Astrophysics. — *The Influence of Collisions on the Formation of the Fraunhofer Lines.* By A. PANNEKOEK.

(Communicated at the meeting of Juni 27, 1931).

1. The formation of absorption lines in the stellar spectra depends on two processes in the stellar atmospheres which usually are designated by the terms of absorption and diffusion. If an atom out of the incident stream of radiation absorbs an energy quantum $h\nu$, is raised to a higher quantum state, and afterwards returns to its former state by emitting the same energy quantum $h\nu$ as radiation, we speak of pure diffusion of radiation. For in this case the atom changes nothing in the character of the radiation, but only in its direction ; it returns exactly the same frequency ν and the same amount, but, in a large number of cases, diffuses it evenly in all directions. The case is different, however, when the excited atom in a hyperelastic collision gives its quantum energy to the colliding electron, increasing the translation energy of it and returning itself to its state of lower energy, without radiation. In the converse process a high speed electron, in an inelastic collision, gives part of its translation energy to the atom that is excited by it to a higher quantum state and afterwards, by falling back, emits radiation which it did not absorb before. The high velocities acquired by the electrons in the hyperelastic collisions as well as the low velocities produced in the converse case are, by the mutual collisions in the electron gas, smoothed into a Maxwellian velocity distribution. Now in thermodynamic equilibrium a Maxwellian distribution of the velocities is in equilibrium with the black body radiation of the same temperature. Hence the interchange of atomic quantum energy with translation energy (i.e. heat motion) by means of collisions has the same effect as if the atoms gave part of their energy to black particles where it is used to increase or maintain their temperature, and receive it back from these particles as blackbody radiation. This process, more specially called absorption, distributes the surplus energy of some special frequency, taken up by one kind of atoms, over all kinds of atoms in the atmosphere, and reduces special high intensities of ν radiation to the rate prescribed by the Maxwell temperature.

There is still another source of black body absorption and emission, viz the absorption and emission of radiation in collisions, by interchange with translation energy, without intervention of the atomic quantum energy (free-free transitions). These processes are much more important than the first named in the production of the continuous spectrum ; only in the darkest centres of the absorption lines the interchange of quantum energy with translation dominates.

In respect to these radiation and absorption phenomena the atmosphere behaves as if it consisted of atoms, absorbing and emitting the frequency ν monochromatically, mixed with a dust of black particles which not only absorb and emit continuous radiation, but also take from and give to the atoms part of their energy of frequency ν. To make the image fit the case still more exactly we must suppose the amount of this black dust, relative to the atoms, to increase with the density, or more exactly, with the electron pressure. The emission of the deeper layers of this dust (i.e. in reality the collisions of electrons with atoms in the denser part of the atmosphere) produces the continuous spectrum. The Fraunhofer line is formed, because the atoms diffuse the ν radiation out of this light, which chiefly comes from below, and weaken it, whereas the black dust radiation tends to add to its intensity and so to lighten up the black centre of the line. From these general considerations we may infer that the larger the diffusion relative to the black body absorption, the smaller will be the residual intensity. This is confirmed by the mathematical treatment of this process.

The intensity produced for some wavelength in the realm of a Fraunhofer line depends on two coefficients, a coefficient of absorption k and a coefficient of diffusion s. If by $d\xi = -\varrho dh$ we denote the elementary layer, by I and I' the upward and the downward stream of radiation (all valid for this frequency ν), by y and z the quantities $I + I'$ (total radiation intensity) and $I - I'$ (net stream), and by E the black body radiation, the equations of the problem are

$$\frac{dy}{d\xi} = (k + s)\, z\,; \quad \frac{dz}{d\xi} = k\,(y - 2E)$$

If by k_0 we denote the general absorption coefficient from free-free transitions and by s_0 the monochromatic absorption coefficient (large in the centre of a line and imperceptible outside its borders), then k consists of k_0 and that part of s_0 which is taken away by collisions $(s_0)_1$, and s is the residual part of s_0. Hence $k + s = k_0 + s_0$ and $k = k_0 + (s_0)_1$. The numerical solution of these equations is given elsewhere [1]). Here we remark only that the darkness in some point of the line with given s_0 (k_0 supposed to be known and constant over small regions) depends on the division of s_0 into its two parts, i.e. on the ratio of the part of the quantum energy of an excited atom taken away in collisions and the remaining part, radiated by spontaneous returns to the lower state. For this ratio only a very rough estimate could be made from effective diameters, collision chances, and average life times.

