If all the **P** are 1, i = n!, the degree of Π_n .

The number of points of intersection of a curve $\{\mathbf{p}_1 \dots \mathbf{p}_n\}$ and a V_{n-1} $\{\mathbf{p}_1 \dots \mathbf{p}_n\}$ is

$$i = p_1 P_1 + p_2 P_2 + \ldots + p_n P_n$$
.

The curve common to (n-1) varieties V_{n-1} , has the indices

$$\widetilde{\mathbf{P}}_{11} \dots \widetilde{\mathbf{P}}_{n1}$$

$$\widetilde{\mathbf{P}}_{1,n-1} \dots \widetilde{\mathbf{P}}_{n,n-1}$$

The H_n are rational, i.e. they correspond birationally to linear spaces S_n (n pencils of S_{n-1} , analogous to § 4); they correspond smoothly to a lineo-linear congruence of linear S_{n-1} (in S_{2n-1}) having one point in common with each of n straight lines in general position (analogous to §6).

The projections $W_n^{n!}$ of the $V_n^{n!}$ on linear spaces of (n+1) or more dimensions, contained in the S_{2^n-1} , have the same properties as the original $V_n^{n!}$, but they may possess singular points to be taken into account.

Thus there exist, in S_4 , varieties W_3^6 containing three systems of straight lines.

Physics. — Measurements of Ultraviolet Sunlight in the Tropics. (I). (Communicated by Prof. P. ZEEMAN). By J. CLAY and T. CLAY—JOLLES.

(Communicated at the meeting of January 30, 1932.)

§ 1. August 1928 we started observations on the intensity of ultraviolet solar radiation in Bandoeng. We were induced by the perfect uncertainty existing on this subject. In "Het Klimaat van Nederlandsch-Indië" in the year 1926 Dr. Braak has expressed the opinion that very little ultraviolet radiation would be present on account of photographic experiences. On the other hand Dorno 1), basing himself on observations by Gorszynski during a voyage through the Dutch East Indies and on some other evidence, supposes that perhaps the tropical sun is very rich in short wave ultraviolet light. However, observations with the neutral wedge photometer on various places in the tropics as well as in the temperate zones show, according to Dorno, that in the tropics the blue violet and the long wave ultraviolet has a smaller intensity than in the

¹⁾ C. DORNO, Meteor. Zeitschrift, Sept. 1924, pag. 247.

temperate zones, for the same altitude of the sun, which fact he contributes to the lense shape of the atmosphere 1).

Dr. D. MULDER, physician at Bandoeng is, on medical grounds, of the opinion, that the proportion of ultraviolet in the Indian sunlight can certainly not be very small, and he thinks that photographic observations do not disagree with this ²).

For this reason it seemed important to determine by accurate measures extending over an appreciable time the strength of the ultraviolet in Bandoeng, for direct sunlight as well as for diffuse light. These observations were for some time accompanied by determinations of the photographic light with the neutral wedge photometer ("Graukeilphotometer").

On account of our unforeseen departure to Europe, these determinations had to be brought to a close in August 1929, and though the time is certainly insufficient for accurate data, these observations extending, apart from certain gaps, over a whole year, are not without importance as a contribution in this unexplored domain.

Bandoeng is situated on 107.5° eastern longitude and 6° 57' southern latitude. The observations took place on the very spacious square surrounded by the buildings of the physical laboratory of the Technische Hoogeschool 760 meters above sea level. The laboratory is a long building of only one storey. For the purpose of observing the ultraviolet a 10 meter high iron tower was erected, so that the entire horizon was free. The nearest mountains were the Prahoe and the Boerangrang, which, having an altitude of about 1300 meters above Bandoeng at a distance of 17 K. M., hardly intercepted the light of the sky, so that the radiation came unobstructed from practically the entire upper hemisphere, which is of great importance for observations of the diffuse light. The more so, since the ground uviol-glass used for determining the illumination of the horizontal plane by the diffuse sky light, diffuses very imperfectly, and for an angle of incidence of about 80° the cell receives much less than would be expected according to LAMBERT'S law. This point will be taken up when discussing the determination of the diffuse sky light.

