For the rest we point out that the lowering in specific refraction is large with the hydrogenation of the cracking-products, smaller with the berginization-fractions, where the change in n and d is also small.

From this investigation it appears clearly how in berginization the formation of cyclic compounds is hindered.

It should be remembered that both series of experiments were performed at $450°$ C., an initial pressure of 110 kg/cm^2 at about $15°$ C. in the Bergin-series, and an initial pressure of 1 kg/cm^2 or vacuum in the Cracking-series. The duration of the experiments was 90 minutes at $450°$ C. In the berginization experiments the maximum pressure was on an average 280—290 kg/cm^2 at $450°$ C., and the final pressure on an average 80 kg/cm^2 at about $15°$; in the cracking experiments the maximum pressure was on an average 100—110 kg/cm^2 at $450°$ C. and the final pressure on an average $15-18$ kg/cm^2 at about $15°$.

By the presence of hydrogen in the berginization, which caused a hydrogen consumption of $1.4°/_0$, the ring-formation is reduced to one seventh of that in the cracking.

By following this method of analysis we have succeeded for the first time in measuring the cyclisation in the cracking quantitatively and in establishing the inhibitory influence of high pressure hydrogen on this cyclisation quantitatively.

<div align="right">

Laboratory for Chemical
Technology of the Technical University.
</div>

Delft, July 1932.

Physics. — *On the Directional Effect of the Single Hot Wire Anemometer.* By M. ZIEGLER (Mededeeling N^0. 25 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hoogeschool te Delft). (Communicated by Prof. J. M. BURGERS).

(Communicated at the meeting of October 29, 1932.)

1. *Introduction.*

In hot wire work still little attention has been given to the fact that **exact** measurements with a single wire anemometer are only possible in a two dimensional field of flow, provided the position of the wire is perpendicular to the plane of the motion. The indications of the wire, calibrated in the so-called normal position, that is perpendicular to the direction of flow, then give the absolute value of the velocity for an arbitrary direction of the velocity vector in that plane. However, as soon as a third component of the velocity, i.e. a component parallel to the wire is present, then neither the absolute value of the velocity, nor the resultant of the two components

perpendicular to the wire can be determined if no other indications are available.

It is well known that the cooling of the wire by an air current is a function of the angle between the wire and the velocity vector. This function, the value of which for the normal position (angle of incidence $= 90°$) is given by KING's formula [1]), presents a sharp minimum in the "zero position" (wire parallel to the flow, angle of incidence $= 0°$). A few measurements concerning the dependence of the cooling effect on the angle of incidence have been published by SIMMONS and BAILEY [2]) and by BURGERS [3]), which clearly show the presence of this minimum.

If a three dimensional field of flow is to be investigated, then for the case of a stationary motion it is yet possible to determine the true value of the air speed at a given point of the field with a single wire, provided it can be turned in all directions in order to find out the position of minimum cooling. In this position the wire has the direction of the flow and the true velocity then follows at once, provided a calibration curve relating to the zero position is available. SIMMONS and BAILEY in their publication point out the practical inconveniences of this method, and give a description of three and four wire "speed and direction" meters which can be kept in a fixed position and work satisfactorily. For reliable experiments all such instruments of course must satisfy the condition of being so small, that the streamlines may be considered as being straight and parallel over the region occupied by the wire system.

As, however, the range of directions for which such instruments can be used is limited, there is always the possibility that in cases of heavy turbulence the velocity vector at a given point varies so strongly both with regard to magnitude and to direction, that a more complicated instrument than the single hot wire has little utility for the determination of the instantaneous velocity.

If f.i. the flow in the turbulent boundary layer is to be investigated, the question thus arises what errors can be caused in the indications of a single wire anemometer, by the presence of a velocity component parallel to the wire. At the same time it can be asked if it would be possible to obtain an estimate of the magnitude of the "cross" component in the boundary layer, by comparing the indications of a single wire which is turned into various positions at the same point of the field.