2. Now there appears to be a possibility to find this ratio in another way, because it determines the contours of a Fraunhofer line also in a quite different way.

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[1]) Monthly Notices of the R. A. S. **91**, 139 (1930).

The ratio of collisions to spontaneous returns is a determining factor in the variation of s_0 with wavelength; and the contours of the Fraunhofer line depend mainly on this variation of s_0 (maximum for $\lambda = \lambda_0$, the theoretical monochromatic wavelength, rapidly decreasing with increasing $\lambda - \lambda_0 = \triangle\lambda$ and then small in the wings, for large $\triangle\lambda$), in its relation to k_0. The central strong part of the function s_0 has a certain width in consequence of the heat motion of the atoms, which by the Doppler principle causes a broadening distributed after a Gaussian curve. Outside this central Doppler part we have the resonance wings, where s_0 is much smaller and varies as $1/\triangle\lambda^2$. This resonance phenomenon — the emission and absorption of wavelengths somewhat different from the true monochromatic wavelength — is an expression of the finiteness of each wave-train at the emission of an energy quantum $h\nu$. The constant determining the resonance part of s_0 depends on the transition probability, which in the absence of collisions is given by Einstein's probability constant A and in denser gases is increased by the effect of collisions. In VOIGT's formula, from which we computed the function s_0, these two parts were expressed by the "damping constant" ν' and the "free path constant" a; the values adopted in our computations are based on the assumption that the collision effect a is negligible compared with the "proper width" ν', in modern theory the effect of spontaneous returns. For ν' the value given by the classical formula

$$\nu' = \frac{8\,\pi^2\,e^2}{3\,mc\,\lambda_0^2}$$

(used by MINKOWSKI, UNSÖLD a.o.) was used. In this way the expression for s_0

$$s_0 = \frac{2\sqrt{\pi}\,e^2 N}{mc\varrho}\left\{\frac{\sqrt{\pi}}{2}\frac{\nu'}{\mu^2} + \frac{\pi}{b}\,e^{-(\mu/b)^2}\right\}; \quad \left(\mu = 2\,\pi\,\frac{c}{\lambda^2}\triangle\lambda\right)$$

was found, where the second term denotes the Doppler part and the first term the resonance wings of the line.

If for ν' the classical value is used, which only contains physical constants, and if the effect of collisions is imperceptible, then the function s_0 and the contours of an absorption line may be computed accurately for different values of the concentration N. If, however, collisions have a perceptible influence, then to ν' in the first term a value is added. The mathematical expression in this case, it is true, becomes somewhat more complicated, but in practice the difference is not so great that we should not be allowed to represent it simply by an increase of the constant ν'. Then the wings of the lines become stronger relative to the centre. Thus the underlying suppositions may be tested by an empirical determination of the figure of the line contours.

Now there are few things so difficult to measure as the true contours of an absorption line; all kinds of instrumental and experimental imper-

