Nevertheless the different vascularisation of the cortical facialis centre in lower mammals (Monotremes, Ungulates) at one side and the Primates at the other side shows that the constancy of relation between bloodvessels and cortical centra is no more valid, when comparing phylogenetically remote animals.

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**Physics.** — *The cosmic corpuscular ultra-radiation.* V. *Ionisation in the Stratosphere and in the highest layers.* By J. CLAY. (Communicated by Prof. P. ZEEMAN.)

(Communicated at the meeting of January 28, 1933).

§ 1. After having found, in our previous communication IV [1]), a basis for an explanation of the variation of intensity of the ultraradiation in the Earth's magnetic field and of the variation in hardness, we now wish to discuss, in how far the remaining phenomena are in agreement with this explanation. In the first place REGENER's [2]) splendid measurements in the stratosphere should be considered, the more so, since at first sight one might think, that for lower magnetic latitudes an ionisation as high as found by REGENER could not be expected, the reason being that, according to STÖRMER's theory, no electrons of lower energy could be incident. We expect, however, that this ionisation near the magnetic equator will be somewhat less, corresponding to the smaller number of primary rays, but that apart from this the ionisation curve will have about the same shape as found by REGENER at 50° magnetic latitude.

In the first place we may convince ourselves, that the high ionisation of e.g. 90 ions which REGENER found at an altitude of 10 km can only to a small extent be due to an increase of the intensity of the primary rays, for the following reason.

Rays from outside penetrating the atmosphere to an altitude of 10 km, have already passed through $^1/_4$ of the atmosphere and therefore originally had a minimum energy which is $^1/_4$ of the minimum energy required to reach the earth's surface or $10^9$ e. Volt. But this shifting of the lowest energy limit from $4 \times 10^9$ to $10^9$ e. Volt does not seem capable of causing an increase of primary rays by a factor 25, as would be necessary if we wish to attribute the ionisation observed by REGENER to primary rays only.

It appears, however, that the existence of such a large increase of ionisation with altitude may be expected on account of the influence of the secondary rays, if we take into account the influence which the pressure in the atmosphere should have.

In dealing with this matter, we shall not enter into the question as to in how far collisions of primary rays in the atmosphere might give rise to

[1]) Proc. Royal Acad. of Amsterdam, **35**, p. 1282, 1932.

[2]) E. REGENER. Die Naturwissenschaften, **20**, p. 695, 1932.

hard gamma rays, although it seems certain that such a process occurs occasionally. Though such gamma rays could of course only ionise by again producing corpuscular rays, some difference in the result may be expected, due to the intervening process of absorption of gamma rays. A more accurate knowledge concerning these collision phenomena will be needed, before this question can be more precisely dealt with.

§ 2. Let us consider first the barometer effect, discovered by MYSSOWSKI and TUWIM [1]). This effect is now accurately known by the researches of the Halle school under the leadership of HOFFMANN, in particular MESSERSCHMIDT's data [2]). Let us now see how we expect the intensity of radiation to vary with varying barometric préssure. We then have to distinguish between rays of such energy that they pass through the whole of the atmosphere, or eventually a much thicker layer of matter, on the one hand, and rays which can only pass through part of the atmosphere on the other hand.

For all rays the equation

$$\frac{dI}{I} = -\mu\, d(D)$$

holds, where $d(D)$ is the increase in thickness of the layer of matter given by an increase in barometric pressure of 1 centimeter of mercury. For the first group of rays, which pass through the atmosphere to its whole extent, $d(D)$ is equal to 0.136, if $\mu$ be expressed per meter of water. From this follows that for such rays the barometric effect increases proportionally to $\mu$.

