Hydrodynamics. — *Contribution to the theory of the vane anemometer.* By B. G. VAN DER HEGGE ZIJNEN. (Mededeeling N°. 24 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hoogeschool te Delft.) (Communicated by Prof. J. M. BURGERS.)

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Introduction.

In investigating as well as in constructing a vane anemometer two factors are of prime importance: the angle at which the vanes are to be set in order that the instrument commences to rotate at the lowest possible wind speed, and secondly the internal friction of the mechanism. So far as we know, only OWER [1]) published a method of calculating these quantities; however, his theory starts from a too much simplified field of velocity to arrive at reliable results, and applies only to a special case. Having regard to the present form of the theory of the screw propeller and the windmill, the treatment of the problem can be brought into a more exact and exhaustive form, as will be shown in the following lines.

Forces acting on the blades.

We suppose the internal friction of the mechanism to be small, though not negligible; consequently a certain amount of power is absorbed by the vane wheel, and this implies that the air passing the vane disc is retarded. In the same time a rotational component of the air speed appears in the slip stream behind the disc; hence the velocity diagram will be as sketched in fig. 1, in which most of the symbols introduced in the following formulae

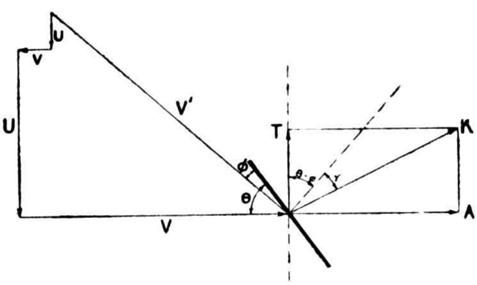

Fig. 1

have been indicated. The velocity V is the original wind speed, supposed to be uniform; the values of v, U and u relate to the centre of pressure of

[1]) E. OWER, A low-speed vane anemometer; Journ. Scient. Instruments, Vol. 3, N°. 4, 1926 The theory of the vane anemometer; Phil. Mag. 1926, p. 881. Measurement of Air Flow (CHAPMAN & HALL, London, 1927).

the vanes, the distance of which from the axis is denoted by r; ϱ is the density of the air, C a coefficient, such that the force K acting under the angle γ with the normal to the relative velocity V' is $\frac{1}{2}\varrho C F V'^2$, where F is the surface of a single vane. The number of vanes in the disc will be denoted by n. The area of the ring which is described by the vanes during their motion be F^*; in general it will differ from nF.

Both C and γ are functions of the effective angle of incidence ϕ. Experimental values have been obtained by EIFFEL [1]), who experimented with square, flat plates, and by OWER [2]), who investigated typical anemometer vanes. The measurements of EIFFEL have been repeated and extended recently by FLACHSBART [3]). However, for the present case the latter do not offer any particular advantage over EIFFEL's investigations; so in the following lines we shall accept the experimental $\gamma - \phi$ curve given by EIFFEL, and the relation between C and ϕ given by OWER (comp. fig. 2). A point of importance is that at a certain value of ϕ, γ attains a minimum value; if ϕ tends to 0, γ increases again to 90°, which is due to the thickness and to the surface friction of the plates. In the region $\phi < 5°$ no experimental results are available and we are restricted here to the dotted part of the curve; as will be shown later on this region is of great importance for the present research and the lacking of more accurate data is severely felt [4]).

Taking account of the axial velocity component v and of the component u in the plane of the vane, the forces acting on a single blade are (comp. fig. 1):

$$K = \tfrac{1}{2}\varrho\{(V-v)^2 + (U+u)^2\}CF \quad \ldots\ldots \quad (1)$$

$$T = K\cos(\theta + \gamma - \phi) \quad \ldots\ldots \quad (2)$$

$$A = K\sin(\theta + \gamma - \phi) \quad \ldots\ldots \quad (3)$$

At the other hand A and T are also determined by the loss of linear momentum and the gain of angular momentum of the air passing through the vane disc. The total mass of air entering the disc in unit time is: $Q = \varrho F^* (V - v)$; as the velocity is slowed down from V far upstream to $V - 2v$ far downstream, the total axial force nA exerted will be:

$$nA = 2\varrho F^* (V - v)\, v. \quad \ldots\ldots \quad (4)$$

In the same way, considering the angular momentum supplied to the mass of air, which downstream obtains a tangential component approximately equal to $2u$, we find for the tangential force:

$$nT = 2\varrho F^* (V - v)\, u. \quad \ldots\ldots \quad (5)$$

[1]) G. EIFFEL, La résistance de l'air et l'aviation (H. DUNOD, Paris, 1911), p. 134.

