Astronomy. — *On the expanding universe.* By W. DE SITTER.

(Communicated at the meeting of May 28, 1932.)

The non-static solutions of the field equations of the general theory of relativity for an isotropic and homogeneous universe, giving a line-element

$$ds^2 = -R^2\, d\sigma^2 + c^2\, dt^2, \quad d\sigma^2 = \gamma_{ij}\, d\xi_i\, d\xi_j \quad . \quad . \quad . \quad . \quad (1)$$

R being a function of t alone, and $d\sigma$ being a line-element of three-dimensional space [1]), have been investigated by FRIEDMANN [2]) in 1922, and independently in 1927 by LEMAÎTRE [3]), who worked out the astronomical consequences in great detail. These solutions have attracted much attention of late in connection with the observed recession of the spiral nebulae. The field-equations are in all these investigations supposed to contain the "cosmical constant" λ; they are thus

$$G_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\, G + \lambda\, g_{\mu\nu} + \varkappa\, T_{\mu\nu} = 0. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The material tensor, or energy tensor, $T_{\mu\nu}$ is supposed to be given by

$$T_{ij} = -g_{ij}\, p = R^2\, \gamma_{ij}\, p\,, \; T_{i4} = T_{4i} = 0\,, \; T_{44} = \varrho = \varrho_0 + 3p, \quad . \quad (3)$$

where ϱ_0 is the "proper" or "invariant" density, ϱ the "relative" density, and p the "pressure", consisting of the average irregular motions of material particles, and the pressure of radiation. The proper mass of radiation is zero, as well is known. Both ϱ and p are supposed to be independent of the space co-ordinates. The assumption (3) then involves isotropic and homogeneous distribution of matter and radiation. We are thus abstracting from all complications introduced by the condensation of matter into stars and stellar systems. We can say that we are studying the field of pure inertia, neglecting gravitation. The tensor $T_{\mu\nu}$ has been so constructed as to make its divergence zero in accordance with the laws of conservation of mass, energy and momentum. It is a consequence of these laws (i.e. of the vanishing of the divergence of $T_{\mu\nu}$) and the independence of ϱ_0 and p of the space co-ordinates, that g_{44} is also independent of the space co-ordinates, and can consequently be taken as equal to unity, as has been done in (1).

FRIEDMANN considers the solutions of (2) for different values of λ,

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[1]) The convention is made throughout this paper that roman indices take the values 1, 2, 3 only, whilst greek indices run from 1 to 4.

[2]) *Zeitschr. für Physik*, **10**, p. 377.

[3]) *Ann. Soc. Scientif. de Bruxelles*, Vol. **47**, A, p. **49**, also *M.N.* xci, p. **483**, **1931**.

positive, negative or zero, LEMAÎTRE considers only positive values of λ. From the mathematical point of view the presence of λ is required in the equation in order to give it the highest degree of generality, $G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G$ and $g_{\mu\nu}$ being the only tensors, containing differential quotients of the $g_{\mu\nu}$ not higher than the second order, of which the divergence vanishes. The constants λ and $\varkappa$ thus have, from the mathematical point of view, entirely the same function, and there is no a priori restriction on their values, they might be positive, negative or zero. Comparison of the first approximation of the solution (for small velocities) with classical mechanics shows that $\varkappa$ is to be identified with the constant of gravitation:

$$\varkappa = \frac{8\pi G}{c^2} = 1.860.10^{-27}\, gr^{-1}\, cm.$$

The constant $\varkappa$ is thus essentially positive: $\varkappa = 0$ would mean that we were investigating pure space, without anything in it, i.e. that we were studying geometry and not physics. The constant λ on the other hand, has no counterpart in classical mechanics. It was introduced into the equations by EINSTEIN in 1917, not from considerations of mathematical generality or elegance, but in order to make possible a finite density of matter in a static universe.

This required a positive value of λ. At the same time three-dimensional space, having the line-element $d\sigma$, was found to be of constant positive curvature, and consequently finite, which was at the time considered to be a great advantage, since it avoided the necessity of boundary conditions at infinity. If the pressure is neglected, the curvature of three-dimensional space in the static universe becomes equal to λ. EDDINGTON [1]) accordingly in 1921 interpreted the meaning of λ as providing a natural standard of length. The idea of a positive λ and a positive curvature of three-dimensional space eventually became so much a part of the accepted theory that, when the static solution was replaced by the non-static ones, it was at first overlooked that in the non-static solutions the sign neither of λ nor of the curvature is prescribed, and both may be positive, negative or zero, independently of each other. Attention was first called to this fact by Dr. O. HECKMANN in July 1931 [2]).