The purpose of the present publication is to describe the results of some

[1]) L. V. KING, On the convection of heat from small cylinders in a stream of fluid; determination of the convection constants of small platinum wires with applications to hot-wire anemometry, Phil. Trans. A **214**, 373—432, 1914.

[2]) L. F. G. SIMMONS and A. BAILEY, Rep. & Mem. N⁰. 1019 of the Aeron. Research Committee (England).

[3]) J. M. BURGERS, Hitzdrahtmessungen § 6 (Handbuch der Experimentalphysik, Band 4, 1. Teil). The curves published are due to VAN DER HEGGE ZIJNEN in Delft.

experiments on the directional effect of the wires used by us and to give an empirical formula for this effect.

2. *Experimental method.*

In the smaller windchannel of the laboratory, the flow of which is very steady, a hot wire was mounted in such a way that it could be turned, in a horizontal plane, through all angular positions between 0° and 360° with regard to the direction of the air current, which also is horizontal. The wire was moved from the outside ; the angle could be read accurately on a dial divided into degrees. The zero position of the wire was determined experimentally as the position of least cooling, with an accuracy of about 0.5 degree. For various air speeds and various angles of incidence the indication of the anemometer arrangement was observed. For each velocity the mean value of readings for equivalent angular positions was taken.

The experiments have been made with hot wires as described in "Mededeeling 21" and "23" of this laboratory [1]). As being the most convenient one, that method was used in which the current through the galvanometer is observed, the electrical tension between the terminals of the Wheatstone bridge arrangement being kept constant. The arrangement of fig. 1, used for the experiments of July 1932, may be given as an example.
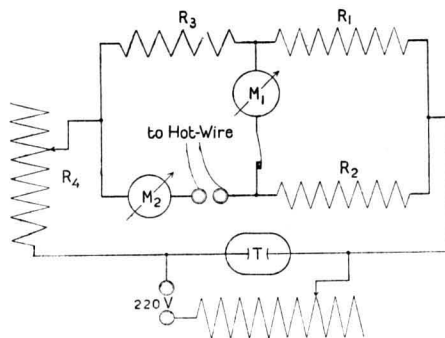

Fig. 1.

Electrical part of the single hot wire anemometer arrangement used for the experiments on the directional effect.

$R_1 = R_2 = 1000 \; \Omega$

R_3 = Hot wire resistance (adjusted for air velocity = 0).

R_4 = Variable resistance for adjusting the heating current at the desired value.

M_1 = Milliammeter the deviation of which is observed.

M_2 = Heating current meter.

T = Gas valve which maintains a constant tension of about 78V between its electrodes independently of input tension variations during the experiment. (Lorenz Stabilisator TRT 10).

The accurate and rapid measurement of very low air speeds in the wind channel gave some difficulty ; therefore the channel was narrowed considerably (in the ratio 1 : 20) at some point downward from the space of measurement. The difference between the static pressure in this throat and that at the place of the wire could be taken as a measure of the velocity.

[1]) M. ZIEGLER, these Proceedings, **34**, p. 663, 1931 : A complete arrangement for the investigation, the measurement and the recording of rapid air speed fluctuations with very thin and short wires; and these Proceedings, **35**, p. 419, 1932 : Oscillographic records of the turbulent motion developing in a boundary layer from a sheet of discontinuity.

3. *Apparent velocity.*

The measurements again showed that the rate of heat loss presents a sharp minimum for the position of the wire parallel to the flow. As to the rate of cooling at other positions the following may be remarked:

We call φ the angle between the wire and the velocity vector and consider an anemometer calibrated for the normal position of the wire; thus for $\varphi = 90°$ the indication of the anemometer arrangement as a function of the velocity is known. Now let V_t be the true velocity of the air which may make an arbitrary angle with the wire. Then we define the apparent velocity V_a as the velocity which, on the normal calibration curve, would correspond to the indication of the instrument. For $\varphi = 90°$, $V_a = V_t$; for $\varphi \neq 90°$, $V_a < V_t$. Taking especially $\varphi = 0$ we shall write $V_a = a_0 V_t$, where a_0 is a coefficient smaller than unity. The value of a_0 depends on the velocity, and will be considered as a function of V_t.