[2]) E. OWER, Measurement of Air Flow p. 111.

[3]) O. FLACHSBART, Messungen an ebenen und gewölbten Platten; Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen IV (Oldenbourg Verlag, München, 1932), p. 96.

[4]) As the blades are very small, and consequently work at rather low REYNOLDS' numbers, it may even be that C and γ to a certain extent will depend on V.

After some reductions the following relations are obtained (putting $\varkappa$ in stead of $n\,CF/4F^{\star}$) :

$$\tan(\theta-\phi)=\frac{U+u}{V-v} \quad \ldots\ldots\ldots \quad (6)$$

$$\frac{u}{U+u}=\frac{\varkappa\cos(\theta+\gamma-\phi)}{\sin(\theta-\phi)\cos(\theta-\phi)} \quad \ldots\ldots\ldots \quad (7)$$

$$\frac{v}{V-v}=\frac{\varkappa\sin(\theta+\gamma-\phi)}{\cos^2(\theta-\phi)} \quad \ldots\ldots\ldots \quad (8)$$

$$\frac{v}{u}=\tan(\theta+\gamma-\phi) \quad \ldots\ldots\ldots \quad (9)$$

Putting $N=\cos^2(\theta-\phi)+\varkappa\sin(\theta+\gamma-\phi)$, we have:

$$\frac{u}{V}=\frac{\varkappa\cos(\theta+\gamma-\phi)}{N} \quad \ldots\ldots\ldots \quad (7^a)$$

$$\frac{v}{V}=\frac{\varkappa\sin(\theta+\gamma-\phi)}{N} \quad \ldots\ldots\ldots \quad (8^a)$$

$$\frac{U}{V}=\frac{\sin(\theta-\phi)\cos(\theta-\phi)-\varkappa\cos(\theta+\gamma-\phi)}{N} \quad \ldots \quad (10)$$

Inserting the values of u and v found in this way into (4) and (5), we obtain :

$$A^{\star}=\frac{n\,A}{2\,\varrho\,F^{\star}\,V^2}=\frac{\varkappa\sin(\theta+\gamma-\phi)\cos^2(\theta-\phi)}{N^2} \quad \ldots \quad (4^a)$$

$$T^{\star}=\frac{n\,T}{2\,\varrho\,F^{\star}\,V^2}=\frac{\varkappa\cos(\theta+\gamma-\phi)\cos^2(\theta-\phi)}{N^2} \quad \ldots \quad (5^a)$$

Now, as the driving torque $n\,T\,r$ just compensates the frictional torque, the magnitude of the tangential force T depends on the internal friction of the mechanism. The latter is partly due to the weight of the spindles, gearing, &c., which part can be taken as constant, at the other hand the internal friction increases with the axial load, therefore with A. In general the relation between the tangential and axial forces acting on the blades can be put into the form :

$$T=p+q\,A \quad \ldots\ldots\ldots \quad (11)$$

where p and q may be assumed to be approximately constant, provided the vanes are rotating.

From this formula at once a conclusion can be drawn about the relation between U and V at very high velocities. Dividing both members of (11) by $2\,\varrho\,F^{\star}\,V^2$ and neglecting $p/2\,\varrho\,F^{\star}\,V^2$ in the limiting case, we have:

$$T^{\star}\overset{\infty}{=}q\,A^{\star} \quad \ldots\ldots\ldots \quad (11^a)$$

from which, in consequence of (4a) and (5a):

$$\cot(\theta + \gamma - \phi)_\infty = q \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (12)$$

The absolute value of V now is eliminated from all relations and only proportions and angles are retained; the latter acquire a constant value.

If q be known (in general a direct determination of q in the case of an arbitrary instrument will not be an easy matter), then (12) gives the value of $(\theta + \gamma - \phi)_\infty$; as θ is known, this leads to $(\gamma - \phi)_\infty$, after which both angles γ_∞ and ϕ_∞ (in the case of extreme high velocities) can be found by means of fig. 2.