Method of observation.

The method making use of the Cadmium cell by means of electrometric discharge was decided upon. The apparatus was a photoelectric photometer after ELSTER and GEITEL, in the special form given to it by GÜNTHER and TEGETMEYER after indications by DORNO.

The Cadmium cell, connected with a single fibre electrometer, was charged up every time by means of a Zamboni pile to a little over 120 volt,

 $^{^{1}}$) C. DORNO, Die ultravioletten Sonnen- und Himmelsstrahlung in tropischen Gegenden, Die Naturw., 18e Jahrg., Heft 12.

²⁾ Dr. D. MULDER, Handelingen van het vierde Nederl.-Ind. Natuurwetenschappelijk Congres 1926, pag. 148.

and the time determined in which the incident sunlight produced a discharge from 120 to 90 volt. In this region the sensitivity curve of the cell is linear, and the same thing is the case for the calibration curve of the electrometer. The latter was repeatedly verified during the research by calibrating the electrometer.

By inserting various diaphragms, the time of discharge was kept within not too wide limits. For very small intensities, discharge took place over a smaller range of potential, taking care that always the time of discharge for the smaller interval was accurately compared with the time observed for the usual drop from 120 to 90 volt. The ratio of the diaphragms was also carefully determined by experiments on the apparatus itself.

Special care was taken to keep cells as well as electrometer dry. This is absolutely necessary in the tropics, and greatly hampers the work with electrometers. Because of the construction of the cells, the chemicals for drying had to be removed during observations, to which there is no objection, since the measurements took place in the sunshine. When not in use, cells and electrometer had to be provided with good drying materials, and moreover the whole apparatus was then enclosed in an electrically heated box, so that the temperature was always above the surrounding. On every observing day, the leakage was repeatedly determined, and, if necessary, allowed for. However, it remained within the limit of observational error in nearly all cases.

In the above mentioned way it was possible to reduce the time of discharge to always the same fall of potential of 120 to 90 volts and the same diaphragm, with an uncertainty of only a few percent.

The intensity was computed according to the formula

$$I = \frac{10000}{t} \log \frac{120}{90} = \frac{1250}{t}$$

where *t* is expressed in seconds.

§ 2. Reduction of the cells to the standard.

The Cadmium cell method has, as is well known, the great advantage of simplicity in treatment and of inexpensive and easily movable apparatus. The main disadvantage is, that the Cadmium cell does not measure the ultraviolet intensity, but rather the integral of the intensity of each spectral element multiplied by the corresponding sensitivity of the cell. The Cadmium cell only responds to ultraviolet rays, and seems, up to about 3200 Angstrom, to be very little sensitive. The sensitivity rises very rapidly for shorter wavelengths, and has a maximum for a wavelength, which is not the same for different cells. For the two cells for which HAUSSER, at DORNO's 1) request, determined the sensitivity curve, the maximum is situated at 2800 and at 2650 Angstrom, therefore in both

¹⁾ C. DORNO, Meteor. Zeitschr., Sept. 1924, pag. 245.

cases outside the spectral region of solar radiation. For both cells the sensitivity at 2900 A is about six times that at 3100 A. The integrated intensity, which is measured, has therefore only relative value, and measures with different cells can only be compared, if the sensitivity of the two cells has been compared for as many solar altitudes, that is as many spectral regions, as possible.

The cell Cd II of DORNO and his collaborators, which is used by them for many years, is generally adopted as a standard. Our observations have been reduced to this cell, and correspond therefore to the Davos scale.

