Hydraulics. — *The cause of periodicity generally occurring with rising mixtures of gas and liquid.* By J. VERSLUYS.

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In a paper of 1928 (1) the author dealt with the factors which interfere with the flow of mixtures of gas and liquid in vertical channels and again in 1929 (2). In these two papers the cause of periodicity or intermittency was already mentioned, while in another paper (3) the causes of this phenomenon were treated regarding hot and gaseous springs.

Periodical flowing often occurs in the case of mixtures of gas and liquid rising in a vertical channel. The phenomenon can be explained if it is accepted that two conditions are possible in the mixture, namely the foam condition and the mist condition.

If there is no stabilizer present in the liquid the foam condition can only exist when the liquid forms a greater portion of the volume than the gas. The mist condition, on the other hand, can only exist when the liquid occupies a smaller part of the volume than the gas. The foam condition occurs when there are gas bubbles in the liquid mass, while in the mist condition drops of liquid are disseminated in a gas-filled space.

For sake of simplicity it has been assumed that the mist condition prevails when 50 % or more of the volume is occupied by gas, while with less than 50 % gas the foam condition will arise. For distilled water and for oils which do not tend to form stable foams this is probably approximately true. If this does not hold good, however, two cases are possible :

1⁰. the fields of the two conditions overlap.

2⁰. they do not meet.

If the fields of both conditions overlap there is an intermediate field where either of the two conditions can exist, depending on the history of the mixture. Suppose the foam condition prevails first and the gas expands ; in this event the foam will break and the liquid will disperse as soon as the ultimate proportion of gas in the mixture has been obtained. Owing to the overlapping of the two fields the mist condition, which has arisen through the breaking of the foam, will persist. If on the contrary the mixture is under the mist condition first, and it is compressed, the conversion to the foam condition will only take place when the gas volume has so far decreased that the limit of mist condition is reached. Due to the overlapping the new condition will not be disturbed.

If the fields of mist and foam conditions do not meet, however, —

whatever the history of the mixture may be — there is no sudden conversion if the volume ratio of the two substances gradually changes. Suppose we start with the foam condition and the gas expands ; when the limit of this condition is reached, the foam would break, but there is not yet sufficient gas to allow of the existence of the mist condition throughout the mass. Probably the mixture will dissociate, viz. it will be converted into a mixture of small bodies of foam and of mist. Not until the gas has sufficiently expanded, so as to allow of the mist condition throughout the mass, will the foam entirely disappear.

In the former considerations, however, the tendency of the two substances, gas and liquid, to segregate owing to the great difference of specific gravity has been disregarded. This would only be correct for a very intimate mixture, that is to say with very small bubbles of gas or very fine drops of liquid.

The dispersion of gas in liquid (the foam condition) or of liquid in gas (the mist condition) generally is not such as to make the particles (either the bubbles or the drops) so small that the bubbles would not rise and the drops would not sink at an appreciable rate. This question of the segregation of the two components, gas and liquid, the effect of which is neutralized by the continuous rising of the mixture in a channel, is of vital importance as to the flow conditions in a channel. The result is that the gas always has a greater velocity than the liquid. Gas bubbles will endeavour to rise in the heavier liquid under all circumstances, while drops of liquid, when distributed in a gas will always have a tendency to drop. In the foam condition the difference in velocity is small, while in the mist condition it is large.

It appears from experiments made by J. S. OWENS (6) that gas bubbles rise in oil at a rate of about 20 cm per second. In water the average velocity is approximately 30 cm per second. Investigations made by P. LENARD (4), have demonstrated that the velocity of the fall of drops of water in air can be as follows :

Diameter of the drops in millimetres	Maximum velocity of fall in meters per second	Diameter of the drops in millimeters	Maximum velocity of fall in meters per second
0.01	0.0032	1.5	5.7
0.02	0.013	2.0	5.9
0.03	0.029	2.5	6.4
0.05	0.080	3.0	6.9
0.1	0.32	3.5	7.4
0.2	1.3	4.0	7.7
0.3	2.7	4.5	8.0
0.4	3.2	5.0	8.0
0.5	3.5	5.5	8.0
1.0	4.4		

Rising air should have the following velocities in order to entrain drops of water :

Diameter of the drops in millimetres	Velocity of the rising air in metres per second
1.28	4.80
3.49	7.37
4.50	8.05
5.47	7.98
6.36	7.80

Figures were found by W. SCHMIDT (5), which do not differ to any appreciable extent from those found by LENARD.

A result of the difference in velocity is that the proportion in which gas and liquid are mixed when the mixture rises is different from that in which they flow through. The liquid always moves slower, and each particle of liquid remains longer in a certain part of the traject than does a particle of gas.

This can be conceived as follows : Suppose pedestrians and motor-cars are moving along a road in the same direction at such a rate that one pedestrian and one motor-car pass a certain point each minute, there will always be 10 pedestrians and 1 motor-car in a stretch of 1 kilometre if the former have a speed of 6 and the latter a speed of 60 kilometres per hour. The ratio of the flow is then 1 : 1 and the ratio of the mixture is 10 : 1.