Inserting the values deduced in this way into (10), we have:

$$\left(\frac{U}{V}\right)_\infty = \frac{\sin(\theta - \phi_\infty)\cos(\theta - \phi_\infty) - \varkappa \cos(\theta + \gamma_\infty - \phi_\infty)}{\cos^2(\theta - \phi_\infty) + \varkappa \sin(\theta + \gamma_\infty - \phi_\infty)} \quad . \qquad (13)$$

It is apparent from (13) that $(U/V)_\infty$ is smaller than the tangent of the blade angle θ, also smaller than $(\theta - \phi_\infty)$, which fact is essentially due to the presence of the disturbing components u and v.

A more complete investigation of the equations deduced here, in which the amount $p/2\varrho F^\star V^2$ is not neglected, may lead to the general relation between U and V, even at the smaller values of V. However, the deductions will be rather complicated and cannot be evaluated in an entirely explicit form; besides, as has been mentioned, eq. (11) is only an approximation. Hence we will not enter upon this question, but deduce below the internal friction for a particular instrument with an empirically determined calibration curve; in the same time the importance of v/V and u/U will be demonstrated.

Optimum blade angle.

The question concerning the blade angle for which a given instrument commences to rotate at the lowest possible wind speed can be answered in a general way from the formulae developed here.

In the initial stage of motion, when the vanes are on the point of commencing to rotate, but still are stationary, U will be zero, whereas the other velocity components will have values differing from zero. Then (7) reduces to:

$$\varkappa \cos(\theta + \gamma_0 - \phi_0) = \sin(\theta - \phi_0)\cos(\theta - \phi_0) \quad . \; . \; . \qquad (14)$$

The velocity components having vanished, for a certain value of $\varkappa$ (14) will give a relation between the angles, from which the initial incidence ϕ_0 can be found.

It is of importance to notice that in (14) the "density" $nF/F^\star$ of the vane circle plays a great part[1]; if the total area of the vanes approaches to the surface of the vane ring, that is, the denser the vane circle is, the

[1]) In the tables and in the diagrams the inverse ratio $F^\star/nF$ has been given.

smaller the incidence ϕ_0 appears to become. Conversily, if the vane ring is infinitely rarefied, which makes $\varkappa$ tend to zero, (14) immediately leads to the case of a single flat plate in an infinite field of flow with uniform velocity, for which $\theta = \phi_0$.

It is not possible to give an explicit solution of (14), if arbitrary values are assigned to the blade angle θ and to nF/F^* ; a solution can only be

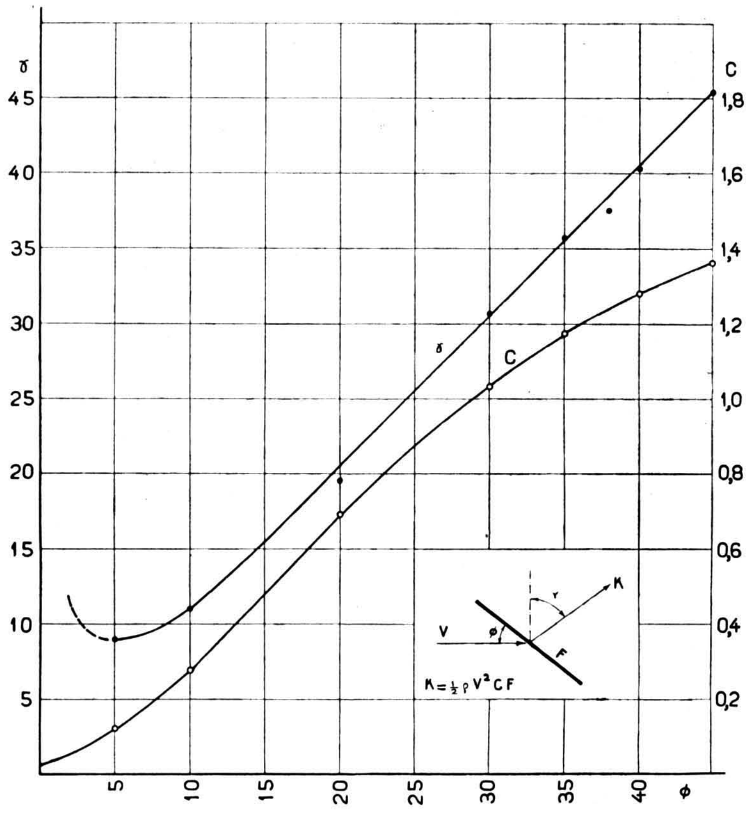

Fig. 2

found by trial (C and γ cannot be put as a mathematical function of ϕ and are to be derived from fig. 2). In this way the following values were determined :

