## Summary.

The absorption spectra of $S m^{+++}$in the hexagonal crystal $\mathrm{Sm}\left(\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{SO}_{4}\right)_{3} \cdot 9 \mathrm{H}_{2} \mathrm{O}$ were taken parallel to the optic axis at $20^{\circ} \mathrm{K}$, $77^{\circ} \mathrm{K}$, and $169^{\circ} \mathrm{K}$. Absorption lines are listed in the region of the spectrum between $4200 \AA$ and $2200 \AA$.

This paper is principally concerned with the various electronic configurations in the basic multiplet, especially as they result from the inter-action of $\mathrm{Sm}^{+++}$and the electric fields of the lattice. In consequence, all the lines whose relative intensities vary with the temperature have been studied in terms of energy level diagrams.

Physics. - The Calibration of a Pressure Balance in Absolute Units. (31 ${ }^{\text {st }}$ Communication of the Van der Waals Fund). By A. Michels. (Communicated by Prof. J. D. van der Waals Jr.). (Communicated at the meeting of September 24, 1932.)

Introduction. For the measurement of pressure in absolute units, as soon as the pressure is greater than some atmospheres, the only method that need be considered is that using a mercury column. This method, however, involves considerable difficulties when the pressures to be measured become appreciable. A few examples are known of high pressure mercury manometers such as those of Amagat in a mine-shaft at Verpilleux near St. Etienne, and of Cailletet in the Eifel Tower, with which it was possible to measure up to about 400 atm . In the same group can be placed the so-called "gebroken manometer" designed by Kamerlingh Onnes, which is still in use for measurements up to 120 atm . We are unaware of any direct measurements with a mercury column besides these.

Of the secondary gauges in use the most suitable for accurate measurement is the pressure balance which also allows measurements to be made at much higher pressures. For the most accurate work it should, however, be calibrated directly. Many types of this apparatus are known (Amagat, Wiebe, Stückrath, Wagner, Lange, Holborn, Bridgman and others) all of which are designed on the same principle: a piston is ground to fit as well as possible in a cylinder; the unknown hydrostatic pressure is applied under the piston and the force is measured necessary to keep the piston in equilibrium. The pressure can then be calculated from

$$
P=\frac{K}{0}
$$

where $K$ is the force applied and 0 the area of the piston. By giving the piston a rotatory or to and fro motion the friction can be kept low.

In two previous publications ${ }^{1}$ ) the necessary conditions were considered, in terms of a theory there derived, to reduce the friction to a minimum and thus to make the sensitivity as high as possible. It has since been possible to increase the sensitivity from $2 \frac{1}{2} \mathrm{~g}$. to less than $1 / 2 \mathrm{~g}$. (on a maximum load of about 300 Kg . in the instrument here used). It follows from the same theoretical treatment that the measured surface of the piston cannot be taken as the effective surface area. This effective area was shown to be dependent on the internal diameter of the cylinder, rate of leaking, pressure etc., so that for really accurate measurements it appeared to be necessary to calibrate any pressure balance against an absolute manometer, thus against a mercury column. In the same publications a differential method was described for carrying out this calibration at higher pressures. The accuracy of the calibration is, however, dependent on the available height of the mercury column which on that occasion was limited to $41 / 2 \mathrm{M}$. On this account, and the then smaller sensitivity of the pressure balance, greater accuracy than $1 / 2400$ could not be obtained in the measurement of the effective area.

In order to put measurements in the field of higher pressures on a better basis, the Van der Waals Fund in 1925 resolved to build up an equipment for the calibration of pressure gauges. Thanks to support received from many quarters, a start was made in the same year.

The use of the "Westertoren" was kindly given by the "Burgemeester en Wethouders" of Amsterdam for the erection of the necessary mercury column.

Although it would have been possible to make use directly of the full length of the tower, this would have made it necessary to bring a large part of the manometer tube through the open air which would have given rise to large temperature changes; it is further impossible in some places to carry the column upwards in a straight line which would involve difficulty in measuring the height. In order to avoid having to overcome so many difficulties, it was decided first to use a height of $271 / 2 \mathrm{M}$. which was available in a vertical line between the first and sixth floors. The intention, however, remains to make use later of the complete height of the tower.