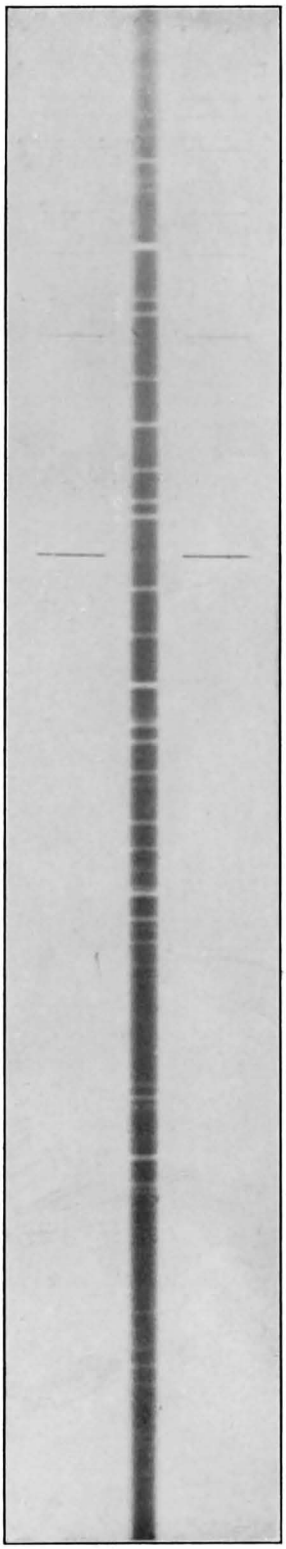
fections tend to broaden the line and to bring spurious light into its dark centre. For this reason MINNAERT and his collaborators at the Heliophysical Institute at Utrecht measure the total strength of the absorption line, i.e. the total amount of energy of the continuous background which is lacking in the line, expressed by the aequivalent width in angström units. This quantity does not change by the instrumental and physical causes which alter the shape of the line contour, after it has been emitted. In the functional dependence of this aequivalent width on the concentration of the atoms producing the line, the peculiarities of the line contours and of the function s_0 ($\triangle \lambda$) are reproduced. If the concentration is very low, only the central Doppler part appears, deepening at first proportionally to the concentration ; then this deepening is retarded and hampered when the central intensity approaches to zero, and only a slight broadening takes place. If the concentration increases still more the resonance wings appear and broaden proportionally to the square root of the concentration. The aequivalent width in this last case, for strong lines, also increases with the square root ; in the first case, for very faint lines, it increases with the concentration itself, and between these there is a transition region (with lines of medium strength), where the aequivalent width increases only little with the concentration. In this way the study of the dependence of the aequivalent width on the concentration may furnish the same information as the study of the contours would give. If ν' by some cause is larger than the classical value, the wings are strengthened relative to the centre, the influence of the wings appears earlier, at a lower concentration, and the transition part of smallest variation is narrowed.

MINNAERT and SLOB have computed curves, giving the aequivalent width as a function of the concentration, for different suppositions on the relative value of the parameters ν' and b [1]). For large ν' the transition part of the aequivalent width-curve is hardly visible, for small ν' it develops into a broad wave, where the curve is nearly horizontal.

MINNAERT and MULDERS determined the shape of this curve from a number of Fraunhofer lines in the solar spectrum of which they had measured the intensity [2]). It yielded a value of ν' nearly 9 times larger than the classical value. This may be caused by collisions, but it is also possible that the real transition probabilities, determined by the Einstein constants A, surpass the classical value of ν'. In a spectrum with many terms we could expect that the ν' for a line emitted from the level a is determined by the mean life time of this level, i.e. by the sum total of all the spontaneous transition probabilities from this to lower levels $\Sigma A_{a \rightarrow x}$. WEISSKOPF and WIGNER have shown, however, that for a line produced by the transition from level a to level b the damping constant is the sum total of all the transition probabilities from level a and from level b, added

[1]) These Proceedings, April 1931 (Vol. 34, p. 547).

[2]) Zeitschr. f. Astrophysik 2, p. 165 (1931).