The proportion in which liquid and gas are mixed is, therefore, always more to the advantage of the liquid (the slowest) than that in which the two substances flow through, and a large difference in velocity with one and the same ratio of flow will change the ratio of the mixture more to the advantage of the liquid than a small difference in velocity.

The ratio of mixture depends on both the ratio of flow and on the difference in velocity and also on the absolute velocity. This is a point of great importance : the ratio of mixture determines which condition will occur, namely the foam condition or the mist condition, but each of these conditions has its own difference in velocity and the difference in velocity has a great influence on the ratio of mixture. As will be seen below, the consequence of this is that with a certain velocity of rise of the mixture and with certain ratios of flow, neither the foam condition nor the mist condition is actually possible.

If a mixture of gas and liquid rises in a vertical channel the pressure then decreases and the gas expands in consequence. As a result of this the ratio of the flow and also the ratio of the mixture vary with height. But the velocity of the mixture also becomes greater. The decrease in pressure has also, therefore, an indirect influence on the ratio of the mixture in consequence of the volume of the gas becoming greater when there is a decrease in pressure, so that the velocity increases. This second, indirect, influence is not as great as the first.

When the mixture of gas and liquid rises in a channel with a uniform cross section the ratio of the mixture will change more and more to the advantage of the gas. It is, therefore, quite conceivable that at the bottom of such a channel the ratio of the mixture is such that the foam condition should exist, while at a certain height, in consequence of the decrease in pressure, there would be too much gas present in proportion to the liquid, so that the foam condition would have to give way to a mist condition.

Should this occur, however, then a much greater difference in velocity would suddenly arise at the height in question and in consequence of this the ratio of the mixture would again change to the advantage of the liquid, so that there would be too much liquid present in the mixture for the mist condition. The mist condition cannot, therefore, arise immediately. This is only possible higher up after the pressure has decreased still more and the ratio of the flow, the absolute velocity and also the ratio of the mixture have changed much more to the advantage of the gas. There will, therefore, be a zone where neither the foam condition nor the mist condition is possible. Should the foam condition arise for a moment in this zone then the accompanying small difference in velocity would cause too much gas to be present in the mixture and the mist condition would have to arise. Similarly, if the mist condition then arose for a moment the liquid would again predominate in the mixture in consequence of the large difference in velocity, and the foam condition would again arise, which, however, as has been described, would have to give way again to the mist condition. There is a vicious circle here.

The result will be that a part of the gas will rise with as much liquid as it can entrain in the mist condition. The remaining liquid will keep as much gas confined as it is possible for it to contain in the foam condition. At first, therefore, larger foam bodies will be entrained in the mist. These will, however, have a tendency partly to remain behind, that is to say to rise slower, and yet at the same time to give off continually more mist. Differentiation, therefore, takes place and in consequence there is periodical flowing. This differentiation more or less conforms with the differentiation described heretofore, which may be caused without difference of velocity in case the fields of foam and mist conditions do not meet.

The larger bodies of foam entrained by the gas, or the mist, when segregation takes place, will have a greater difference of velocity with the surrounding gas than the small drops in mistcondition. In many cases even the difference of velocity would be greater than the velocity of the gas and the larger bodies would not be lifted. In this event they are thrown up from time to time and this can cause real intermittence.

The question of the intermediate condition between the foam condition and the mist condition, due to difference in velocity, can be summarized as follows. — The great difference in velocity existing in the mist condition causes a lower ratio of gas to liquid. In consequence of this, if the ratio of the flow of gas and liquid is not very high, the ratio of mixture will arise at which the foam condition must result. As soon, however, as this arises the difference in velocity becomes small and the ratio of mixture of gas to liquid becomes high again, so that the foam condition becomes at once impossible.

According as the absolute velocity is greater, the influence of the difference in velocity on the ratio of the mixture is smaller. As the difference in velocity always transposes the ratio of the mixture towards that of the foam condition, a great velocity of rise, which itself is promoted by a small cross section, will in general have a tendency to make the ratio of the mixture favourable for the mist condition.

With certain quantities of gas and liquid flowing through per unit of time, two critical cross sections exist, for the foam condition is only possible up to a certain maximum velocity, thus to a certain minimum diameter. The foam condition only exists, therefore, when the cross sections are larger than the larger of the two critical ones, and the mist condition only when the cross section is smaller than the smaller critical one.

It must, however, be borne in mind that the mist condition is not possible with every ratio of flow. This condition is not tenable when more liquid flows through than gas. The difference in velocity can never make the ratio of mixture more favourable to the gas than the ratio of flow. The foam condition can take place theoretically with every ratio of flow if the cross section is large enough. An extreme case would be when the quantity of liquid rising is 0.

Every channel and every pipe in which a mixture of gas and liquid rises will show a certain periodicity in flow when in one part the cross section lies between the two critical ones for the ratio of flow prevailing there (which, as said, depends i.a. on the pressure).