The preparatory work is at present largely finished and a start could be made with the definite measurements. Calibrations using the differential method have, indeed, not yet been carried out for lack of certain pieces of apparatus, but the direct measurements against an open mercury manometer, which therefore only go to low pressures (about 40 atm.) have been completed. It was decided to publish these results for the following reasons:

1. All measurements carried out in this laboratory on the influence of pressure on physical constants published since 1924 are based on the

[^0]calibration made in that year. The accuracy obtained then was, as has been stated, $1 / 2400$, while it is by no means impossible that by wear or ageing effects the effective area then measured has altered. To this must be added the fact that the piston originally calibrated has been out of use for several years as the result of wear and there are only available comparison figures between the pistons at present in use and the calibiated one in its original state. All published results have therefore up to the present an element of uncertainty so that verification was very desirable.
2. The change of the effective area with pressure is very small, as appears from the theoretical treatment. The values


Fig. 1 measured at present at low pressures, can therefore temporarily be used also for higher pressures. Corrections for the effect of pressure can then be applied later when the differential method has been used.

## The equipment of the "Westertoren".

A description will only be given of such apparatus as was used for the first measurements.

In order to carry out the work, laboratories were built and equiped on the first, fourth and sixth floors at heights of 10,24 and 38 M . respectively and were connected by a vertical wooden case ( $30 \times 50 \mathrm{~cm}$.). In the lowest room, which is also the largest, the main part of the apparatus was set up.

For the direct calibration of a piston against an open mercury manometer it is in principle only necessary to have a pressure balance, press and mercury column of which the height and temperature can be accurately defined.

Fig. 1 shows a diagrammatic sketch of the arrangements, while fig. 2 shows better the apparatus on the first and sixth floors. The mercury is in a steel tube $(A B)$ of about 11 mm . diameter. A short glass tube $(F)$ is joined to the top of $(A B)$ in which the mercury surface can be observed with a cathetometer. The second mercury surface is at (C) in a thick-walled glass tube. This surface is in direct contact with the oil which transmits the pressure through the press $(D)$ to the effective surface of the piston $(O)$. The steel vessel $(E)$, the nitrogen cylinder $(R)$ and the taps (G), (H) and ( $I$ ) are only used in filling the apparatus with mercury. The steel tube $(A B)$ is hung in the
wooden case $(Y Z)$. Where this case passes openings in the tower with the consequent possibility of rapid temperature changes, it is double walled. The height of the mercury column is measured with the invar


Fig. 2
tape $(K L)$, the invar metre $(M)$ and the cathetometers $(N)$ and $(P)$. The invar tape is divided in half metres and is suspended from one
end in the groove of a bronze wheel (Q) by which the height of the tape can be adjusted. The other end hangs free and is loaded with a weight of 10 Kg .

A blackened brass tube is pushed over the glass tube $(F)$. The invar tape is so adjusted with the wheel ( $Q$ ) that one of the divisions on it is at the same height as the bottom of the brass tube. During the measurements the distance of the mercury meniscus from the tube is read, this distance being always kept less than a few millimeters.

The position of the zero stripe on the invar tape is read with the cathetometer $(P)$ on the invar metre $(M)$. The position of the bottom meniscus in $(C)$ is measured on $(M)$ with the same
 cathetometer. In this way the height of the mercury column is found. Since ( $O$ ) and ( $C$ ) are not at the same height a correction must be applied for the pressure due to the oil column OC.

In order to measure the temperature of the mercury, thermometers are placed through the wall of the case at intervals of 1 M . These thermometers can be read from outside the case and measurements were made simultaneously by six observers.

It is simpler to use a secondary thermometer consisting of a platinum wire stretched up and down along the whole length of the case so that the two halves lie on opposite sides of the mercury column. The wire is in a glass tube which in turn is placed inside a brass tube. By giving the walls suitable dimensions the temperature lag of the wire is made as nearly as possible the same as that of the mercury column. To remove strains due to its own weight the wire is wound every two metres on an ebonite rod in the manner shown in fig. 3.

From the resistance of the platinum wire it is possible to calculate its mean temperarure and therefore that of the mercury in the manometer since the two are always nearly enough alike. From this the mean density of the mercury follows directly, which can be multiplied with the height to give the pressure. In an appendix a deduction is given of the temperature limits within which this method is allowable; in the present case an allowable temperature variation of $14^{\circ}$ was found for a final accuracy of $1 / 100000$. The readings of the platinum wire were calibrated by measurements with the ordinary mercury thermometers.

Calibration of the Invar Tape.
The division and temperature coefficient of the invar tape were compared with an invar metre calibrated at the Bureau International de

Poids et de Mesures at Paris. For this comparison an apparatus was constructed as shown in fig. 4.