—4464.47 Ti +
—4468.49 Ti +
—4472.93 Fe +

—4481.23 Mg +

—4491.41 Fe +

—4501.27 Ti +

—4508.29 Fe +

—4515.34 Fe +

—4520.24 Fe +
—4522.64 Fe +

—4534 Ti + Fe +

—4541.52 Fe +

—4549 Fe + Ti +

—4555.90 Fe +
—4558.66 Cr +
—4563.77 Ti +

—4571.98 Ti +
—4576.31 Fe +

—4583.84 Fe +
—4588.21 Cr +
—4592.06 Cr +

—4618.87 Cr +

—4629.33 Fe +
—4634.12 Cr +

—4656.98 Fe +

—4663.05 Al +
—4666.75 Fe +
—4670.40 Sc +

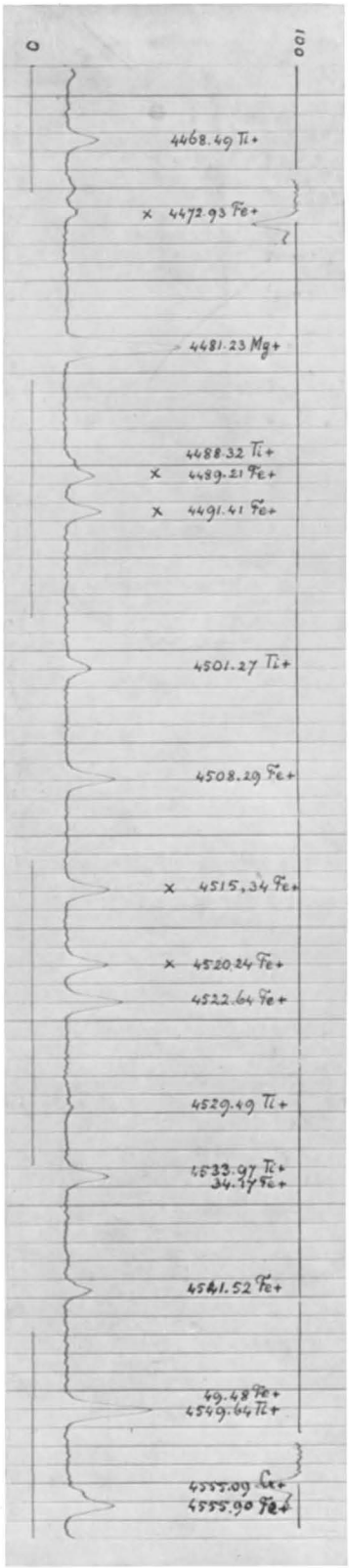


Fig. 1. Fig. 2.

Spectrum of α Cygni, photographed with the Victoria telescope.
Fig. 1. Part of the spectrum, enlarged 10 times; Fig. 2. Part of the tracing at $^1/_2$ size (one division corresponds to 0.1 mm. in the spectrum).

together: $\nu' = \Sigma A_{a \rightarrow x} + \Sigma A_{b \rightarrow y}$ [1]). Now the quantities A can be computed only in a few simple cases (e.g. for hydrogen); in more complicated cases they are entirely unknown. MINNAERT and MULDERS, in their paper, computed them for some multiplets by combining intensity measures and theoretical relations, and they found 4.5 times and 2.9 times the classical value. Thus it seems not quite impossible that the factor 9 may be simply produced by a large average value of $\Sigma A_a + \Sigma A_b$, without contribution of a collision effect. But as long as we are not able to deduce values of A in an independent way, it is not possible to derive a definite conclusion on the influence of collisions in the formation of Fraunhofer lines from measures of their intensity in the solar spectrum.

The matter becomes different, however, as soon as we make use of stellar spectra. In the atmosphere of a dwarf star, as our sun, the pressure is higher and so the effect of collisions is stronger than in the atmosphere of a giant star, where the pressure is lower. If there is a perceptible collision effect, it must appear in this way, that from the lines in the spectrum of a giant star the quantity ν' is found to be smaller than from the solar spectrum.

3. Among the stellar spectra which, by the kindness of Director J. S. PLASKETT, I was able to photograph in 1929 with the 72 inch telescope of the Dominion Astrophysical Observatory at Victoria B. C., there are some of the supergiant *α Cygni*. It is especially one negative N⁰. 17780, taken on a fine grained Eastman Process plate (exposure 110^m), where the contours of the different lines are shown beautifully. The quality of this plate may be judged from the enlargement of a part of it (Fig. 1) and from a part of the record curve made with our MOLL registering apparatus (Fig. 2).