From the equations (2) with the value (3) of the energy tensor it follows at once that the three-dimensional line-element $d\sigma$ must be one of a space of *constant* curvature. This curvature, however, may be either positive, negative or zero. If it is not zero, its numerical value can be taken equal to unity without any loss of generality. Thus, if k be a quantity which can have the values $+1$, 0, -1, and if we denote by s the function

$$\mathrm{s}(x) = \frac{\sin x}{x},$$

[1]) Cf. *The Mathematical Theory of Relativity*, §§ 65, 66.

[2]) *Göttinger Nachrichten*, July 1931, p. 127.

and therefore

$$\mathbf{s}\,(i\,x) = -\,\frac{\sinh\,x}{x},$$

we have

$$d\sigma^2 = d\varrho^2 + \varrho^2\,\mathbf{s}^2\,(\varrho\sqrt{k})\,(d\psi^{\,2} + \sin^2\psi\,.\,d\theta^2). \quad (k = +\,1,\,0,\,-1)\;. \quad (4)$$

The equations (2) then give the further conditions, differential quotients d/cdt being denoted by dots:

$$\left.\begin{aligned} 2\,\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} &= \lambda - \varkappa p \\ \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} &= \tfrac{1}{3}\,(\lambda + \varkappa\varrho), \end{aligned} \quad (k = +\,1,\,0,\,-1)\right\} \quad . \; . \; . \quad (5)$$

and the equation of energy becomes

$$\dot{\varrho} + 3\,\frac{\dot{R}}{R}\,(\varrho + p) = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

If we call ε the instantaneous curvature of three-dimensional space, thus:

$$\varepsilon = \frac{k}{R^2},$$

and if we denote the coefficient of expansion by h, thus:

$$h = \frac{\dot{R}}{R},$$

the equations (5) are equivalent to

$$\left.\begin{aligned} \lambda + \varkappa\varrho &= 3\,(\varepsilon + h^2) \\ \varkappa\,(\varrho + p) &= 2\,(\varepsilon - \dot{h}) \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

In the static case, with $h = \dot{h} = 0$, it is evident that, if ϱ and p are positive, both λ and ε must be positive, and if $p = 0$ we have $\lambda = \varepsilon = \frac{1}{2}\,\varkappa\varrho$, as in EINSTEIN's universe. But in the non-static case the equations (7) are insufficient to determine the values, or even the signs, of λ and ε, since $\dot{h}$ is entirely unknown.

In the actual universe the pressure density p, consisting of the radiation pressure and the irregular motions of stars and stellar systems, is very small as compared to the material density ϱ_0, the ratio p/ϱ_0 being probably of the order of 10^{-6}. We can therefore as a good approximation neglect the pressure, taking

$$p = 0, \quad \varrho = \varrho_0.$$

This will be done in the present paper, though the general conclusions remain the same if the pressure is not neglected. In this case, the equation of energy (6) gives at once

$$\varkappa \varrho = \frac{3 R_1}{R^3},$$

R_1 being a (positive) constant of integration. Further we put

$$y = \frac{R}{R_1}, \quad \tau = \frac{ct}{R_1}, \quad \gamma = \tfrac{1}{3} R_1^2 \lambda.$$

The second equation (5) then becomes

$$\left(\frac{dy}{d\tau}\right)^2 = \frac{1}{y} - k + \gamma y^2, \quad \dots \dots \dots \quad (8)$$

the first giving simply the derivative of this, viz.:

$$\frac{d^2 y}{d\tau^2} = -\frac{1}{2 y^2} + \gamma y \quad \dots \dots \dots \quad (9)$$

By introducing y and τ instead of R and ct we have, of course, excluded from consideration the case $R_1 = 0$, or $\varrho = 0$, corresponding for $\lambda > 0$, $k = +1$ to the well known "solution B" or "empty universe", and giving similar solutions for other values of λ and k. But these are only limiting cases, none of which does occur in nature, and they are of mathematical interest only.