Four wheels ( $A, B, C$ and $D$ ) supported on ball bearings, are hung


Fig. 4
in an iron frame. $B$ and $C$ have one, $A$ and $D$ two grooves cut in the circumference. At the beginning of the comparison the tape is rolled up on $A$ and the end made fast as shown in the figure to the circumference of $D$, the tape lying in the grooves of $B$ and $C$. In the second grooves of both $A$ and $D$ a steel tape is fitted from which hangs a weight of $10 \mathrm{Kg} .(E$ and $F)$ keeping the invar tape under tension. By means of a cathetometer (allowing measurements to be made to 0.001 mm .) the tape is now metre by metre compared with the standard metre. In the same way the distances $0-1 / 2,1 / 2-1^{1} / 2,1 \frac{1}{2}-21 / 2 \ldots .28^{1 / 2}-29^{1} / 2$,
$29^{1} / 2-30 \mathrm{M}$. were measured. These measurements were carried out at two different temperatures.

## Filling of the apparatus.

The filling is carried out as follows: with the tap $G$ (fig. 2) shut, the mercury to be used is poured into the vessel $E$ and a pressure of a few atmospheres is applied to the surface of the mercury from the gas cylinder $R$. The coupling $U$ is loosened. By opening $G$ the mercury is allowed to rise slowly in the tubes $S$ and $V$ till the surface reaches the top of $V . G$ is now closed, the tube $T$ filled with oil from the press and the coupling $U$ again tightened. The press is then used to force the mercury so far back that the meniscus is about half way down $V$. $H$ is now shut and $G$ opened. The pressure in the vessel $E$ is so adjusted with the tap $I$ that the mercury slowly rises in the manometer tube till it is visible in $F . G$ is then shut and the gas pressure let off.

## The measurements.

The effective areas of the pistons $A_{2} 250$ and $A 50$ were determined with the following results:

$$
\begin{aligned}
& \text { Effective area of } A_{2} 250 \text {. . . . } 1.03210 \mathrm{~cm}^{2} \\
& \text { " , } A 50 \ldots 5.5112^{5} \text {,, at } 18^{\circ} \mathrm{C} \text {. }
\end{aligned}
$$

The reproducibility of the measurements was found to be better than $1 / 50000$ corresponding with $\pm 0.5 \mathrm{~mm}$. mercury or a variation of $0.1^{\circ}$ in the temperature of the column.

The temperature of the wooden case was not the same over its whole length, being $\pm 0.7^{\circ}$ higher at the top than at the bottom. The gradient was very regular. Apart from this, during the first hour of the measurements the temperature rose steadily about $0.25^{\circ}$ and then remained constant within $0.02^{\circ}$.

Both pistons had been previously compared with the old piston $A_{1} 250$ calibrated 8 years ago. Taking the average of a large number of determinations of this piston as $0.9988^{5}$ the values found by comparison were:

$$
\begin{aligned}
& A_{2} 250 \ldots 1.0318 \mathrm{~cm}^{2} \\
& A 50 \ldots 5.5096 \quad \ldots
\end{aligned}
$$

The difference amounts to $3 / 10000$. Since the accuracy of the old measurements was limited to $1 / 2400$ the agreement is satisfactory.

It may be of interest to mention that during the measurements the sun began to shine with the result that the tower increased in height.

Although the temperature rise inside remained less than $0.25^{\circ}$, the measured length in the tower, $\pm 27 \mathrm{M}$. increased by 0.8 mm . in 2 hours.

It may also be mentioned that the calibration could only be carried out in calm weather. Before the experiments were started the period and the amplitude of the movement of the tower were measured during a storm. The maximum double amplitude of the top was found to be 26 mm . with a period of $\pm 1$ second $^{1}$ ).

## Appendix.

The use of the platinum thermometer is based on the following argument:
The total mercury pressure at the bottom of the mercury column is $P=\int_{0^{\circ}}{ }^{H} \varrho d h$, where $\varrho$ is the local density of the mercury. This is dependent on the temperature.

The total resistance of the platinum wire can also be written as $W=\int_{0^{0}}^{H} w d h$. where $w$ is the resistance per cm . length.

Assuming that $\varrho$ and $w$ are linearly dependent on the temperature it is possible to measure the mean temperature by a simple calculation from the second integral, and from this the mean value of $\varrho$ which determines the first integral.