A great difficulty, however, is, that the sensitivity curve of our two cells 5450 and 5480 of GÜNTHER and TEGETMEYER deviates very appreciably from the one of Cd II. This seems to be the case for the majority of newer cells. It follows from the fact that the sensitivity ratio of our cells to Cd II strongly depends on the solar altitude. Before we took the cells to India, they were in February 1928 compared with the Davos cell for solar altitudes up to 35°. The very pronounced dependance of the reduction factor on the sun's altitude made us hesitate to extrapolate the calibration to higher altitudes, as would be necessary, since we worked with solar altitudes up to 90°. Therefore publication was delayed, until recalibration had taken place in Davos for the maximum solar altitude.

At first, calibrations at various times of the year, did not agree satisfactorily, if observations at the same solar altitude were compared. Dr. MÖRIKOFER and Dr. LEVY, Director and collaborator of the Physical-Meteorological Observatory in Davos, have taken great pains in thoroughly investigating this matter. Theoretical considerations made them express the sensitivity ratio, not as a function of solar altitude, but as a function of the minos ratio. This is defined as the intensity ratio of unfiltered and filtered sunlight as directly given by the Cadmium cell, the filter being minos glas, which kind of glass lets through rays down to 3130 A, the latter part however much weakened.

Reduction factors determined in this way agree well, even for different seasons. It is to be regretted, that the calibrations of 1928 cannot be arranged according to minos ratios, since at that time only unfiltered observations were taken. Fig. 1 gives the various calibrations, arranged according to solar altitudes, Fig. 2 the same, arranged according to minos ratios. Both figures were kindly put at my disposal by Dr. Levy.

Dr. Levy further derived, that in general reduction factor E and minos ratio Q are in first approximation related by $EQ^a = \text{constant}$, where a is a constant.

This first approximation consists in the assumption, that the Cadmium cell enables us to measure the intensity of a definite wavelength in a relative measure, in other words, that the effective wavelength is independent of solar altitude. The same thing is assumed for the cell with filter, in which case the effective wavelength is larger and the relative measure a different one.

Though this condition, either with or without filter, is certainly not fulfilled for our cell 5450, with which our measurements took place, the

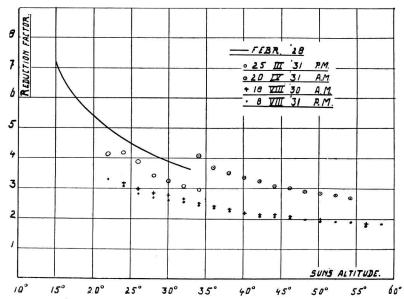


Fig. 1. Calibrations of the cell 5450, arranged according to solar altitudes.

results of the calibration agree rather well with the formula $EQ^a = \text{constant}$, and we find with a = 1 a maximum deviation of about 5%. It is true that for very small minos values the deviations are somewhat larger, but for Davos the observations have little accuracy in this region, and in Bandoeng they are still less accurate, for which reason very few observations were taken at these small minos values.

In Fig. 2, the line representing the means of the reduction factors as well as the line EQ = constant is drawn. From the fact that these two lines nearly coincide follows therefore, that our cell with minos filter has about the same effective wavelength as the Davos cell CII without minos.

Our observations in Bandoeng, which always were accompanied by filtered observations, indicate minos ratios of 4 to 15. Comparisons with the cell Cd II only took place up to the minos ratio 10. Extrapolation over a fairly large region is therefore necessary. Because of the rather good agreement of the reduction factors with the line EQ = constant, we could assume that this line gives the correct values and thus extrapolate. At the same time this would give the advantage of a simple reduction to the Davos scale. We simply would have to multiply our filtered observations by a constant factor.