Values of ϕ_0

θ	30	35	40	45	50	55°
$F^*/nF = 1.0$	20.9	24.7	28.8	33.4	38.2	43.5°
$F^*/nF = 1.5$	23.3	27.5	31.9	36.8	41.9	47.4°
$F^*/nF = 2.0$	24.6	29.1	33.7	38.6	43.8	49.2°

Now in the commencement of the rotation T is entirely determined by the constructional details of the mechanism and can be treated as a given quantity; therefore the velocity at which the instrument starts will be minimum provided T^* (comp. (5a)) is maximum. A series of values of T^*, corresponding to the angle ϕ_0 found above, are collected in the accompanying table:

Values of T^*

θ	30	35	40	45	50	55°
F^*/nF = 1.0	0.133	0.141	0.144	0.140	0.134	0.123
F^*/nF = 1.5	0.103	0.110	0.113	0.110	0.105	0.093
F^*/nF = 2.0	0.085	0.090	0.092	0.091	0.085	0.077

In the case considered here the optimum blade angle appears to be 40°, independently of F^*/nF, as will be seen from fig. 3, where T^* is plotted

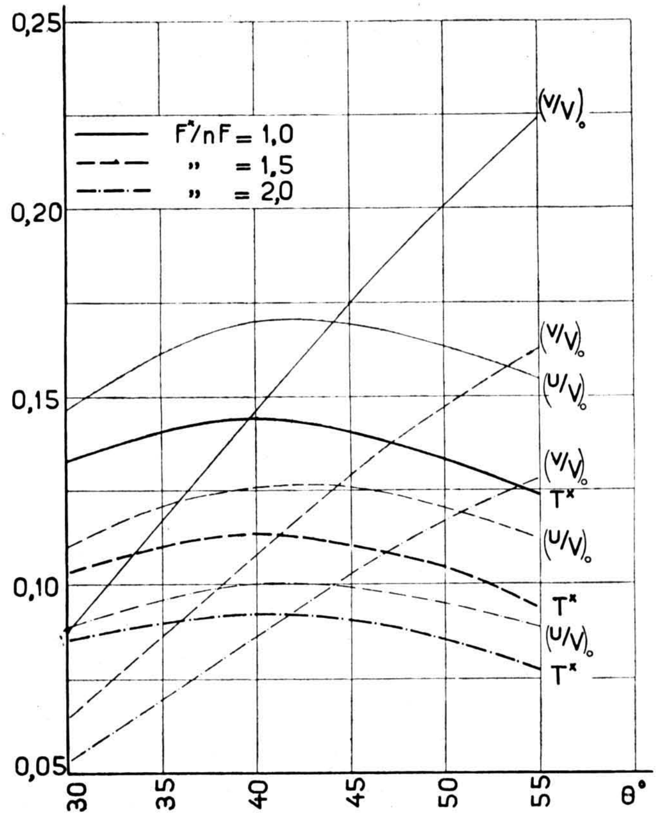

Fig. 3

against θ. It is easily verified that in the case of a single vane ($\varkappa = 0$) the optimum of the tangential force also occurs at about $\theta = 40°$. Therefore this value seems to hold in general for this type of vane; the result is confirmed by the experiments of OWER [1]).

Magnitude of the disturbing components u, v.

Finally the disturbing components u and v in the initial stage of motion, expressed in terms of V, were calculated from (7a) and (8a):

θ		30	35	40	45	50	55°
$F^*/nF = 1.0$	$(v/V)_0$	0.088	0.116	0.146	0.176	0.200	0.224
	$(u/V)_0$	0.147	0.161	0.170	0.169	0.164	0.154
$F^*/nF = 1.5$	$(v/V)_0$	0.065	0.086	0.107	0.129	0.147	0.163
	$(u/V)_0$	0.110	0.121	0.125	0.126	0.121	0.112
$F^*/nF = 2.0$	$(v/V)_0$	0.053	0.069	0.086	0.102	0.116	0.128
	$(u/V)_0$	0.089	0.096	0.100	0.100	0.095	0.088

These values are also represented in fig. 3. It is evident that the disturbing components in the case of the usual instruments (θ about 40 to 50°, F^*/nF between 1.0 and 1.5) are rather considerable and that it is not allowed — at least not in the initial stage of motion — to neglect them. What part they play at the higher velocities cannot be shown in the general case; it is possible, however, to calculate them for any given instrument as soon as the calibration curve is known.

