$$

and

$$pv = RT\left(1 + \frac{B}{v}\right), \quad \cdots \cdots \cdots \quad (2)$$

it follows that

$$W = \sqrt{\frac{c_p}{c_v} \cdot \frac{R_M T}{M}\left(1 + \frac{2B}{RT}p\right)}, \quad \cdots \cdots \quad (3)$$

where $B = 0.504 \times 10^{-3}$ [1]). With $W = 559.1$, $T = 90.20$, $p = 1$, $\frac{c_p}{c_v}$ becomes 1.662. We put the accuracy of this at 0.4 % (comp. § 1).

SCHEEL and HEUSE [2]) obtain at —180° C. $\frac{c_p}{c_v} = 1.671$ (they give as accuracy 0.7 %).

§ 6. *Calculation of* $\frac{c_p}{c_v}$ *for a pressure* $p = 0$. From known thermodynamical formulae and formula (2) in which the term with B is supposed to be sufficiently small, it follows that:

$$\frac{c_p}{c_v} = \left(\frac{c_p}{c_v}\right)_{p=0}\left[1 + \frac{2}{\lambda R}\left\{\frac{dB}{dT} + \frac{1}{2(\lambda+1)}T\frac{d^2B}{dT^2}\right\}p\right], \quad \cdots \quad (4)$$

when

$$M(c_v)_{p=0} = \lambda R_M.$$

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[1]) See G. P. NIJHOFF. Comm. Leiden Suppl. N⁰. 64c.
[2]) See Wärmetabellen, Physik. Techn. Reichsanstalt, p. 56.

From the values for B, given by NIJHOFF l.c. follows:

$$\left(\frac{dB}{dT}\right)_{90,20^\circ} = 1,9 \times 10^{-6} \quad \text{and} \quad \left(\frac{d^2B}{dT^2}\right)_{90,20^\circ} = 3,6 \times 10^{-8}.$$

We thus obtain

$$\left(\frac{c_p}{c_v}\right)_{p=0} = 1,661.$$

§ 7. *Influence of the wall of the resonator on the velocity of sound.* We saw in § 2, how it is possible to eliminate with our method the correction of the end-openings on the velocity of sound.

In order to estimate the influence of the wall of the resonator on the velocity of sound, we have made use of the formula of KIRCHHOFF-HELMHOLTZ [1]

$$\frac{\triangle W}{W} = \frac{1}{D\sqrt{\pi \nu \varrho}}\left[\sqrt{\eta} + \left\{\sqrt{\frac{c_p}{c_v}} - \sqrt{\frac{c_v}{c_p}}\right\}\sqrt{\frac{\varkappa}{c_v}}\right] \quad \ldots \quad (5)$$

where:

$\triangle W =$ correction of the velocity of sound,
$D =$ diameter in cm, 2.6 cm,
$\nu =$ frequency, 1800 osc./sec,
$\varrho =$ density in gr./cm³,
$\eta =$ coefficient of viscosity in C.G.S. units
$\quad \eta = 918.6 \times 10^{-7}$, [2]

$\frac{c_p}{c_v} = 1.66$, $c_v = 0.74$ cal/gr.,

$\varkappa =$ heat conductivity in cal/gr. cm sec.,
$\varkappa = 1.48 \times 10^{-4}$ at -191.7° C: (EUCKEN 1913).

(5) gives $\dfrac{\triangle W}{W} = 7.0 \times 10^{-5}$. This can therefore be neglected in measurements of this degree of accuracy.

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We also thank Mr. J. A. v. LAMMEREN, phil. nat. cand., for his helpful assistance.

[1] Comp. R. E. CORNISH and E. D. EASTMAN, Phys. Rev. (2) **33**, 90, 1929.
[2] H. KAMERLINGH ONNES and S. WEBER. Comm. Leiden N⁰. 134*b*.