We may expect the spectrum of this star, of class *A2c*, to give accurate results, because it is not rich in lines so that the lines, though they are rather broad, are well separated, with only few blends among them. Though the reductions to absolute intensities by means of the standard spectra impressed upon the plate are only provisional, the results as to the present problem probably will hardly change.

Besides the strong and broad hydrogen lines, the $Mg+$ line 4481 and the $Si+$ lines 4131 and 4128, we find in this spectrum a number of lines produced by the ionized atoms of titanium, iron and chromium, of which the multiplet relations have been investigated; a considerable number of other, mostly faint lines, have not yet been identified. For the lines of $Fe+$, $Ti+$, $Cr+$ the apparent contour curves have been measured and by integration the aequivalent width was derived. A list of the lines which were used in the present investigation, and their total intensities measured is given in Table 1.

[1]) Zs. f. Physik **63**, 54 (1930); Cf. also MINNAERT and MULDERS, Zs. f. Astrophysik **2**, 174; UNSÖLD ib. p. 199.

TABLE 1. Aequivalent width of lines in the spectrum of α Cygni.

	λ	j	w	EW
	Ionized Titanium			
$^2D-^2F$	4450.49	3—3	1	.097
	4443.80	2—3	14	.27
	4395.04	3—4	20	.33
$^2D-^2D'$	4344.31	3—2	1	.076
	4337.92	2 2	9	.154
	4294.10	3 - 3	14	.187
	4287.88	2—3	1	.056
$^2G-^2F$	4501 27	4—3	27	.22
	4468.49	5—4	35	.23
	4444.56	4—4	1	.026
$^2P-^2D'$	4589.96	2—2	1	.078
	4563.77	1—2	5	.207
	4533.97	2—3	9	(.30)
$^2H'-^2G'$	4571.98	5—4	44	.24
	4549.64	6—5	54	(.58)
	4529.46	5—5	1	.050
$^4P'-^4D'$	4330.71	3—2	3	.059
	4320.95	2—1	5	—
	4314.98	1—1	25	.129
	4312.88	3—3	27	.163
	4307.89	2—2	32	.167
	4301.93	1—2	25	.122
	4300.05	3—4	120	.268
	4290 23	2—3	63	.203
	Ionized Chromium			
$^4F-^4D$	4555.09	4—4	100	.095
	4558.66	5—4	875	.34
	4558.84	3—4	5	
	4588 21	4—3	600	.29
	4589.89	2—3	7	—
	4592.06	3—3	128	.166
	4616.72	2—2	98	.135
	4618.82	3—2	392	.23
	4634.12	2—1	245	.205

	λ	j	w	EW
	Ionized Iron			
$^4P-^4D'$	4416.81	1—2	25	.27
	4385.39	1—1	25	.31
	4351.77	2—3	63	.43
	4303.18	2—2	32	.29
	4273.33	2—1	5	.144
	4233.16	3—4	120	.46
	4173.48	3—3	27	.355
	4128.74	3—2	3	.116
$^4F-^4F'$	4666.75	4—5	175	.126
	4629.33	5—5	1925	.36
	4582.84	3—4	225	.174
	4555.90	4—4	1280	.39
	4534.17	2—3	168	(.30)
	4520.24	5—4	175	.28
	4515.33	3—3	867	.31
	4491.41	2—2	672	.28
	4489.23	4—3	225	.23
	4472.93	3—2	168	.085
$^4F-^4D$	4648.82	3—4	5	.04
	4620.52	4—4	100	.165
	4595.6	2—3	7	.073
	4583.84	5—4	875	.55
	4576.31	3—3	128	.216
	4549.48	4—3	600	(.58)
	4541.52	2—2	98	.204
	4522.64	3—2	392	.38
	4508.29	2—1	245	.365