Now it might be supposed that for an arbitrary angle of incidence the apparent velocity would not differ very much from the value given by the expression

$$V_a = \sqrt{V_n^2 + a_0 V_p^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where V_n is the component of the true velocity perpendicular to the wire and V_p the component parallel to it. This expression may also be written:

$$V_a = V_t \sqrt{sin^2 \varphi + a_0^2 cos^2 \varphi} \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

It has been found that this formula is in satisfactory agreement with the behaviour of the wire, as may be seen e.g. from the results of the following
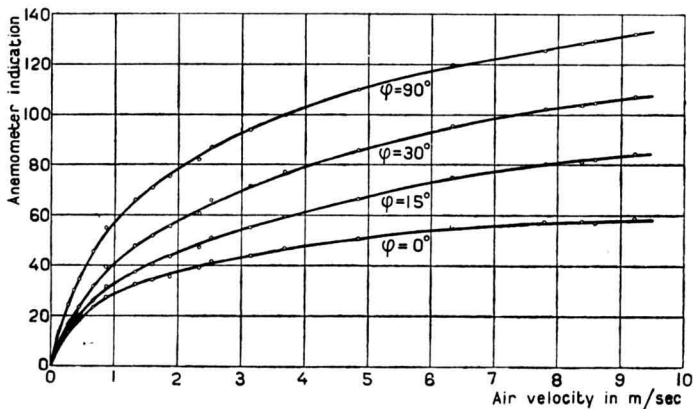

Fig. 2

Calibration curves of the hot wire arrangement for four angular positions of the wire.

experiments (July 1932). The electrical arrangement was that of fig. 1. Platinum hot-wire, diameter 0.005 mm, length 2 mm. For $V = 0$: heating current $= 32$ mA; resistance of the wire $=$ ca. 60 Ω. The bridge was balanced for $V = 0$.

The curves of fig. 2 for $\varphi = 90°$, 30°, 15° and 0° give the anemometer indication as a function of the velocity (calibration curves), while those of fig. 3 show the variation of the apparent velocity with the angle of incidence for the air velocities of 8.60, 4.85 and 1.60 m/sec.
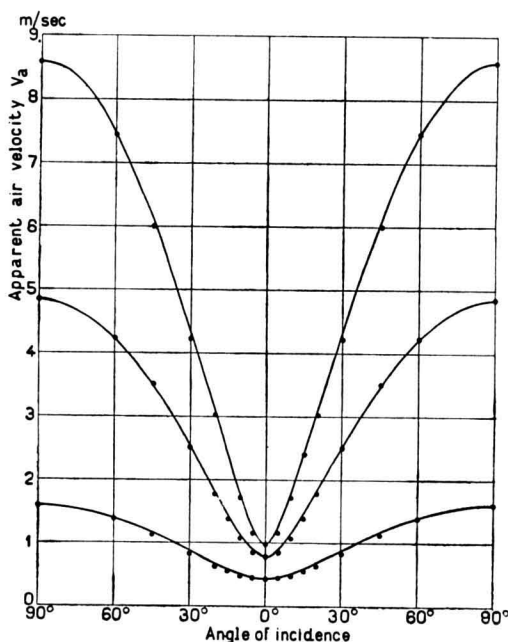

Fig. 3.

Variation of the apparent velocity V_a with the inclination of the wire to the wind direction for true wind speeds V_t resp. equal to 8,60, 4,85 and 1,60 m/sec. The full drawn curves have been deduced from the formula; the small circles represent the values deduced from the measurement.

The values of a_0 which belong to these velocities can be obtained from fig. 2; they are respectively:

$$\frac{0.97}{8.60} = 0.113 \qquad \frac{0.78}{4.85} = 0.161 \qquad \frac{0.44}{1.60} = 0.275.$$

For an arbitrary angle of incidence the apparent velocity is given by formula (1a).