Let us now, however, discuss the second group of rays, which are formed inside the atmosphere and of which the penetrating power is only a fraction of the atmosphere. Denote by $\mu_1$ the absorption coefficient for this kind of rays which corresponds to the thickness $D$ of the atmosphere. Then secondary radiation having an absorption coefficient $n\mu_1$ will have a maximum path of length $D/n$ in the atmosphere, and the maximum variation of intensity of such radiation for a variation in pressure of 0.136 meters of water will be $\frac{dI}{I} = -n\mu_1 \frac{0.136}{n} = -0.136\,\mu_1$.

If therefore the increase in pressure had no other influence than indicated above, the rays of less high energy (less hard rays) would at most have an equal, but in general a smaller barometer coefficient than the harder rays. Something quite different is observed. The barometer effect is *highest*, when the shielding is least, that is for rays of lower energy. This indicates that for higher pressure the *number* of rays of low energy decreases. This assumption is confirmed by the fact that for the same shielding but at higher altitude, that is at smaller pressure, the barometer coefficient

[1]) L. MYSSOWSKI and L. TUWIM. Zeitschrift für Physik, **39**, 146, (1926).
[2]) W. MESSERSCHMIDT. Z. f. Ph., **78**, p. 668, (1932).

increases appreciably, which means that at this lower pressure the number of secondary corpuscular rays found is greater.

For primary radiation having an absorption coefficient of 0.020 per meter of water and the hardest secondaries, of which the absorption coefficient is 0.075 per meter of water, the intensities are in the ratio 0.12 to 0.7 [1]), which leads to an average absorption coefficient of 0.066 per meter of water. For this radiation the barometer coefficient $\frac{dI}{I} = -0.066 \times 0.136 = -0.0090$ is to be expected. Accurate measurements by MESSERSCHMIDT yield a barometer coefficient of 0.0104 with a shield of 20 cm of lead. However, this shield does not exclude radiation with a somewhat larger $\mu$.

Let us further compare the measurements by MESSERSCHMIDT [2]) in Halle (110 m), in a surrounding of 75 cm pressure, with those by LINDHOLM at Muottas Muraigl at an elevation of 2456 meters and a pressure of 56.5 cm [3]). For the same shielding, the *values* found in the low pressure surrounding are largest, as is shown by the following figures. In 10 cm of lead, the barometer coefficient in Halle is 0.0184, and in Muottas Muraigl 0.0400, and without shielding 0.0192 in Halle and 0.0845 in Muottas Muraigl.

Without shielding MESSERSCHMIDT's observations of intensity deviate from proportionality for a decrease in pressure of 3 cm of mercury, in the sense that the ionisation increases more rapidly. We think that also the reason for this is that the number of secondary (tertiary etc.) corpuscles accompanying one primary *and having sufficient energy to penetrate the ionisation chamber* decreases with increasing pressure, which assumption was stated above. This may be due to the chance for the formation of a new secondary becoming smaller, as the energy of the primary decreases, which may be made plausible by the following reasoning. Assuming the effective area for a nucleus and an electron to be the same, its ratio to the effective area of the atom as a whole is as 1 to $10^8$. Since for oxygen the number of extra-nuclear electrons is 8, for nitrogen 7, we assume that the probability for the formation of a secondary ray and the probability for an ionisation have the ratio 1 to $10^7$. For the ionisation of $10^7$ atoms $3.2 \times 10^8$ e. Volt must be lost, after this also a secondary ray will be produced, on the average. That is: for a primary corpuscle of $3.2 \times 10^8$ e. Volt the probability of the formation of a secondary is 1. Further: the number of corpuscles of high energy accompanying a primary ray must be independent of the density of the gas, since, for a higher density, the number of secondaries produced increases, but their ranges diminish in the same ratio. However, this is different for corpuscles of low energy. We

[1]) The values are taken from a private communication by Prof. REGENER and differ only slightly from those published by him in Nature **127**, 233, (1931).

[2]) W. MESSERSCHMIDT. Zeitschrift für Physik, **78**, 668, (1932).