We put

$$P = 1 - k y + \gamma y^3 = \frac{3 h^2}{\varkappa \varrho}.$$

Then (8) can be written

$$\left(\frac{dy}{d\tau}\right)^2 = \frac{P}{y}, \quad \dots \dots \dots \dots \quad (8')$$

and consequently real solutions are only possible for positive values of P, since y by its nature is necessarily positive. In Fig. 1, in which the co-ordinates are y and γ, the full drawn lines represent the curves $P = 0$ for the three cases $k = +1$ (spherical or elliptical space), $k = 0$ (euclidean space) and $k = -1$ (hyperbolical space). P is positive above these curves and negative below them, the real solutions thus correspond to the part of the semi-plane above the curves. It is seen by inspection of the diagram that there are three possible types of solution, which may be called the oscillating universes, and the expanding universes of the first and of the second kind.

In the oscillating solutions the value of y oscillates between zero and ε

maximum value y_1. In the expanding solutions of the first kind the value of y increases from zero to infinity, and in those of the second kind it

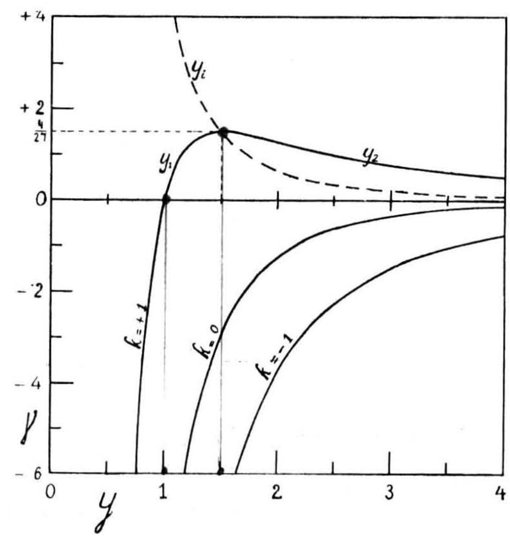

Fig. 1

increases from a certain minimum value y_2 to infinity. It is clear from the diagram that the occurrence of the different kinds of solutions is as given in the following little table.

λ	Curvature		
	$k=+1$	$k=0$	$k=-1$
positive	Osc. Exp. I Exp. II	Exp. I	Exp. I
zero	Osc.	Exp. I	Exp. I
negative	Osc.	Osc.	Osc.

Only for a positive λ and a spherical (or rather "elliptical") three-dimensional space all three kinds of solutions are possible; for all other combinations of the signs of λ and k we have either only oscillating universes, or only expanding universes of the first kind.

The general type of the variation of y with τ in the different cases is represented in Fig. 2. For $y=0$ we have $P=1$, therefore $dy/d\tau=\infty$: all solutions leave the axis of τ perpendicularly: the expansion in the case of the oscillating universes and the expanding universes of the first kind starts by an explosion. Of course for very large densities, i.e. for very small values of y, the simplifications made in deriving the equations are no longer allowable. The actual value $y = 0$ is impossible in nature, and presumably in the actual universe, if it is of one of these two types, there will be a minimum value of y somewhat as shown by the dotted lines in Fig. 2. In

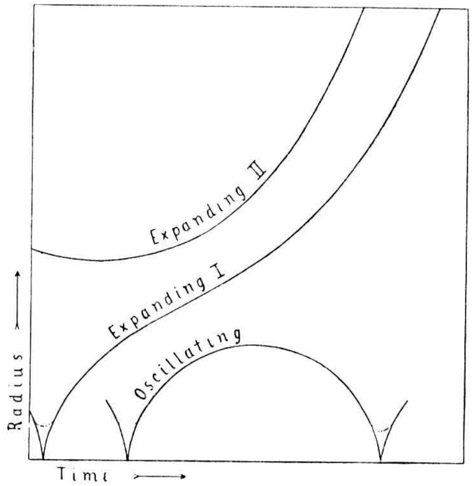

Fig. 2

the expanding solution of the second kind there is a minimum value y_2. These solutions are only possible for the limited range of λ given by $0 < \gamma < 4/27$. For the limiting value $\gamma = 4/27$ we have either the limiting member of the family of expanding universes of the second kind, viz.: LEMAÎTRE's universe, in which y varies from $y_2 = 1.5$ to infinity, the minimum radius being that of the static universe or "EINSTEIN's universe", and corresponding to $t = -\infty$, or the limiting member of the family of oscillating universes, having the same value $y_1 = 1.5$ as a maximum, which is only reached at the time $t = +\infty$. The expanding solutions of the first kind have a point of inflexion, which according to (9) occurs for the value