The following table gives the course of the calculation for the three velocities mentioned (corresponding respectively to column I, II, III) and for various angles:

φ	$\alpha = \sqrt{sin^2\varphi + \alpha_0^2 cos^2\varphi}$			V_a calculated $=$ $= V_t \times \alpha$			Anemometer Indication			V_a observed		
	I	II	III	I	II	III	I	II	III	I	II	III
0°	0.113	0.161	0.275	0.97	0.78	0.44	56.7	50.3	34.1	0.97	0.78	0.44
5°	0.146	0.186	0.289	1.26	0.90	0.46	61.5	52.5	34.7	1.15	0.85	0.45
10°	0.207	0.236	0.323	1.78	1.14	0.52	71.4	58.9	36.9	1.71	1.07	0.49
15°	0.281	0.302	0.371	2.42	1.46	0.59	82.2	66.4	40.3	2.40	1.38	0.56
20°	0.358	0.374	0.429	3.08	1.81	0.69	91.4	74.1	44.1	3.02	1.78	0.64
30°	0.509	0.519	0.554	4.38	2.52	0.89	104.6	86.1	51.8	4.22	2.50	0.83
45°	0.711	0.716	0.734	6.12	3.47	1.17	117.1	97.9	60.6	6.00	3.51	1.13
60°	0.868	0.870	0.876	7.47	4.21	1.40	124.5	104.9	66.5	7.47	4.23	1.38
90°	1.000	1.000	1.000	8.60	4.85	1.60	129.7	109.9	70.9	8.60	4.85	1.60

It will be seen from fig. 3 that the values of V_a deduced from the observation are very close to the calculated curve. Other experiments performed (January 1932) with hot wires of the same type agree equally well with the formula.

It is possible also to check the formula on results of other investigators. SIMMONS and BAILY in the publication mentioned above describe experiments performed with a wire of 3.07″ = 78 mm length and 0.00105″ = 0.027 mm diameter. Fig. II[1]) of their paper gives the measured wire resistance in ohms, for a heating current of 0.215 A and an air velocity of 40 ft/sec, as a function of the angle with the direction of stream.

For the same wire two empirical formulae are given expressing the amount of energy carried off by the wind:

1°. for the case the wire is perpendicular to the air flow:

$$H = 6.68 \cdot 10^{-5} \, V^{1/2} + 9.60 \cdot 10^{-5} \quad . \quad . \quad , \quad . \quad . \quad . \quad (2)$$

2°. for the case the wire is parallel to the flow:

$$H = 1.673 \cdot 10^{-6} \, V + (1.0 \cdot 10^{-4} + 7.5 \cdot 10^{-8} \, t) \quad . \quad . \quad . \quad (3)$$

where $H = \dfrac{0.24 \, i^2 R}{(t - t_0) \, l}$ is the rate of heat loss in calories, per unit of length of the wire[2]), and per 1° C difference of temperature with the air (t being the temperature of the wire, t_0 that of the air).

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[1]) To avoid confusion figures of SIMMONS and BAILEY's paper wil be indicated here by Roman numerals.

[2]) The authors indicate "per foot". It appears from comparison with values read off from the graphs that the length of the wire has to be taken in cm and not in feet.

From the graphs of fig. II we read:

$$\text{for } \varphi = 90°: \text{............} \quad R = 19.33 \ \Omega$$
$$\text{for } \varphi = \ \ 0°: \text{............} \quad R = 29.20 \ \Omega$$

Inserting the first value of R into equation (2), V being equal to 40 ft/sec, the temperature difference $t-t_0$ is found to be 53.1° C (fig. I gives $t-t_0 = 54.5°$, which differs not exaggerately from the calculated value in view of the inexactitude inherent to the readings).

Inserting the second value for R into equation (3), taking t_0 equal to 22° C as indicated in fig. I, gives $t-t_0 = 224.2°$ C (fig. I gives 225°). V_{a0} now can be calculated easily by substituting into (2): $R = 29.20 \ \Omega$; $t-t_0 = 224.2°$ C. We get $V_{a0} = 1.8$ ft/sec, and thus finally $a_0 = V_{a0}/V_t = 1.8/40 = 0.045$.