Another way of extrapolating is, that we prolong the line representing the means of the reductions, and thus extrapolate graphically. In the present case we have chosen the latter procedure, because the experimental calibration curve, just at Q = 10 has the tendency of rising a little

above the line EQ = constant, so that the deviations of the latter line from the real values become perhaps somewhat larger than in the region

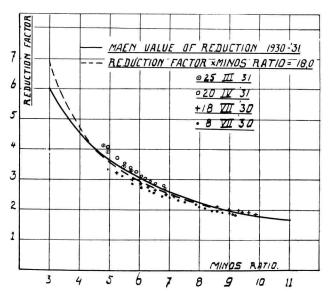


Fig. 2. Calibrations of the cell 5450, arranged according to minos ratios.

up to Q=10. The advantage of simple calculations was not applicable to us, since computations had already been completed by means of graphical extrapolation, when Dr. Levy's considerations came to our knowledge. An uncertainty remains in any case for the reduction factors at minos ratios above 10, which could only be removed by calibration with the standard cell up to about 85° solar altitude, which is practically impossible.

Since the reduction factor for our cell is diminishing to one third of its value for minos ratios from 4 to 15, reduction to the Davos scale for a cell as ours is only possible if calibrations are done with extreme care, and only for observations accompanied by determinations with filter.

In conclusion we remark, that we never experienced any fatigue in our cells.

§ 3. Results of observations of ultraviolet solar intensity.

These are given in Tables I and II, in Table I arranged according to the sun's altitude, in Table II according to the hours of the day, both reduced to the Davos galvanometer scale (unit 3,5.10—II Amp.).

Observations were carried out exclusively in the morning and the early afternoon. No measurements were ever taken after 2 p.m. Observations rarely begin at a solar altitude below 20°. On account of the small intensity and its rapid change the observations for lower solar altitudes have little accuracy. For the same reason the minos ratio for these low

TABLE I.
Ultraviolet intensity and Suns altitude.

Altitude	15°	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°	90°
Month September 1928 . (3)				84	103	135	164	192	220	245	270	294	310	3 2 7		
October (5)			67	99	129	159	191	217	245	269	292	310	327	333	340	346
January 1929 (9)	7.5	22	5 5	97	136	161	196	233	263	294	32 6	353	375			
February		37	77	108	139	170	200	222	252	284	314	342	364	384	39 4	
March				114	157	195	223	247	268	291	314	333	352	366	379	
April (3)			42	79	134	174	202	231	2 58	280	299					
May(1)			47	77	114	154	191	224	252	278	301					
June (4)	9.6	25	56	85	120	155	180	20 6	229	245						
July (4)		39	67	98	134	166	200	235	262	292						
August				95	131	166	203	239	267	299	326	355				

altitudes can only be imperfectly determined. Since in Bandoeng the sun rises almost perpendicularly, these difficulties are greater there then in more temperate regions, where the altitude of the sun changes less rapidly.

The observations cover an entire year, the first observing day being 23^d Aug. '28 the last 17th Aug. '29. The observations for these two days are in such good agreement, that a change in sensitivity of the cell in the course of the observing year becomes improbable.

Expecting nearer data from Davos, under more on the reduction factor, and not thinking that our observations should be discontinued so soon on account of our departure from India, no observations were done during November and December. The number of observing days for every month is added in parentheses. Mostly this is not very large, in certain months observations could only be done on very few days. These are the months when it rains in the morning and clouds usually begin to gather at nine. Frequently observations had to be discontinued for this reason. Perfectly cloudless days do not occur, practically, there are always clouds near the horizon, often also small white clouds in spots. Some of the

TABLE II.

Ultraviolet intensity and time of the day,

Time of the day	7h	8h	9ь	10h	11h	12h
Month September 1928 .		80	158	240	299	327
October		99	191	270	327	346
January 1929	10	99	193	281	352	382
February	15	110	200	284	362	394
March		114	223	291	348	379
April		64	188	263	310	340
Мау		50	146	230	278	296
June		50	134	195	230	245
July		67	154	228	280	302
August		74	174	261	320	350

observing days were left out afterwards, since the results were clearly too small, which in those cases was confirmed by an abnormally high intensity of the diffuse light. When a few clear, almost cloudless observing days occur in the same month, their agreement is striking, e.g. for February. A little before noon, often a certain haziness occurs, by which the solar intensity diminishes somewhat. In the afternoon there usually are clouds, often showers, and, also for other reasons, this time of the day usually is less suited for observations than the morning.