Mathematically the principle can be deduced simply as follows :

As has already been said, no matter which condition prevails, the foam condition or the mist condition, there is a difference in velocity between gas and liquid, which will be represented by b.

If the velocity of the gas is u, then that of the liquid is $u-b$. If per unit of volume of liquid at the prevailing pressure n volume units of gas flow through at a pressure prevailing in a certain cross section, then in that cross section, for each unit of volume flowing through per unit of time, a surface of :

$$\frac{1}{u-b} \quad \text{is occupied by liquid} \quad \ldots\ldots \quad (1)$$

and

$$\frac{n}{u} \quad \text{by gas} \quad \ldots\ldots\ldots\ldots \quad (2)$$

If the cross section per unit of volume liquid flowing through per unit of time is S, then

$$S=\frac{n}{u}+\frac{1}{u-b} \quad \ldots\ldots\ldots \quad (3)$$

It follows from (1) and (2) that although gas and liquid flow through in the ratio n they are mixed in the cross section in the ratio :

$$\varphi = \frac{n}{u} : \frac{1}{u-b} = \frac{n\,(u-b)}{u} \quad \ldots\ldots \quad (4)$$

As always is: $\frac{u-b}{u} < 1$, consequently must be:

$$\varphi < n, \quad \ldots\ldots\ldots\ldots \quad (5)$$

that is to say that the ratio of the mixture is higher than the ratio of the flow.

If then, the velocity is taken from (3) and (5), S may then be expressed as follows :

$$S = \frac{(n-\varphi)(\varphi+1)}{\varphi\, b} \quad \ldots\ldots\ldots \quad (6)$$

It is known from observations that in the foam condition $b = 20$ to 30 cm, and in the mist condition 600 to 800 cm per second.

In order to be on the safe side (that is to say not to take the limits of the intermittency too narrow) it is taken here that :

$$\left.\begin{array}{l}\text{for the foam condition } b = 20,\\ \text{for the mist condition } b = 1000,\end{array}\right\} \quad \ldots\ldots \quad (7)$$

both in centimeters per second.

If the foam condition exists and the volume of the bubbles increases in proportion to that of the liquid, it may then be assumed that if there is no stabilizer present (in water this may be soap, for instance) the bubbles will touch one another and unite when the gas volume is as large approximately as the liquid volume. This means that the liquid disperses in drops.

In the same way, when the mist condition exists, the drops will touch one another and have a tendency to unite when their total volume becomes as large as that of the gas. A continuous liquid mass then forms which contains gas bubbles. Although it is not quite correct to do so, it will, as it has been stated heretofore for sake of simplicity, be taken that $\varphi = 1$ is the limit of the possibility of the existence of either the mist condition or the foam condition. If $\varphi > 1$, the mist condition then prevails, while if $\varphi < 1$, the foam condition prevails. This applies to water and to many grades of oil. It might be thought that φ may be greater for some kinds of oil. Provisionally, however, φ is taken as equal to 1.

Further the fact that the difference b becomes smaller both in the foam condition and in the mist condition if the volumes of gas and liquid in the mixture become approximately equally large, will be left out of consideration.

By taking $\varphi = 1$ in (6) the value of S is obtained for the transition state from foam to mist :

$$S_l = 2\,\frac{n-1}{b} \quad \ldots\ldots\ldots \quad (9)$$

When the transition state is reached from the foam condition then $b = 20$ and

$$S_{lf} = \frac{n-1}{10} \quad \ldots\ldots\ldots\ldots \quad (10)$$

and when reached from the mist condition, then $b = 1000$ and

$$S_{lm} = \frac{n-1}{500} \quad \ldots\ldots\ldots\ldots \quad (11)$$

As was already concluded by reasoning, there are, therefore, two critical cross sections, which are expressed per unit of volume of liquid for a certain ratio of flow n by (10) and (11).

It follows from (10) and (11) that:

$$S_{lf} = 50\, S_{lm} \quad \ldots\ldots\ldots\ldots \quad (12)$$

In order to make the mist condition at once possible, when rising takes place, where the foam condition is no longer possible, the cross section of a channel would, therefore, have to be 50 times as small and in case of a circular cross section, the diameter $\sqrt{50}$, or about 7 times as small.

For a great number of oil wells which practically produce a mixture of gas and oil it appeared that they flowed regularly when the diameter of the flow pipe as to the volumes of gas and oil could be computed to be outside the two critical ones, as indicated in this paper, throughout the height of the well. If it appeared from a calculation that in a portion of the well, the cross section was lying between the critical ones, for the volumes of liquid and gas flowing through, some periodicity generally could be observed, in many cases real surging or intermittence took place.

It is quite probable that various phenomena of volcanic eruptions can also be explained in this way. The difference of velocity in the foam condition may practically be 0 with volcanos, while instead of drops of liquid and a mist condition, often larger blocks (in many cases of solid foam) are entrained by the gases, which would mean a greater difference in velocity.

LITERATURE.

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