Actually, however, neither $\varrho$ nor $w$ are strictly linear temperature functions and can be written

$$
\begin{equation*}
\varrho=\varrho_{0}(1+\alpha t+X) \quad w=w_{0}(1+a t+Y) . \tag{1}
\end{equation*}
$$

where $\varrho_{0}$ and $w_{0}$ are the specific quantities at the mean temperature and $t$ is the local variation from this temperature. In order to see how great is the quantity omitted in considering the functions as linear (removal of $X$ and $Y$ ) the following method may be used:

$$
\begin{align*}
P & =\int_{0}^{H} \varrho_{0}(1+a t+X) d h= \\
& =\varrho_{0} \int_{0}^{H} d h+\varrho_{0} a \int_{0}^{H} t d h+\varrho_{0} \int_{0}^{H} X d h  \tag{2a}\\
& =\varrho_{0} H+\varrho_{0} \alpha \int_{0}^{H} t d h+\varrho_{0} \int_{0}^{H} X d h
\end{align*}
$$

[^1]\[

$$
\begin{align*}
& W=\int_{0}^{H} w_{0}(1+a t+Y) d h= \\
& =w_{0} \int_{0}^{H} d h+w_{0} a \int_{0}^{H} t d h+w_{0} \int_{0}^{H} Y d h= \\
& w_{0} H+w_{0} a \int_{0}^{H} t d h+w_{0} \int_{0}^{H} Y d h \quad \text { or } \\
& \int_{0}^{H} t d h=\frac{W-w_{0} H-w_{0} \int_{0}^{H} Y d h}{w_{0} a} . \tag{2b}
\end{align*}
$$
\]

Substituting (2b) in (2a) gives

$$
\left.\begin{array}{rl}
P & =\varrho_{0} H+\frac{\varrho_{0} \alpha}{w_{0} a}\left\{W-w_{0} H-w_{0} \int_{0}^{H} Y d h\right\}+\varrho_{0} \int_{0}^{H} X d h  \tag{2c}\\
& =\varrho_{0} H+\frac{\varrho_{0} \alpha}{w_{0} a}\left(W-w_{0} H\right)+\varrho_{0} \int_{0}^{H} d h\left[X-\frac{\alpha}{a} Y\right]
\end{array}\right\}
$$

If linearity is assumed therefore, the third term of the right hand side is omitted while the first is by far the largest. Then, if the error in $P$ may not be for example greater than ${ }^{1 / 100000}$

$$
\frac{\varrho_{0} \int_{0}^{H} d h\left[X-\frac{\alpha}{a} Y\right]}{\varrho_{0} H}<10^{-5}
$$

or

$$
\begin{equation*}
\int_{0}^{H} d h\left[X-\frac{\alpha}{a} Y\right]<10^{-5} H . \tag{3}
\end{equation*}
$$

$X$ and $Y$ can be written in the following way

$$
X=\beta t^{2}+\gamma t^{3}+\ldots ; \quad Y=b t^{2}+c t^{3}+\ldots ;
$$

as is seen by comparison of the expressions (1) with the full series development of $\varrho$ and $w$ with respect to temperature. If $\tau$ is the maximum -
deviation from the mean temperature, then expression (3) will certainly be satisfied if

$$
\begin{equation*}
\int_{0}^{H} d h\left[\left(\beta-\frac{\alpha}{a} b\right) \tau^{2}+\left(\gamma-\frac{\alpha}{a} c\right) \tau^{3}+\ldots .\right]<10^{-5} H . \tag{4}
\end{equation*}
$$

$\tau$ is now a constant so that (4) can be integrated to give

$$
\left(\beta-\frac{\alpha}{a} b\right) \tau^{2} H+\left(\gamma-\frac{\alpha}{a} c\right) \tau^{3} H+\ldots<10^{-5} H
$$

or

$$
\begin{equation*}
\left(\beta-\frac{\alpha}{a} b\right) \tau^{2}+\left(\gamma-\frac{\alpha}{a} c\right) \tau^{3}+\ldots<10^{-5} . \tag{5}
\end{equation*}
$$

Here $\alpha$ is of the order ${ }^{1}$ ) $-1.8 \times 10^{-4}$

| $\beta$ | $+2.4 \times 10^{-8}$ |
| :---: | :---: |
| $\gamma$ | $1.0 \times 10^{-10}$ |
| $a$ | $3.8 \times 10^{-3}$ |
| $b$ | $0.6 \times 10^{-6}$ |
| $c$ | - |

(These figures are correct for $0^{\circ}$ and can therefore only be approximately applied for the mean temperature of the column).

Neglecting the terms in $\tau^{3}$ on account of the small values of the coefficients $\gamma$ and $c$ expression (5) becomes

$$
\left(2.4 \times 10^{-8}+3.0 \times 10^{-8}\right) \tau^{2}<10^{-5} \quad \text { or } \quad \tau<14^{\circ}
$$

${ }^{1}$ ) These figures are taken from Landolt-Bornstein: Physikalische Tabelle.


[^0]:    ${ }^{1}$ ) A. Michels, Ann. d. Phys. 72, 285, 1923: 73 577, 1924.

[^1]:    ${ }^{1}$ ) Our thanks are due to Prof. Schermerhorn of the Technische Hoogeschool Delft, for the loan of a theodolite used to measure the movement of the tower.