[3]) F. LINDHOLM. Gerland's Beiträge zur Geophysik, **22**, 141, (1929).

only need to consider rays having an energy larger than $10^6$ e. Volt, since $1.6 \times 10^6$ e. Volt was required in order to pass through the brass well of thickness 0.5 mm of REGENER's ionisation chamber. Suppose we have a ray of only $3.2 \times 10^6$ e. Volt, then the chance for the production of one secondary has gone down to $^1/_{100}$, and in this case the range will be diminished at higher pressure, but the chance for a replacement by a new secondary will be practically negligible. In this way one may possibly understand, why for a higher pressure the number of corpuscular rays passing through a space of 1 cc becomes less. We have, at any rate, that for lower pressure this number must be much larger, as is found experimentally (LINDHOLM's and MESSERSCHMIDT's results of barometer-effect as seen above).

Independently of the above consideration, which perhaps does not sufficiently elucidate the mechanism producing the secondary ray intensity, it is therefore experimentally certain that the number of secondaries of lower energy depends on the pressure, in the sense that it decreases for increasing pressure. For the higher layers we shall now assume, that this number is indirectly proportional to the pressure [1]), and this is the only postulate which we now use in order to discuss quantitatively REGENER's ionisation curve in the higher layers.

§ 3. Let us consider the whole of the atmosphere to be reduced to normal pressure, which is expressed in meters of water. Passing from the upper boundary downward, the intensity of primary radiation will decrease according to the well known law

$$I = I_0\, 2\pi \int_0^{\pi/2} \sin\theta\, e^{-\mu \sec\theta}\, d\theta.$$

In this formula, $I_0$ is the intensity of primary radiation incident at the upper boundary, $\mu$ the absorption coefficient of this radiation, which, according to REGENER's experiments is equal to 0.0206 per meter of water, and $H$ the depth below the upper boundary in meters of water.

The value of this integral may be obtained from GOLD's tables [2]). This primary radiation produces secondaries, with which, however, it becomes saturated only after it has passed through a certain thickness of the atmosphere. In our previous communication we found that the secondaries have an average range of 2.6 k.m. of normal air. We shall therefore assume that the primary rays are saturated with secondaries, after they have passed through a portion of the atmosphere equivalent to 3 meters of water. We may add, that the results are changed but little, if 3.5 meters or 2.5 meters is assumed, but in any case further investigation will have fix this value more precisely.

[1]) For the layers near the earth, the variation seems to be even more than this.

[2]) E. GOLD. Proceedings of the Royal Society A 82, 62, (1918).

If now, we consider a layer which is higher, we cannot have saturation, but the secondary ionisation expressed in its saturation value may then be calculated in the following manner.

Let $A$ be the point in question, at a distance $h$ below the top of the atmosphere. We surround $A$ by a sphere of radius $r$, which represent the range necessary for producing saturation. Both $h$ and $r$ are expressed in meters of water.

For secondary rays originating within an elementary solid angle $d\omega$ having its vertex at $A$, the ionisation produced in a closed vessel at $A$ is

$$k \varrho \, d \omega$$

if $\varrho$ the distance from $A$ within which the secondary rays are produced, and $k$ a factor of proportionality.

If $h \geqq r$, there is saturation at $A$, and the intensity of ionisation measured at this point is

$$2 \pi k r$$

as is found by integrating the above expression over the hemisphere.

If $h = a r$ and $a < 1$, the sphere intersects the upper boundary of the atmosphere, which is considered as a plane. The vertically shaded region represents radiation saturated at $A$, and the corresponding portion of the integral is $2 \pi k r a$. The horizontally shaded region represents radiation which is only partly saturated at $A$, and the corresponding portion of the integral is

$$k \int_0^{\alpha_1} \frac{a r}{\cos \alpha} 2 \pi \sin \alpha \, d\alpha = - 2 \pi k r a \lg a.$$

In total, the intensity of ionisation at $A$ is therefore $2 \pi k r a \ (1 - \lg a)$, and the desired ratio of secondary ionisation to its saturation value is therefore

$$a \, (1 - \lg a).$$

The assumptions involved in the above are that the secondary radiation accompanying a primary ray is proportional to the distance travelled in the atmosphere, as long as this distance is smaller than $r$, that the emission of secondaries from a volume element occurs uniformly over a hemisphere, downward, and that there are no primaries with a range smaller than $r$.