$$y_i = (2\gamma)^{-1/3} . \qquad (10)$$

This curve is represented by the broken line in Fig. 1. In the case $\lambda = 0$ there is no point of inflexion. The curve giving y as a function of τ in the case $k = -1$ (hyperbolical space) is of hyperbolical character (without an asymptote, however), the limiting value of $dy/d\tau$ for $y = \infty$ being unity. In the case $k = 0$ (euclidean space) it is of parabolical character, the limiting value of $dy/d\tau$ being zero. In the case $k = +1$ (elliptical space) we have an oscillating universe. The solutions for $\lambda = 0$ are best expressed by means of an auxiliary variable ψ, thus:

$$\left.\begin{aligned} \lambda = 0,\ k &= +1 : \tau = \psi - \tfrac{1}{2} \sin 2\psi \quad , \quad y = \sin^2 \psi \ , \\ k &= \ \ 0 : \tau = \tfrac{2}{3} \psi^3 \quad , \quad y = \psi^2 \ , \\ k &= -1 : \tau = \tfrac{1}{2} \sinh 2\psi - \psi \ , \quad y = \sinh^2 \psi . \end{aligned}\right\} \qquad (11)$$

The different solutions for $k=+1$ have been given in considerable detail by the present writer in $B.A.N.$ 223, using the same approximation as in the present paper, viz.: $p=0$, i.e. assuming the universe to contain only matter at rest, and no radiation. Dr. HECKMANN [1]) has discussed the solutions for all values of λ and k for the case $\varrho_0=0$, i.e. for a universe without matter, but filled with radiation. The general character of the solutions, however, depends very little on the contents of the universe; it is practically the same in the two extreme cases considered by Dr. HECKMANN and myself.

There are no astronomical data of observation, which enable us to make a decision regarding the value of λ, or the sign of the curvature. The only data that can be derived from astronomical observations are the rate of expansion h, i.e. the ratio of the radial velocity and the distance of the spiral nebulae ($h=v/cr=\dot{R}/R$), and the density ϱ. It is convenient to express all data by quantities of the dimension of a length, thus:

$$h=\frac{1}{R_B}, \quad \varkappa\varrho=\frac{2}{R_A^2}, \quad \lambda=\frac{3}{R_C^2}, \quad \nu=\frac{R_1^2}{R_C^2}.$$

We can adopt as the most probable value of the coefficient of expansion:

$$h=500 \text{ km/sec per } 10^6 \text{ parsecs.}$$

The determination of h depends on the measured redshifts in the spectra of nebulae, and on the scale of distances for these same nebulae. The first does not introduce a larger uncertainty than about 10 %. The second, however, is still very uncertain. There are only two or three of the larger and nearer spirals, in which cepheids or novae have been discovered. The adopted distances of a few others depend on the measured brightness and assumed absolute magnitude of the so called brightest stars in them. Even if it were certain that these objects are actually stars, their assumed absolute magnitude is still extremely uncertain. Of the great majority of the nebulae the distances are derived from the total magnitudes. On the one hand the determination of the scale of apparent magnitudes is as yet not very accurate, on the other hand the adopted absolute magnitude of an average spiral, which is based on those of the few of which the distances have been determined from cepheids, novae or "brightest stars", is also subject to considerable uncertainty. Taking all this into consideration I think the uncertainty of h is not overestimated if we take it to correspond to a factor of 2 both ways, and thus adopt for the limits of R_B:

$$10^{27} < R_B < 4.10^{27} \quad \ldots\ldots\ldots \quad (12)$$

The density is still much more uncertain. HUBBLE [2]) adopted in 1926 as a lower limit 1.5×10^{-31}. To be quite safe, we shall take as the lower

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[1]) *Göttinger Nachrichten*, Febr. 1932, p. 181.