These lines all belong to the quartet systems, with some doublets in the case of $Ti+$. The weights w according to the multiplet rules are given in the 3d column; the concentrations N occurring in the formula for s_0 are proportional to these weights multiplied by $(\lambda_0/\lambda)^4$, where for λ_0 the value 4500 A. was adopted. The aequivalent width of each line according to the measures, is given in the 4th column (EW); plotting its logarithm against the logarithm of $w\lambda_0{}^4/\lambda^4$, we get the desired aequivalent width-curve. In order to make use of the published curves computed by MINNAERT and SLOB, we had to make some slight reductions of scale. Their curves depend on one parameter $a = \nu'/b$, and they were computed for $\lambda_0 = 4500$ A and $b_0 = 1.70 \times 10^{10}$. For other wavelengths and other b (variable with element, temperature and wavelength) the concentration coordinate (called C in their paper) is obtained by multiplying $w\lambda_0{}^4/\lambda^4$ by b/b_0, and their quantity A, the vertical coordinate, is found from the aequivalent width measured by multiplication by $(\lambda_0/\lambda)^2$ (b_0/b). The Doppler constants b are given in their paper in Table I for different elements and wave lengths and for $T = 5000°$; assuming $T = 8500°$ for α *Cygni*, the factor b/b_0 is found to be 1.58 λ/λ_0 for $Ti+$, 1.51 λ/λ_0 for $Cr+$ and 1.45 λ/λ_0. In this way the curves for the different elements and wavelengths could be expressed in the same units and thus made identical.

In order to superpose the different multiplets into one curve, we have to displace them horizontally till they fit in the best way. At first the multiplets ${}^4P'$—${}^4D'$ of $Ti+$ and ${}^4F$—${}^4D$ of $Fe+$ were superposed as well as possible; then for other multiplets the lines with $log\ A$ above 9.20 or 9.10 (where the curve is clearly determined by the wings, $d\ log\ A = \frac{1}{2}\ d\ log\ C$) were used to determine the horizontal displacement, and so the points for the fainter lines were found. The curve obtained in this way, with all the single points representing the different multiplet lines, is shown in Fig. 3.

For the faint lines the deviations in logarithm are of course larger than for the strong lines. Because the lines are broad, the faintest are blotted out and the limit of intensity, below which they are invisible, is higher here than in the solar spectrum. Hence the lower part of the curve is somewhat uncertain. There are, besides, some curious deviations. The largest discrepancies appear in the multiplet ${}^4F$—${}^4F'$ of $Fe+$. They cannot be ascribed to errors of measurement or to uncertainties in the spectrum itself; they appear, moreover, in the same way in the spectrum of the solar chromosphere [1]). In order to show their reality just this part of the α *Cygni* spectrum is reproduced in fig. 1, and a part of the tracing is added in Fig. 2. There cannot be any doubt that the line λ 4520.24, which after the theoretical weights should have an intensity comparable with 4472.93, is in reality of the same order as 4491.41 and 4515.33. The same holds

[1]) PANNEKOEK and MINNAERT, Results of observation of the total solar eclipse of June 29, 1927, I, p. 94 (Verhand. K. A. v. W. XIII. 5).

in a less degree, for 4489.23. If not, by chance, these two lines blend with two unknown strong lines, we have to assume that in this ^4F—$^4F'$ multiplet the intensities are abnormal and unsymmetrical.