We now wish to determine the amount of primary radiation which may penetrate in the layer at $A$ from below. For this we first have to know, at which point horizontal rays may penetrate, in other words the point where the horizontal direction is equivalent to $2 \times 100$ meters of water.

To this end, we first determine the density of the air at the point in question, and then imagine all of the atmosphere above this point to be compressed to this density. This point $B$ be at $x$ meter water pressure below the top of the atmosphere, so that $x$, in general might have all values from

0 to 10.33. The density of the air at $B$ is therefore $\frac{x}{10.33}$ 0.00129. The air above $B$, which corresponds to $x$ meters of water has, if compressed to this density, a thickness of 8000 meters assuming $t = 0°$ C.

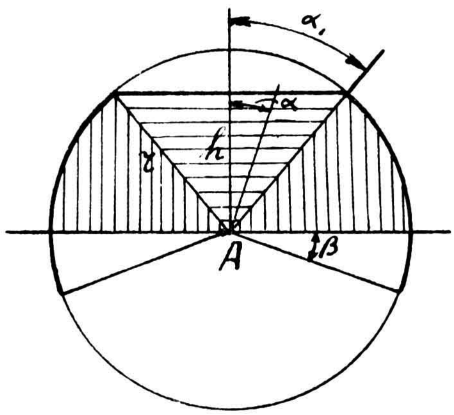


Fig. 1.

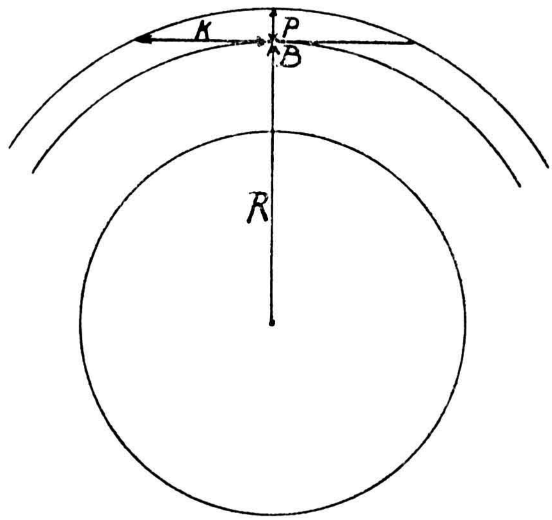


Fig. 2.

Now $p \times 2R = k^2$, where the thickness of the atmosphere has been neglected with respect to $R$. Since $R = 6300$ km, we have $k = \sqrt{2 \times 8 \times 6300} = 318$ km. The water-equivalent of these 318 km is found by multiplying this length with the reduced density assumed above, and is therefore

$$318 \times \frac{x}{10.33}\, 0.00129 = 40\,x \text{ meters.}$$

Since this has to be equal to the range of 100 meters, we find $x = 2\frac{1}{2}$. The desired layer, at which horizontal primary rays may penetrate, and which we shall name the "layer of zero dip"; is therefore situated at an altitude of about $2\frac{1}{2}$ meters of water below the top of the atmosphere, if an average energy of $3 \times 10^{10}$ e. Volt for primaries is assumed. We shall adopt the value of 10 kilometer for its altitude above the Earth's surface.

§ 4. Proceeding to greater altitudes, a certain fraction of the primary radiation comes from below, the intensity of this fraction being determined by the dip $\beta$ of the horizon. We have $\sin \beta = \sqrt{\frac{2H}{R}}$, where $H$ the altitude above the saturation layer mentioned above, and $R$ the radius of the Earth, so that $\sin \beta = 0.0178 \sqrt{H}$ if $H$ be expressed in kilometers. Since the ratio of the solid angle subtended by to the hemisphere is equal to $\sin \beta$, the intensity of the primary radiation incident from below is given by the intensity in the layer of zero dip, multiplied by *sin* $\beta$. This amount is to be added to the intensity of the primary radiation incident from above.