[2]) *Mt. Wilson Contrib.* 324.

limit 10^{-31}. An upper limit is given by assuming the whole of intergalactic space to be filled with gas and computing the maximum density this could have without giving rise to greater absorption than is reconcilable with astronomical data. Dr. MENZEL [1]) in this way arrived at an upper limit of 10^{-26}, corresponding to one atom of hydrogen per 100 cm^3. To these limits corresponds:

$$3.10^{26} < R_A < 10^{29} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

From (12) and (13) we find for the limits of the value of $P = 3\,h^2/\varkappa\varrho = 3R_A^2/2R_B^2$ at the present stage in the evolution of the universe:

$$0.01 < P < 15000 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If we adopted the limits sometimes given for ϱ, viz.: $10^{-28} > \varrho > 10^{-30}$, and took no account of the uncertainty of h, the possible range of P would become more restricted, viz.:

$$4 < P < 400, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14^*)$$

but I think that (14) represents the uncertainty of our present knowledge better than (14*). Even if the value of P were known accurately, the values of γ and k would still be indetermined, though the choice would be limited. Thus, if we could be sure that P exceeded unity, all oscillating families would be excluded, with the exception of those for $\gamma < 0$, $k = -1$, for which P reaches a maximum amounting to $P_{max} = 1 + (-6.75\,\gamma)^{-\frac{1}{2}}$ for the value $y = (-3\,\gamma)^{-\frac{1}{2}}$. On the other hand if P were smaller than 1 all the expanding universes of the first kind would be excluded, excepting those for $\gamma > 4/27$, $k = +1$, for which P reaches a minimum of $P_{min} = 1 - (6.75\,\gamma)^{-\frac{1}{2}}$ for the value $y = (3\,\gamma)^{-\frac{1}{2}}$. The expanding universe of the second kind, which is only possible for $k = +1$, $0 < \gamma \leqq 4/27$, assumes during its course of evolution all values of P from zero to infinity. If we wish with EINSTEIN, to remove the term with λ from the field equations, we have the choice between the three solutions (11), and we will have to take $k = +1, 0, -1$ for $P < 1$, $P = 1$, $P > 1$ respectively. With a view to the great uncertainty of P, these and similar statements are, however, of little practical interest.

In the present state of our knowledge it is very well possible to assume that both λ and k are equal to zero.

It would be different if the value of λ, or the value of the curvature $\varepsilon = k/R^2$, were known from some other source. Sir ARTHUR EDDINGTON has recently published [2]) the formula

$$\frac{\sqrt{N}}{R} = \frac{m\,c^2}{e^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

m being the mass, and e the charge of an electron. N is the number of

[1]) Privately communicated.

[2]) *Proc. Royal Society*, A. Vol. **133**, p. 605 (August 1931).

protons in the universe, and consequently only has a meaning for a closed universe, with a finite mass, i.e. in the case $k = +1$. EDDINGTON further satisfies himself that for R the value denoted above by R_C must be taken, by reasonings based on the assumption that the actual universe is LEMAÎTRE's one, the limiting case of the expanding series of the second kind, $\gamma = 4/27$, $k = +1$. If this, or a similar, formula could be proved to hold independent of *a priori* assumptions about λ and the curvature, this would give an independent determination of either λ or ε, and we would be able to determine the other, as well as $\dot{h}$, from (7) as soon as the determination of ϱ (and p) and h from astronomical observations would be sufficiently accurate, and thus the whole problem of the grand scale model of the universe would be solved. An important remark should, however, be made. The whole theory of relativity, including the equations for the expanding universe, is a pure abstraction or generalisation from observations, after NEWTON's own heart, without any hypotheses. But (15) is based on considerations belonging to the quantum theory and wave mechanics, and introduces the hypothesis. Also in my opinion the "universe" itself is an hypothesis. Our observations cover only a very limited part of space and time, which I have been in the habit of calling "our neighbourhood". The "universe" is an hypothetical concept, arrived at by extrapolation beyond this neighbourhood involving the applicability of the values (3) of the energy tensor to all space and time. A complete theory of the universe is not possible without hypotheses. Consequently, as has been explained by DINGLE in his recent book [1]) we must be prepared in the theory of the universe to meet with paradoxical results. The universe, like the atom, may do things that would be impossible for a finite mechanical system.

A well known paradox, which however I think is only apparent, connected with the theory of the expanding universe is the shortness of the time scale. The differential coefficient $dy/d\tau$, or dR/cdt, being finite, it is evident that the time $\tau - \tau_0$ elapsed since the minimum of y, the time required for y to double its value, and similar intervals of time, must be roughly of the same order [2]) as y itself.

The order of magnitude is given by the observed value of $h^{-1} = R_B$, which is 2.10^9 lightyears. The interval $t - t_0$ elapsed since the beginning of the expansion is thus of the order of a few thousand million years. Now a thousand million lightyears is a very large distance indeed, but a thousand million years is a short time. The interval $t - t_0$ is of the order of the age of the earth. Astronomers have been in the habit lately, and I think on good grounds, of reckoning the ages of the stars, and of stellar systems such as double and multiple stars, or star clusters, in periods which are at least a thousand times longer. The paradox arises from the identification of the beginning of the expansion with the beginning of this

[1]) Science and Human Experience, 1931.

