Physics. —*The conduction of heat of lead-thallium at low temperatures.* By W. J. DE HAAS and H. BREMMER. (Communication N⁰. 220c from the Kamerlingh Onnes laboratory Leyden.)

§ 1. After having investigated the thermal conductivity of the pure supraconductors lead, tin and indium at low temperatures [1]) we now used the same apparatus to determine the change of the thermal resistance with the temperature for the alloy $Pb\,Tl_2$. This alloy was chosen, because it is a supraconductor (transition point $4°.09$ K), consisting of two supra-conductors (transition point $Tl\ 2°.37$ K, transition point $Pb\ 7°.2$ K). We intended to find out, whether here also a connexion exists between supra-conductivity and conduction of heat, as we have found for pure indium, lead and pure tin. Moreover the transition point of the alloy ($4°.09$ K) lies favourably in the temperature region of liquid helium.

For the measurements we used a short thick rod long 9 mm, thick 7.4 mm (which had been annealed during 180 hours slightly below the melting point). We chose these dimensions in order to obtain at the low temper-atures a thermal resistance of the same order as in the measurements with the pure metals. A rod of the alloy of the same dimensions as of those of the pure metals would have had a much greater thermal resistance, so that a stationary temperature difference between the extremities of the rod would only have been reached after a long time.

For our measurements in the neighbourhood of the melting point of ice the described form of the little rod had still another advantage. In other cases at such temperatures the loss of heat by radiation within the apparatus becomes very large. As the rod was short and thick the conditions were now better. In these measurements the ratio of the heat passing through the rod by conduction to the heat radiated by the bulb of the gas thermo-meter and the heating body was about 5:1. By an estimation we found, that the radiation from the surface of the rod might be neglected.

Tables I and II give the results of the measurements, the thermal resistance (w) in Watt^{-1} at different temperatures.

Table II, which gives the measurements in the helium region, also contains the (longitudinal) magnetic field (H) in Gauss.

In fig. 1 and 2 the w-T-curves have been plotted. Above $4°.09$ K the thermal resistance evidently increases, when the temperature is lowered. This is also the case probably at room temperature, so that a minimum if it exists, may be expected above room temperature.

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[1]) These Proceedings **34**, 325, 1931, **35**, p. 131, 1932, Comm. Leyden, 214d and 220b.

TABLE I

$T\,°\mathrm{K.}$	W Watt^{-1}
276	15.6
95	22.0
82	24.9
20.9	38.7
18.2	42.7
16.2	46.7
15.1	49.2

TABLE II

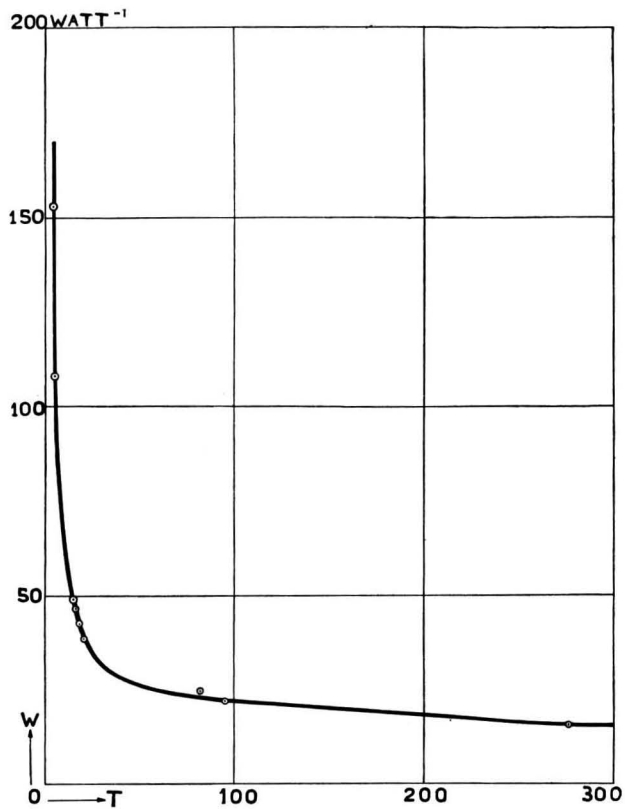| T	H Gauss	W Watt^{-1}	T	H Gauss	W Watt^{-1}
5.69	0	108	3.17	642	189
5.69	749	107	3.15^5	481	174
4.34	0	153	3.12	0	133
3.88	481	161	2.61	0	132
3.88	749	164	2.21	481	157
3.87	0	153	2.17	0	120
3.84	0	151	2.02	749	191
3.66	642	175	2.00	481	168
3.66	835	179	1.97	0	133
3.65	481	170	1.97	214	142
3.63	0	142			