A pronounced annual period at a given solar altitude does not appear from the observations. It might seem that September and June are lowest, but the differences are at any rate much less than at higher latitudes.

This shows especially if means over three months intervals are taken. These are plotted in Fig. 3 as a function of solar altitude, however it should be remarked, that the quarter of November, December and January contains only observations in January for the reason stated. The half year from May till October has a somewhat lower mean than the other half year, the difference however amounting to only 7%. We may therefore

conclude that for the same solar altitude the ultraviolet radiation at Bandoeng shows a much smaller annual variation than e.g. at Davos.

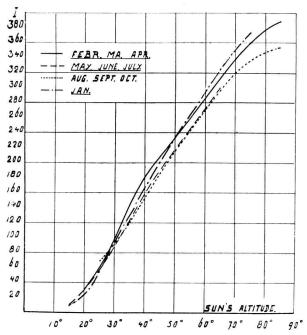


Fig. 3. Ultraviolet solar intensities, arranged according to solar altitudes.

Comparing the monthly means, arranged according to hours of the day, the differences throughout the year are of course larger, but at noon the minimum value, occurring in June, is still 62 % of the maximum value, occurring in February.

For January, March, June and September the dependance on the hour of the day is represented in Fig. 4.

For comparison of the strength of the ultraviolet in Bandoeng with temperate regions, Davos is obviously most suited, since for this location very accurate data taken with cell Cd II and extending over a long time are available. However the difference in altitude between the two locations comes in as an unknown factor. We have no accurate data on the dependance of the ultraviolet radiation on geographical elevation, and those we have diverge. We have therefore made a comparison between Arosa (1860 M.), Davos (1560 M.), Bandoeng (760 M.) and Chur (590 M.). For Arosa Götz¹) has collected accurate data. From these we have computed, by means of the ratio Arosa to Chur, as given on p. 48 of the paper mentioned under ¹), the annual mean for Chur, and have drawn in Fig. 5 the four annual means, arranged according to solar altitude. Comparison of these four curves shows, that, at a given solar altitude,

¹⁾ Dr. F. PAUL GÖTZ, Das Strahlungsklima von Arosa, pag. 47.

Bandoeng's ultraviolet intensity, averaged over the year is less above that of Chur than corresponds to the difference in elevation. Some values for

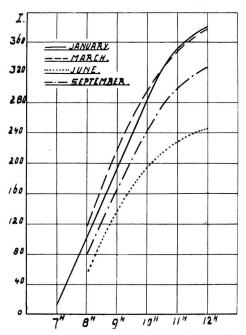


Fig. 4. Ultraviolet solar intensities arranged according to hours of the day.

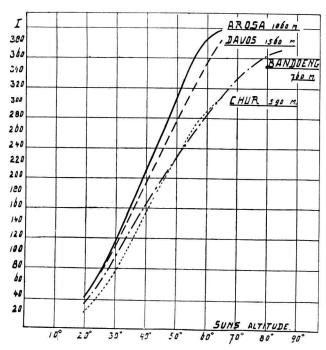


Fig. 5. Annual means of ultraviolet solar intensity for Arosa, Davos, Bandoeng and Chur.

Chur are even a little above those for Bandoeng. One might conclude from this, that the value for Bandoeng would turn out somewhat lower than for a place in Switzerland at the same elevation. The differences, however are only a few percent and the insufficient material for Bandoeng makes this conclusion uncertain; the accuracy of our observations certainly does not reach beyond a few percent. At any rate it seems that the annual mean for Bandoeng does not appreciably deviate from that of a place of observation in Switzerland at the same elevation and for the same solar altitude. We shall return to this in the next paragraph 1).

If we take the total sum of light over a year, Bandoeng even fairly exceeds Davos, which is easily understood on account of the much higher solar altitude. We shall return to this when discussing the strength of the diffuse ultraviolet.