[2]) Two quantities may still be considered to be of the same order of magnitude if they differ by a factor of 10, but not of 1000.

evolution. This identification, however, is entirely gratuitous. We will have to accustom ourselves to the idea that the evolution of the stars and stellar systems on the one hand, and the expansion of the universe on the other hand, are two processes going on side by side, but independent of each other, apart from possible interaction at critical epochs, e.g. when y is so small that the simplified equations of the expanding universe are no longer applicable.

The line-element is

$$ds^2 = -R^2\,d\sigma^2 + c^2\,dt^2, \quad d\sigma^2 = \gamma_{ij}\,d\xi_i\,.\,d\xi_j \quad . \quad . \quad . \quad . \quad (1)$$

The space co-ordinates in natural measure are, however, not ξ_i, but

$$x_i = R\,.\,\xi_i\,.$$

An observer, in interpreting his observations, refers them to a system of co-ordinates which is galilean for his particular position in space and time. He will naturally choose this point for his origin of co-ordinates, and count his spatial radius vector and his time from there. A transformation to galilean co-ordinates is always possible, but the differentials dr and du of the galilean radius vector and time will as a rule not be total differentials. Since we wish to investigate not only the immediate neighbourhood of the observer, but the whole universe, we must introduce new variables r and u so chosen that dr and du will be total differentials, co-inciding with the galilean dr and du for $r=0$, $u=0$. It is convenient to make the transformation in two steps, first transforming the radius vector alone by putting

$$r = R\,.\,\varrho \quad , \quad R\,d\varrho = dr - h\,r\,.\,c\,dt.$$

This transforms the line-element to

$$ds^2 = -dr^2 - br^2(d\psi^2 + \sin^2\psi\,.\,d\theta^2) + 2h\,r\,.\,dr\,.\,c\,dt + (1 - h^2 r^2)\,c^2\,dt^2, \quad (1^*)$$

showing its essential non-static character. The term with $dr\ c\ dt$ can always be removed by introducing a new time u making the line-element:

$$ds^2 = -a\,dr^2 + br^2\,(d\psi^2 + \sin^2\psi\,d\theta^2) + f\,.\,du^2, \quad . \quad . \quad . \quad (1^{**})$$

where a, b and f now are functions of both r and u. The variable u can always be so chosen that a and f become equal to unity for $r=0$, $u=0$. The factor b in (1^*) and (1^{**}) is not essential, it only serves to take care of the (eventual) curvature of three-dimensional space, and becomes unity for $r=0$. It depends on y, which is now also a function of both r and u. changing not only with the time u, but also with the radius vector r. This second transformation from (1^*) to (1^{**}) is, however, not essential, since (1^*) already is galilean for $r=0$.

We have now to consider the motion of material particles, or galactic systems, i.e. the geodesics in the space-time (1^*). In the case of the line-element (1) the track of the particle is a geodesic in the three-dimensional space (ϱ, ψ, θ), but this track is *not* described with constant velocity. The velocity and its radial component are [1]:

$$\left(\frac{d\sigma}{c\,dt}\right)^2 = \frac{\varphi_0^2}{R^4 + R^2\varphi_0^2}, \quad \left(\frac{d\varrho}{c\,dt}\right)^2 = \frac{\varphi_0^2 - \Gamma_0^2/\varrho^2}{R^4 + R^2\varphi_0^2}, \quad . \quad . \quad . \quad (16)$$

where φ_0 and Γ_0 are constants. A special case is, of course, $\varphi_0 = 0$, $\Gamma_0 = 0$, the particle being at rest in the space (ϱ, ψ, θ). The actual values of φ_0 and Γ_0 of course are not exactly zero, but they are presumably small and different for each individual spiral.