With the value of a_0 thus obtained the apparent velocity V_a can be calculated for various positions of the wire. In the expression for H instead of $t-t_0$ we write $(R-R_0) \ \beta$; for β we find easily 17.34 degree/ohm, while $R_0 = 16.26 \ \Omega$. Equation (2) now assumes the form:

$$\frac{0.24 \ i^2 \ R}{17.34 \ (R-R_0) \ l} = 6.68 \cdot 10^{-5} V \sqrt{V_a} + 9.60 \cdot 10^{-5} \quad . \quad . \quad . \quad (2a)$$

from which the value of R corresponding to any value of V_a can be found.

The following table shows the course of the calculation. The last column gives the values which can be read directly from fig. II.

φ	$a = \sqrt{\sin^2 \varphi + a_0^2 \cos^2 \varphi}$	$V_a = a \ V_t$ in ft/sec	R calculated	R observed
0°	0.045	1.80	29.2	29.2
5°	0.104	4.15	25.2	25.0
10°	0.179	7.17	23.2	23.0
20°	0.345	13.78	21.3	21.3
30°	0.501	20.06	20.5	20.6
45°	0.709	28.31	19.9	19.8
60°	0.866	34.64	19.6	19.6
90°	1.000	40.00	19.3	19.3

We see that here again the agreement is very satisfactory.

It is of importance to investigate whether the relation between V_a and V_t, defined by eq. (1), is independent of the particular arrangement of the anemometer circuit. For any particular arrangement, equality of the indication of the anemometer, be it the position of the bridge galvanometer,

or the current through the wire, ensures equality of temperature of the latter. Hence eq. (1) expresses that for a particular value of the temperature (the same in both cases), the cooling effect of the true air current V_t acting under the angle φ and that of the air current V_a acting under 90° are equal. This equality, however, will also hold for other temperatures of the wire as long as we can neglect radiation effects and the influences of the temperature upon the heat conductivity and the viscosity of the air, for then the cooling effect of the air current as well as that of the supports of the wire, can be assumed to be proportional to the temperature difference between wire and air. Under these circumstances the value of a_0 will depend uniquely on V_t, and will be the same function of V_t for all kinds of circuits.

4. *Variation of the coefficient a_0 with V_t.*

It has already been mentioned that the value of a_0 depends on the velocity of the air in which the wire is placed. From the calibration curves for the normal and the zero position of the wire, we can obtain this coefficient as the quotient of the velocities corresponding to the same indication of the anemometer. The curves A and B from fig. 4 show its
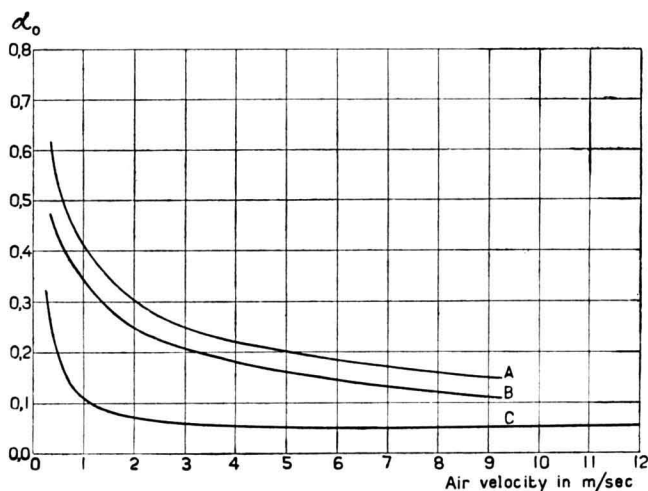

Fig. 4.

Variation of the factor a_0 with the true velocity V_t:

A = For a wire of 1,2 mm length (diam. 0,005 mm).
B = For a wire of 2,0 mm length (diam. 0,005 mm).
C = As deduced from Simmons and Bailey's results for a wire of 78 mm length (diam. 0,026 mm).

dependence on the true velocity (V_t) as deduced respectively from experiments of January and of July 1932 (for the latter, see fig. 2).