We have refrained from giving separately the intensity of the long wave ultraviolet, which minos glass transmits, since, in our opinion, this has little value. In this matter we are entirely in accord with a consideration by Dr. Levy in a paper on his experiences in calibrating Cadmium cells, which is to appear shortly in the "Meteorologische Zeitschrift", and of which he kindly sent us a copy. That our cell, also when used with filter, has no constant effective wavelength, follows moreover from the fact, that the effective wavelength of our cell with filter is equal to that of cel Cd II without filter, as resulted from the calibrations. And of the latter the effective wavelength certainly is not constant. This greatly lessens the value of filtered observations with our apparatus as a means of determining intensities uninfluenced by extension of the spectrum into the shorter wavelengths. The glass introduced by Götz as a filter which cuts off at larger wavelengths, was better in this respect.

Though our filtered observations are not of much value as intensity determinations, they are of much importance for the determination of the reduction factor, and, besides, they make it possible to carry out an estimate of the thickness of the ozone layer, with which the following paragraph deals.

Transmission coefficients were determined for the unfiltered light after reduction to the Davos scale. The values obtained are very large: for January till March on the average 0.53, for June and July 0.52, for Aug. till Oct. 0,54.

The thickness of the Ozone layer.

§ 4. In order to obtain information on the thickness of the ozone layer,

 $^{^{1}}$) At the time when our cell had only been compared with $Cd\,II$ up to 35° solar altitude, and we could not yet foresee, that the reduction factor would diminish to such an extent as afterwards turned out to be the case, our observations seemed to point out that the ultraviolet intensity of sunlight in Bandoeng exceeded even that of Davos for the same solar altitude. We made a statement to that effect in a communication before the Natuurwetenschappelijke Vereeniging in Bandoeng, however making a reserve in case the calibration might yield different results.

we have followed the method indicated by GÖTZ¹). We may assume that the intensity ratio of unfiltered to filtered radiation as given by the Cadmium cell, that is the minos ratio, is a continuous function of the length of the path in the ozone. Without entering into the nature of this function, we may say that, if, for two different measures with the same apparatus, the minos ratio is the same, then the length of the path in the ozone is also the same. The latter, in its turn, is related in a simple manner to the sun's altitude and the thickness of the ozone layer, and thus we may determine the ratio of the thickness of the ozone layer for two observations having the same minos ratios.

By this simple method we may determine.

- A. The ratio of the thicknesses of the ozone layer for various periods in Bandoeng.
- B. The ratio of the thickness of the ozone layer in Bandoeng and Davos at corresponding times of the year, making use of observations with our apparatus in Davos by Dr. Levy. From the mean ozone values which are known for Davos, we may therefore make an estimate of the thickness of the ozone layer at Bandoeng.

Of course the accuracy of this method cannot come anywhere near that of the ultraviolet spectrograph of DOBSON, but nevertheless it gives a certain estimate which may provide a useful contribution on the thickness of the ozone layer in the tropics and its variation throughout the year.

As regards A, we have plotted in Fig. 6 the four means of the minos

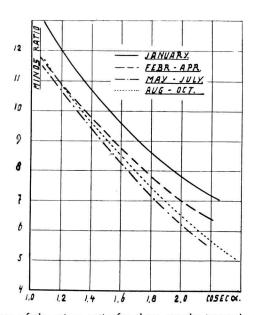


Fig. 6. Means of the minos ratio for three months intervals as functions of the cosec. of the sun's altitude.

¹⁾ Dr. F. PAUL GÖTZ, Das Strahlungsklima van Arosa, pag. 56.

ratio for three months intervals as functions of the cosec. of the sun's altitude (a). If we find that the minos ratio at a, for the one curve corresponds to that of a_2 for the other, then we must have $d_1 \csc a_1 = d_2 \csc a_2$ (d is the thickness of the ozone layer).