To this end a common type of vane anemometer (constructed by FUESS, Berlin-Steglitz, number 1207) was calibrated in the Laboratory for Aero- and Hydrodynamics of the Technical University, Delft, after all vanes were set carefully at the same blade angle. The data of this instrument are the following:

Number of vanes . . . . $n = 8$
Total surface of vanes . . . . $nF = 19.2 \cdot 10^{-4}$ m²
Area of vane ring . . . . $F^* = 25.4 \cdot 10^{-4}$ m²
Radius of centre of pressure . . . . $r = 0.0235$ m
Blade angle . . . . $\theta = 53°15'$

The calibration curve (fig. 4) is linear from $V = 0.5$ m/sec upwards and satisfies in the region investigated the equation

$$U = 1.1360\, V - 0.230 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

[1]) E. OWER, Measurement of Air Flow p. 123.

where U and V are expressed in m/sec (U being found from the number of revolutions).

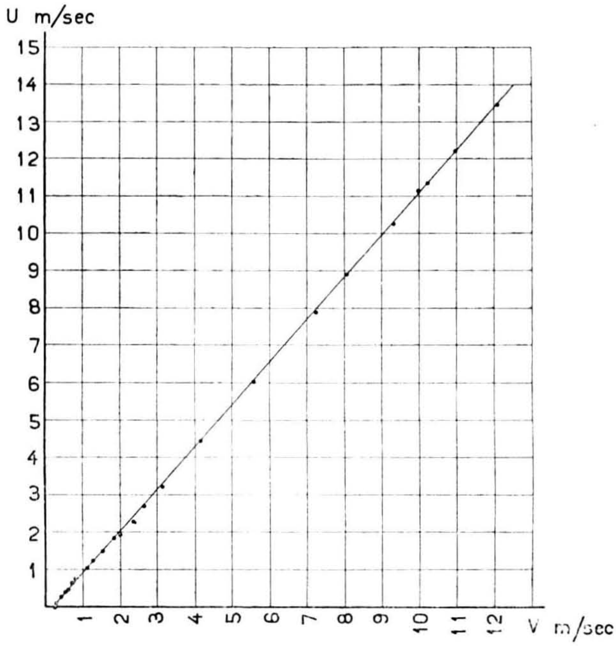

Fig. 4

ø	γ	C	v/V	u/U	U/V	V m/s	U m/s	$nA/2\varrho F^*$	$nT/2\varrho F^*$
42°12′	42°42′	1.320	0.172	—	0	(0.25)	0	—	—
10° 0′	11° 0′	0.278	0.0744	0.0656	0.8171	0.72	0.59	0.0358	0.0258
8° 0′	9°36′	0.210	0.0614	0.0479	0.9036	0.99	0.89	0.0564	0.0397
6° 0′	9° 6′	0.148	0.0480	0.0321	0.9979	1.66	1.66	0.1261	0.0841
5° 0′	9° 0′	0.122	0.0419	0.0258	1.0464	2.57	2.68	0.264	0.170
4° 0′	9° 9′	0.098	0.0357	0.0200	1.0972	5.93	6.50	1.210	0.744
3°50′	9°12′	0.094	0.0346	0.0191	1.1059	7.64	8.45	1.950	1.191
3°45′	9°13′	0.092	0.0340	0.0186	1.1105	9.02	10.02	2.67	1.62
3°40′	9°14′	0.090	0.0334	0.0182	1.1148	10.8	12.1	3.80	2.31
3°35′	9°15′	0.088	0.0328	0.0177	1.1194	13.9	15.5	6.08	3.67
3°30′	9°16′	0.086	0.0323	0.0173	1.1238	18.9	21.2	11.11	6.69
3°25′	9°17′	0.084	0.0318	0.0168	1.1283	29.9	33.7	27.5	16.4
3°20′	9°18′	0.082	0.0311	0.0163	1.1333	85.2	96.5	219	130
3°17′	9°19′	0.081	0.0308	0.0161	1.1377	∞	∞	∞	∞

For a number of values of the angle of incidence the corresponding values of C and γ were read off from fig. 2, then with the aid of (10) the ratio U/V could be found. Introducing this ratio into (15), absolute values of U and V have been deduced.