The same diagram also shows the a_0 curve calculated from SIMMONS and

BAILEY's results for a wire of 78 mm length (curve C). As, according to what has been said in the foregoing section, the same indication of the anemometer means the same heat loss per degree of temperature difference between wire and air, and thus equal values of H in equation (2) and (3), the relation between V_a and V_t is given by:

$$H_{(eq.\ 2)} = H_{(eq.\ 3)}$$

or

$$6.68 \cdot 10^{-5} V_a^{1/2} + 9.60 \cdot 10^{-5} = 1.673 \cdot 10^{-6} V_t + 10^{-4} + 7.5 \cdot 10^{-8} t.$$

For a_0 we obtain:

$$a_0 = \frac{V_a}{V_t} = \frac{(0.1673 \ V_t + 0.40 + 7.5 \cdot 10^{-3} t)^2}{6.68^2 \ V_t}.$$

It must be remarked that this formula cannot be applied to very low velocities (a_0 would grow indefinitely); this can be explained by the fact that the formulae upon which the calculation is based, have but a limited range of validity. For the calculation of curve C in fig. 4 a temperature of $400°$ C has been chosen; V_t is in ft/sec.

We see that the general character of this curve is the same as that of curves A and B. Most typical is the rapid increase of a_0 with decreasing velocities. It is not possible to determine experimentally which limiting value will be reached if V decreases infinitely to zero; it may be that in that case a_0 is equal to unity, as it might be supposed that for very low velocities the rate of heat loss does not depend on the angular position of the wire with regard to the velocity vector.

The value of a_0 calculated from the equation given by SIMMONS and BAILEY reaches a minimum for $V = 6.19$ m/sec and then increases again indefinitely. The minimum occurs in the range of velocities investigated by the authors and thus must exist in reality. We have not further given attention to this point as it was of little interest for our work; besides experiments are complicated by the circumstance that vortices are formed behind the needles between which the wire is stretched if the REYNOLDS' number is higher than about 50. A calibration curve for the hot wire of 2 mm length used for the experiments in July, in zero position and for high velocities, indeed shows a fairly sharp bend in upward direction at $V = 21$ m/sec.

There must also be an important effect of the length of the wire on its cooling if placed in the zero position. With increasing length the temperature of the air at the end of the wire will assume a higher value, and thus the mean rate of heat transfer per unit of length of the wire will decrease. This must cause a decrease of a_0 with increasing length of the wire. It can also be expected that the shorter the wire is, the sooner a_0 will increase to unity with decreasing velocities. These relations are clearly confirmed by experiment (see fig. 4).

5. *Discussion of results.*

From the results of the preceding pages can be deduced that the errors which were signalized in the first section of this communication readily may be neglected in ordinary hot wire work. If the length of the wire is not below 1 cm and the velocities which have to be measured do not decrease below a few meters per second, the presence of a velocity component parallel to the wire will not sensibly affect the indication of the anemometer. In fact under these conditions we may assume that a_0 will be at most 0.1 (comp. fig. 4). From the formula for V_a etc. it follows that the component of the velocity parallel to the wire may increase to 1.42 times the normal component, before an error of 1 % in the indication of the normal component will be introduced. This means a rotation of the velocity vector over 55°, its absolute value being in the ratio 1.75/1 to the normal component. We thus may safely say that the single hot wire anemometer within a large range is only sensitive for the value of the air speed component perpendicular to the wire.

The case is different with short wires if low velocities are to be measured. The value of a_0 then may rise e.g. to 0.4; in this case an error of 1 % in the value of the normal component would be obtained with a parallel component equal to 0.35 times the normal one, corresponding to an angle of deviation of 19°. These values are mentioned only in order to show the order of magnitude of the errors to be expected; the formula is only an approximation and the actual differences may be either smaller or greater.