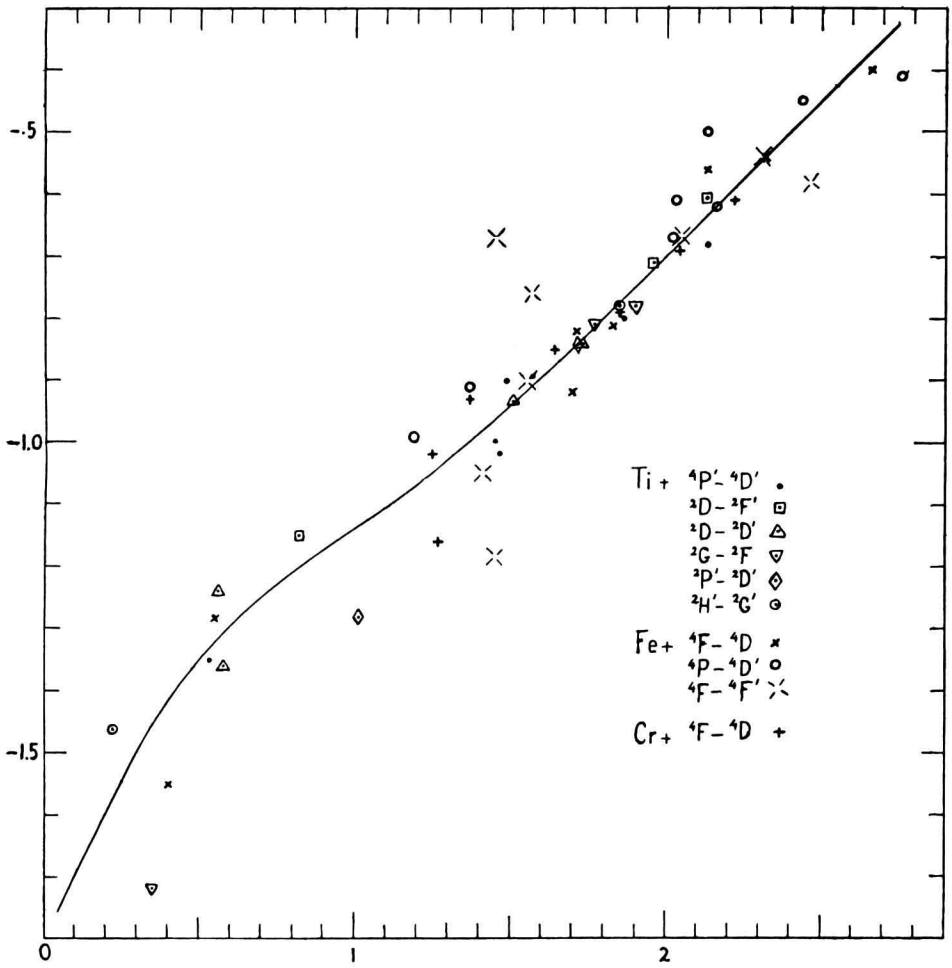


Fig. 3. Aequivalent width-curve for lines of α Cygni. Horiz. *Log C* (concentration); Vert. *Log A* (aequivalent width).

A comparison of this curve with the diagram of MINNAERT and SLOB shows that its figure nearly corresponds to $a = 0.1$. From the minimum slope, which is 0.50 for $a = 1$, 0.29 for $a = .1$, 0.14 for $a = .01$ and 0.30 for our curve, we find $log\, a = -0.95$. For the mean of the three elements used and $T = 8500°$ we had $log\, b/b_0 = 0.13$, where $b_0 = 1.70 \times \times 10^{10}$; since for the classical ν' $log\, \nu' = 8.14 = log\, b_0 - 2.09$, we have

$$log \frac{\nu'\,(curve)}{\nu'\,(class)} = log\, a \cdot \frac{b}{b_0} \cdot \frac{b_0}{\nu'\,(cl)} = -0.95 + 0.13 + 2.09 = +1.27$$

Hence the damping constant is found here 19 times its classical value.

Comparing it with the result of MINNAERT and MULDERS from the solar lines, viz. 9 times the classical value, we see that the expectation, that in the giant star this constant, by lack of collisions, should be smaller than in the solar atmosphere, is not fulfilled. That it is found even larger may possibly be ascribed to a wide variability of this constant for different multiplets ; a comparison of curves based on identical multiplets could supply a control. The conclusion reached from the data so far is that collisions, compared with the spontaneous processes, do not play a perceptible role in the energy interchanges of the atoms in the solar atmosphere.

I wish to express my thanks to Mr. A. VAN ZUTPHEN for his care in making the tracings and his assistance in the discussion.