As has been remarked, the determination of C and γ for small values of ϕ (corresponding to high velocities) is rather inaccurate; however, as the factor $\varkappa$ appears to be small, the influence of this inaccuracy on the value of U/V is much reduced. The results have been tabulated in the accompanying table, together with the values of v/V, u/U, $nA/2\varrho F^{\star}$ and $nT/2\varrho F^{\star}$, though it will be understood that these latter quantities are affected with a greater relative uncertainty.

It will be seen that the limiting value of the angle of incidence is $3°17'$, the corresponding values of V and U becoming infinite — at least if it is assumed that the empirical calibration curve may be extrapolated indefinitely. That in this case the disturbing velocities u and v still are of importance, can be shown by observing that if they were neglected, the angle of incidence would be given by:

$$\theta - \text{arc tg}\, U/V = 53°15' - 48°39' = 4°36'.$$

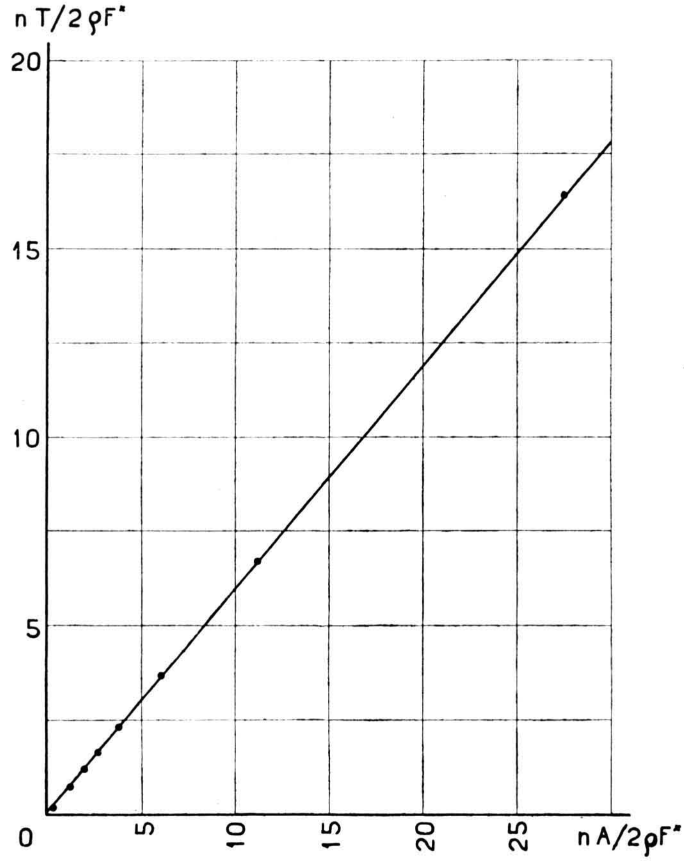

Fig. 5

The value of C corresponding to $\phi = 4^\circ 36'$ is 0.112; hence neglecting u and v the value of the force components would become about 38 % too high.

Determination of internal friction.

From the limiting case we deduce $(T/A)_\infty = \cot\ (\theta + \gamma - \phi)_\infty = \cot\ 59^\circ 17' = 0.594$. Hence this must be the value of q (at least for the higher velocities) for this particular instrument.

In order to check the relations expressed by (11) the values of $nA/2\,\varrho F^*$ and $nT/2\,\varrho F^*$ given in the table have been plotted in fig. 5. It will be seen that the points approximately arrange themselves on a straight line. The value of p in (11) is too small to be read off directly from the diagram fig. 5.

ERRATUM.

Proc. Royal Acad. Amsterdam, Vol. **35**, N°. 6, 1932 (p. 750).

Chemistry. — *Oxidation of phenol with peracetic acid. (Contribution to the knowledge of the substitution of benzene).* By Prof. J. BÖESEKEN and R. ENGELBERTS.