Fig. 1

On the whole alloys are known as well with thermal resistances that increase as with such that decrease when the temperature is lowered.
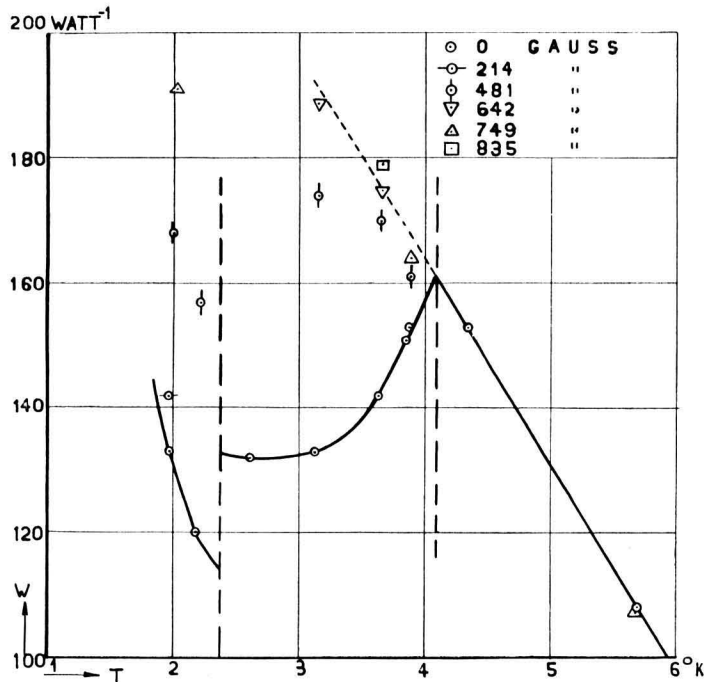

Fig. 2

§ 2.   Interesting is the form of the curve in the region of liquid helium.

Let us first consider in fig. 2 the points measured without a magnetic field. We see, that above the transition point $4°.09$ of $Pb\,Tl_2$ the thermal resistance increases with sinking temperature in the same way as in the measurements of the pure metals. For these we found namely at a definite temperature a minimum of the thermal resistance, which may be due to impurities.

In the case of $Pb\,Tl_2$ we see that the normal increase of the thermal resistance is disturbed. At $4°.09$ K the w-T-curve shows a change of direction; the thermal resistance decreases even. At the transition point of $Tl\ 2°.37$ another discontinuity occurs. Further downward the resistance increases in the usual way.

Should the discontinuity at $2°.37$ be due to the occurrence of free thallium, than the thallium must show a great change in thermal resistance at its electrical transition point.

The change of direction at $4°.09$ must be a property of the alloy lead-thallium.

The magnetic field is always found to increase the thermal resistance, except at $5°.69$ where a very small decrease is found which is of the same order of magnitude as the accuracy of the measurements. Perhaps this has

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something to do with the fact, that one of the components of the alloy is lead, (transition point 7°.2) and that we measured here in a magnetic field higher than the threshold value of lead.

At hydrogen temperatures we could not detect any change in the same field, though the accuracy of the measurements was greater here.

At all temperatures below 4.°09 K we see a strong increase of the thermal resistance in magnetic fields.

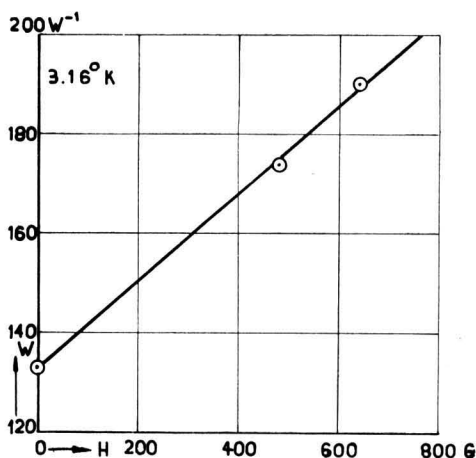The phenomena in this temperature region are illustrated by figures 3, 4
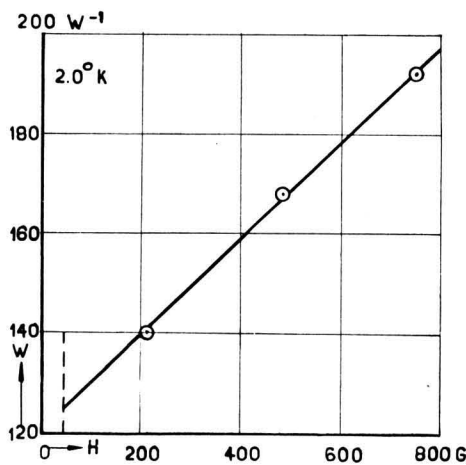

Fig. 3

Fig. 4

and 5, where for a definite temperature w has been plotted as a function of the magnetic field H. Most times the temperatures, at which the measurements in the different magnetic fields were made, are not quite the same, as is seen from table 2. We then estimated how great w would have been at the slightly different temperature, to which the figure refers.

