Hydraulics. — *Motion of gasbubbles in a horizontal flow of liquid.* By J. VERSLUYS.

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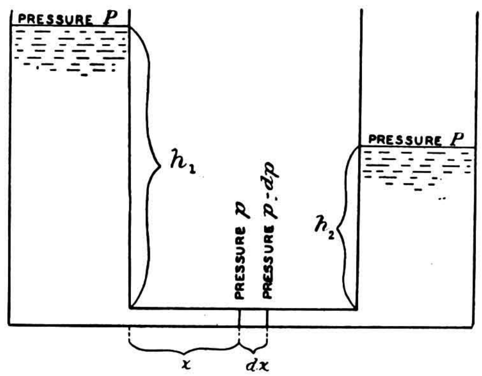
In case a liquid moves horizontally and there is a certain resistance, there must be differentials of pressure in order to maintain the motion, and due to these differentials of pressure, a gas bubble in the liquid will tend to move in the same direction as the liquid but at a greater speed.



Suppose two reservoirs, as shown in the accompanying figure, are connected at the bottom by means of a horizontal tube. These reservoirs are filled with liquid to different levels and consequently the liquid will flow through the tube from one reservoir to the other. It will be accepted that the water levels in the reservoir do not change and that kinetic energy of the water may be neglected. There is turbulence in the tube and this causes resistance. According to this, there must be a drop of pressure head in the tube between the two reservoirs. We take dp for the drop of pressure on the interval of length dx. A gas bubble which would have a volume v_0 at the pressure p_0 moves a distance dx to the right, the direction in which the pressure head decreases. At this displacement, the volume of the bubble increases from

$$\frac{p_0 v_0}{p} \text{ to } \frac{p_0 v_0}{p - dp}.$$

So the increment of volume is:

$$\frac{p_0 v_0}{p^2} dp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The gas in the bubble has expanded and owing to this it has performed a work:

$$\frac{p_0 v_0}{p} dp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

At the same time a volume of liquid $\frac{p_0 v_0}{p^2} dp$ is lifted to the surface. If at first we do not venture any speculations about the reservoir in which this liquid rises but suppose that this volume of liquid is lifted to a height h, then gravity performs a work:

$$-\gamma_l \frac{p_0 v_0}{p^2} h \, dp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

if the specific weight of the liquid is γ_l. Besides, the liquid raises the surface of one of the reservoirs or of both reservoirs on which the atmosphere exerts a pressure P, and consequently the pressure of the atmosphere exerts a work:

$$-\frac{p_0 v_0}{p^2} P \, dp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

So regardless of the entraining action of the motion of liquid on the bubble, this will tend to move to the right in case:

$$\frac{p_0 v_0}{p} dp > \gamma_l \frac{p_0 v_0}{p^2} h \, dp + \frac{p_0 v_0}{p^2} P \, dp \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or if:

$$p > \gamma_l h + P \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If we presume $h = h_2$, this formula is converted to:

$$p > \gamma_l h_2 + P \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The right member of this formula represents the pressure at the right end of the tube and the pressure p is greater than this pressure so that this formula is accomplished. Consequently, the free gas will tend to move horizontally at a greater speed then the surrounding liquid.

This phenomenon, the tendency of gas to migrate horizontally faster than a surrounding liquid is of some importance for the study of flow of gas and liquid in a porous medium.

The difference of speed between gas and liquid in a vertical channel has been dealt with by the author in former papers, especially in "The cause of periodicity generally occurring with rising mixtures of gas and liquids" presented to the Academy in the May meeting, and in the papers mentioned at the foot of that paper.