In the case (1^*) the track of the particle in the three-dimensional space (r, ψ, θ) is a curve of hyperbolic character, approaching the origin to a certain minimum distance, which is reached at a time t differing from the time t_0 corresponding to the minimum of R by a quantity of the order of φ_0 and Γ_0. The radial component of the velocity in this curve is

$$\frac{dr}{c\,dt} = \sqrt{\frac{\varphi_0^2 - R^2\,\Gamma_0^2/r^2}{R^2 + \varphi_0^2}} + rh\,, \quad . \quad . \quad . \quad . \quad . \quad (16^*)$$

and consequently for large distances r, and not too small values of R, is practically proportional to the distance, as observed. All the spirals have thus passed very near to the origin of co-ordinates, and consequently to each other, at a time which is a few thousand million years ago. If we assume the actual universe to be one of the expanding family of the second kind, their minimum distances were probably still considerably larger than their diameters, if it is of the oscillating type or of the expanding type of the first kind, the minimum distances were probably much smaller, and they may have partly penetrated each other. It should be remarked that the size of the galaxies themselves is not influenced by the change of R [2], at least not so long as the equations for the expanding universe remain applicable.

This near approach was, however, not the "beginning of the world". The galactic systems and the stars existed before that time. Still it is to be expected that it has not been entirely without influence on their development. I think the effects of this influence can still be traced.

The spirals and our own galactic system are all rotating, with periods of the order of a few hundred million years. They are all very inhomogeneous in structure, consisting of condensations, or star clouds, separated by regions of smaller density. If the rotation had been going on undisturbed for a great many revolutions, this inhomogeneity could not subsist. But if only a small number of revolutions (of the order of ten) has been completed since a strong perturbation occurred, the inhomogeneity is of comparatively recent date, and has not yet had time to be smoothed out. Also the spiral structure itself is most readily explained as an effect of tidal forces resulting from a near approach. If, however, we compute the frequency of near approaches of spiral nebulae on the basis of their average peculiar random

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1) Cf. *B. A. N.* **193**, p. 217.

2) Cf. *B. A. N.* **223**, p. 146.

motions, and average distances apart at the present time, taking no account of the change of size of the universe, we find that they should be very rare, the time between encounters being more nearly of the order of 10^{12} years, instead of 10^9 or 10^{10}.

Also it is a significant co-incidence that the minimum value of R occurred about at the date of the birth of the planetary system. Modern theories ascribe the origin of the planets to a near approach, or even a collision, of the sun and another star. Evidently the chances that such a collision should occur were very much larger at the epoch of minimum size of the universe than they are now.

The time used in our theories of the evolution of stars, of the planetary system and of double stars and stellar systems, is, of course, the time u of the line-element (1^{**}) co-inciding with the galilean co-ordinates here and now. It is easy to choose this time so as to relegate the epoch of the minimum of y to the infinite past, where it is already in LEMAÎTRE's universe for the time t of the line-element (1). Of course the infinity is only logarithmic, and it does not make the time during which anything really happens any longer. This introduction of another time is only a mathematical trick, providing no solution for the paradox of the time scale. But the fact that it is possible in all cases is another illustration of the fact that our present knowledge does not contain the necessary data to choose between the different families of expanding universes.

Physics. — *La courbe de fusion de l'hydrogène jusqu'à 610 kg/cm²*. Par MM. W. H. KEESOM et J. H. C. LISMAN. (Communication N⁰. 221*a* from the KAMERLINGH ONNES Laboratory at Leiden.)

(Communicated at the meeting of May 28, 1932.)

§ 1. *Introduction.* Les déterminations de la courbe de fusion de l'hydrogène, déjà faites dans ce laboratoire jusqu'à une pression de 450 kg/cm² et jusqu'à une température de 24.67° K. [1]), ont été continuées jusqu'à 610 kg/cm² et jusqu'à 27.65° K., celle-ci étant la température maximale réalisable avec le cryostat employé [2]).

§ 2. *Méthode et appareils.* La méthode est la même que celle décrite dans la Comm. N⁰. 184*a* [3]). Avant le commencement des observations le thermomètre à résistance *Pt*-64 était tombé hors de service par suite d'un dérangement ; il fallut donc mesurer les températures à l'aide de *Pt*-24' ; les températures mesurées correspondent d'une manière très satisfaisante

1) W. H. KEESOM and J. H. C. LISMAN, These Proceedings, 34, 598, 1931. Comm. Leiden N⁰. 213*e*.

2) W. H. KEESOM and J. H. C. LISMAN, These Proceedings 34, 602, 1931. Comm. Leiden N⁰. 213*f*.

3) H. KAMERLINGH ONNES and W. VAN GULIK, These Proceedings 29, 1184, 1926. Comm. Leiden N⁰. 184*a*.