For the thickness of the ozone layer expressed in that for May to June as a unit, we find from August to October at minos ratios 7, 8, 9, 10, 11, the thicknesses 0,98, 0,97, 0,97, 0,97, 0,97, mean 0,97. For January (Nov. and Dec. are left out), we find in the same way 0,82, 0,85, 0,87, 0,86, 0,85, mean 0,85, for Febr. to April resp. 0,93, 0,94, 0,95, 0,95, 0,97, mean 0,95.

These observations point therefore, in agreement with DOBSON's 1) results to a small variation of the thickness of the ozone layer throughout the year and fit in rather well between the observations at 10° northern and 22° southern latitude.

B. From Davos we obtained means of minos ratios from January to May and from August to October taken there with our cell and minos glass. For the sake of comparison we have also taken means of Bandoeng observations for the same intervals. Only for part of the minos ratios comparisons of the ozone layer can be made, since the large minos ratios do not occur in Davos, and the small ratios in Bandoeng have little accuracy, whereas moreover minos ratios in Bandoeng are higher than in Davos for the same solar altitudes.

Comparing the curves of January to May for Davos and for Bandoeng, we conclude that for each of the minos ratios 5, 6 and 7 the ozone layer in Bandoeng is 68 % of that in Davos.

Comparison of the curves for August to October gives for minos ratios 8, 9 and 10, an ozone layer at Bandoeng which is 94 %, 91 % and 90 % of that in Davos, average 92 %.

If we take for the mean thickness of the ozone layer in Davos the value determined by Götz in Arosa¹) we find as mean for January to May 3.33 m.M. and for August to October 2.79 m.M. For Bandoeng these two values become therefore 2,26 and 2,57 m.M. Taking into account the inaccuracy of our method, these values also agree rather well with those published by Dobson¹).

Indeed, in spring the ozone layer in Davos turn out to be appreciably thicker than in Bandoeng, in the autumn the difference is not large.

In Fig. 7 the annual mean of the ultraviolet intensity for Bandoeng and the September mean for Chur are drawn, arranged according to solar altitudes (derived in the same way as described above). By this procedure periods of approximately the same thickness of ozone are compared. The month of September was chosen for Chur, because this month being in the period of thinner ozone layer, yet has rather large solar altitudes. In this case the line for Chur runs appreciably higher than that of Bandoeng. We may conclude that in periods of about equal ozone layer,

¹⁾ Dr. G. M. B. DOBSON, Nature, May 2, 1931, pag. 668.

Bandoeng has, for a given sun's altitude, somewhat less ultraviolet than exists in Switzerland at the same elevation. Perhaps this may be explained by the lenze shape of the atmosphere, as DORNO supposes.

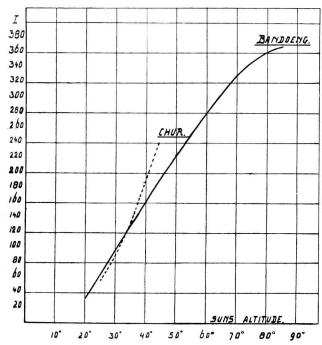


Fig. 7. Annual mean of the ultraviolet solar intensity for Bandoeng and the September mean to Chur, arranged according to solar altitude.

In-conclusion we wish to express our very sincere thanks to Professor DORNO for his generous help and advice, and to Dr. MÖRIKOFER and Dr. Levy for the great pains they have taken for the calibration of our cells.

Physics. — On the ZEEMAN-effect in the arc spectrum of Nickel. By C. J. BAKKER. (Communicated by Prof. P. ZEEMAN.)

(Communicated at the meeting of January 30, 1932.)

 \S 1. Introduction. In his important paper on the extension of the analysis of the Nickel arc spectrum H. N. Russell 1) remarks in his conclusions, that the Zeeman effect in this spectrum offers an extensive field of work; it may be anticipated that many g-values will be abnormal, especially for the higher terms, but this should be important in studying the changes of coupling of the vectors which are involved.

¹⁾ H. N. RUSSELL, Phys. Rev. 34, 821, 1929.