To begin with fig. 3. Here we have only to do with magnetic fields lower than the field, in which for lead-thallium the electrical resistance begins to come back and which we shall call the initial field. We are below the transition point of lead-thallium and above that of thallium.

The curve shows a linear increase of w with H.

Fig. 4 represents measurements at 2°.0. Here we are under both the transition points of lead-thallium, and of thallium. The magnetic field, though still below the initial field, is above the threshold field of thallium. Again the curve shows a linear increase of w with H.

The point for the field O does not lie on the same straight line. This may have the same reason as the discontinuity at the transition point of thallium in the w-T-curve without a magnetic field.

Fig. 5 gives the relation between w and H at a temperature so little below the transition point of $Pb\,Tl_2$, that we could also reach the threshold values of the field for $Pb\,Tl_2$ and for lead. The region between the value of H for which the electrical resistance begins to come back and that for

which it has regained its normal value has been hatched. The threshold value of H for $Pb\,Tl_2$ has been indicated by a thin line, that for Pb by a
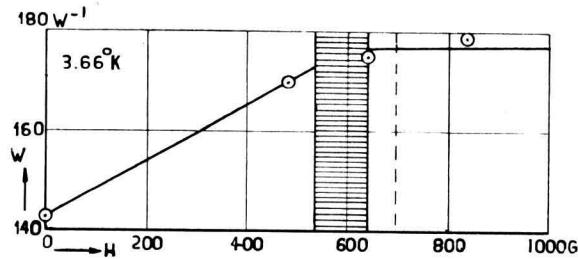

Fig. 5

dotted line. From the figure it is evident, that for ·high fields w does no longer increase linearly with H, but becomes practically constant when the

TABLE III 3.16°			TABLE IV 2.00°			TABLE V 3.66°	
H Gauss	W Watt^{-1}		H Gauss	W Watt^{-1}		H Gauss	W Watt^{-1}
0	133		0	131		0	143
481	174		214	140		481	170
642	190		481	168		642	175
			749	192		835	179

threshold field of $Pb\,Tl_2$ has been reached. The passage of the threshold value of Pb is of no consequence. The change observed here is of the order of the accuracy of the measurements.

In analogy with the figures 3 and 4 we have assumed a linear relation between w and H down to the initial field.

§ 3.   Resuming we can say, that below the transition point of lead-thallium the thermal resistance increases linearly with magnetic fields under the threshold value of lead-thallium and remains practically constant, when that value has been reached. This result is interesting in two respects :

1⁰.   According to the rule of LIPPMANN no magnetic field can penetrate into a supraconductor when starting from a field zero we stay below the threshold value ; at a change of the field the intensity of the persisting currents along the surface only will be altered. We therefore might expect no influence of the external field on the thermal resistance. This expectation was proved right by our measurement with pure metals, as here we did not find such variations or at least very small ones.

2⁰. In magnetic fields above the threshold value the thermal resistance is greater than without a field, while for pure metals just the opposite influence of the field was found.

The latter phenomenon cannot be ascribed to the eventual occurrence of free thallium.

We must remark, that for tin we also found an increase of w in magnetic fields above the transition point; this change is however proportional with H^2 and much smaller.

In the case of lead-thallium the mentioned change is not observed above the transition point of lead-thallium; it must therefore be a property of the lead-thallium and not of the thallium.

To our opinion the phenomenon of the linear increase of w with H in fields below the threshold value of $Pb\,Tl_2$ must be interpreted as a gradual penetration of the field in the rod.

We might suppose the alloy to contain spots where the concentration of the lead atoms is very high and of others where the thallium atoms are dense. At the first places, where the field is lower than the threshold value of lead, it would hardly be possible for the field to penetrate. At the places with the great concentration of thallium atoms the field would be able to penetrate easily as long as we stayed above the transition point or above the threshold value of thallium.

Then we should have an intermediate case between the homogeneous supraconductive metal, for which the rule of LIPPMANN holds and into which the magnetic field cannot penetrate at all and the case of totally free electrons, the orbits of which the field can cross.

We intend to extend our investigations of these phenomena to other alloys.

Finally we express our thanks to Mr. J. DE NOBEL for his help during the measurements and with the calculations.

Mathematics. — *Ueber die Matrixgleichung $XX' = A$.* Von R. WEITZENBÖCK.

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Wir gehen aus von einer nicht-singulären symmetrischen Matrix $A = \|\,a_{ik}\,\|$ und fragen nach einer Matrix $X = \|\,x_{ik}\,\|$, die mit ihrer transponierten $X' = \|\,x_{ki}\,\|$ multipliziert, die Matrix A ergibt, die also die Gleichung

$$XX' = A. \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

befriedigt. Hier ist A wegen $(XX')' = XX'$ notwendig symmetrisch. Wir geben im Folgenden zwei Wege an, die zur Lösung X von (1) führen.