In regions of the turbulent boundary layer near to the wall where the mean value of the velocity component in the general direction of flow is very low, it may happen (as the fluctuations in x, y and z direction are not necessarily in phase), that a maximum of the cross component coincides with a minimum of the other components and thus of their resultant[1]). Assuming the wire to be parallel to the wall and normal to the general flow, this means that at certain moments the component parallel to the wire may obtain a value greater than that of the component perpendicular to the wire. In that case the recorded value will differ both from the velocity in a plane normal to the wire and from the absolute value of the velocity in space. The greatest percentage differences always will be found near the wall; however, compared to the value of the velocity in the free stream, the errors will appear not very important. Oscillograms of the velocity fluctuations recorded with short wires in regions near to the

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[1]) In a work of A. FAGE and H. C. H. TOWNEND, An examination of turbulent flow with an ultra-microscope Proc. Roy. Soc. A. Vol. **135**, p. 656 1932, some experimental data are given on the distribution of turbulent velocities near a wall. The fact that the cross component of the velocity fluctuations attains a higher value than the other components is in agreement with our own observations with hot wires. It must be remarked that in some cases the magnitude of the velocity fluctuations observed by us was still greater than the values given by these authors for turbulent flow in a square pipe.

wall thus still may be considered as representative of the fluctuations of the air speed in the plane perpendicular to the wire[1]).

At the same time it can be remarked that the use of a wire mounted parallel to the direction of the general flow may give valuable information concerning the magnitude of the cross component. We hope to come back to this in a future paper.

The investigation of the directional properties of the single hot wire has not been extended further for the present, as it was considered that the data obtained were sufficient for a general discussion of its behaviour, while on the other hand the inaccuracies inherent to experimental work with thin wires, especially with the short ones, as yet do not allow the deduction of a more accurate relation.

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[1]) Such oscillograms have been reproduced in Mededeeling 23 (these Proceedings, 35, p. 419, 1932).

Mathematics. — *Ueber die henkelfreien Kontinua.* Von Dr. W. HUREWICZ. (Communicated by Prof. L. E. J. BROUWER.)

(Communicated at the meeting of October 29, 1932.)

Ein kompaktes Kontinuum C heisst *henkelfrei* ("unicoherent"), wenn bei jeder Zerlegung von C in zwei Teilkontinua ihr Durchschnitt zusammenhängend ist. Der Begriff der Henkelfreiheit und seine Verallgemeinerungen spielen eine bedeutende Rolle in den topologischen Untersuchungen der letzten Jahre[1]). Im Falle eines *lokal zusammenhängenden Kontinuums C* bedeutet nach KURATOWSKI[2]) die Henkelfreiheit, dass in C der klassische BROUWER-PHRAGMEN'sche Satz gilt, d.h. für jedes Teilkontinuum M von C sämtliche Komponenten der offenen Menge C-M zusammenhängende Begrenzungen[3]) haben. Wir wollen daher lokal zusammenhängende henkelfreie Kontinua kurz als BROUWER'*sche Kontinua* bezeichnen.

Wir beweisen nun: *Für $n \geq 3$ ist jeder n-dimensionale (separable und metrisierbare) Raum R mit einer Teilmenge eines n-dimensionalen* BROUWER'*schen Kontinuums homöomorph.*

Beim Beweis können wir R als kompakt voraussetzen, da doch jeder Raum R in einen kompakten Raum von derselben Dimension topologisch einbettbar ist[4]). Wir stützen uns auf das folgende Ergebnis des Ver-

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[1]) Vgl. u.a. KNASTER, C. R. du Premier Congrés des Mathématiciens des Pays Slaves, p. 287. Der Ausdruck "henkelfrei" stammt von VIETORIS (diese Proceedings **29** (1926) S. 440).

[2]) Vgl. Fund. Math. **8**, S. 148.

[3]) Selbstverständlich handelt es sich hier um Begrenzungen in Bezug auf C als Raum.

[4]) Vgl. meine Arbeit in den Monatsheften f. Math. u. Phys., **37**, S. 199.