TABLE I

Ionisation in a closed vessel in the stratosphere.

| meter water aequivalent below top of atmosphere | pressure in mm. Hg | altitude km. above sealevel | (*a*) primary intensity calculated (Gold). | (*b*) $a(1-\lg . a)$ | (*c*) dip of horizon | $ab+c$ | $\frac{1}{p}$ | ionisation calculated in arbitrary units | observed ionisation (Regener) | ratio $\times$ 100 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 760 | 0 | 0.574 | | | 0.574 | 1 | 0.574 | | |
| 9 | 684 | 0.7 | 0.602 | | | 0.602 | 1.1 | 0.662 | | |
| 8 | 608 | 1.6 | 0.630 | | | 0.630 | 1.25 | 0.787 | | |
| 7 | 532 | 2.6 | 0.657 | | | 0.657 | 1.43 | 0.940 | | |
| 6 | 456 | 3.7 | 0.690 | | | 0.690 | 1.67 | 1.152 | | |
| 5 | 380 | 5.1 | 0.722 | | | 0.722 | 2.0 | 1.444 | | |
| 4 | 304 | 6.7 | 0.761 | | | 0.761 | 2.5 | 1.902 | 53 | 36 |
| 3 | 228 | 8.9 | 0.804 | 1 | 0 | 0.804 | 3.33 | 2.677 | 77 | 35 |
| 2.7 | 205 | 9.7 | 0.818 | 0.995 | 0.016 | 0.830 | 3.70 | 3.071 | 90 | 34 |
| 2.4 | 182 | 10.5 | 0.833 | 0.978 | 0.023 | 0.838 | 4.17 | 3.494 | 109 | 32 |
| 2.1 | 160 | 11.5 | 0.849 | 0.949 | 0.029 | 0.835 | 4.76 | 3.975 | 133 | 30 |
| 1.8 | 137 | 12.7 | 0.865 | 0.906 | 0.035 | 0.819 | 5.56 | 4.554 | 177 | 26 |
| 1.5 | 114 | 14.1 | 0.882 | 0.846 | 0.041 | 0.787 | 6.67 | 5.249 | 203 | 26 |
| 1.2 | 91 | 15.8 | 0.901 | 0.766 | 0.047 | 0.737 | 8.33 | 6.139 | 233 | 26 |
| 0.9 | 68 | 18.0 | 0.920 | 0.661 | 0.054 | 0.662 | 11.11 | 7.355 | 248 | 30 |
| 0.6 | 46 | 21.0 | 0.942 | 0.522 | 0.062 | 0.554 | 16.67 | 9.235 | 264 | 35 |
| 0.3 | 23 | 26.2 | 0.964 | 0.333 | 0.074 | 0.392 | 33.33 | 13.065 | 270 | 48 |
| 0.15 | 11 | 31.9 | 0.975 | 0.120 | 0.085 | 0.203 | 66.66 | 13.534 | 273 | 49 |

The intensity ratios, calculated at different heights, are given in column 9 of Table 1, the ionisations observed by REGENER are given in column 10, however for the first value in this column KOLHÖRSTER's observation [1]) has been used, since REGENER's values do not extend so far downward. For the lower altitudes, our calculation yields too small values. For the highest altitude the calculated value is too high [2]), which may be due, either to unknown factors, or to the fact that the saturation with secondary rays decreases more rapidly than has been computed. However, for the whole of

[1]) W. KOLHÖRSTER. Verh. d. Phys. Ges. **16**, p. 719, (1914).

[2]) Note added during the correction:

According to a private communication by REGENER, his most recent measurements yield for the ionisation in the highest layer the values 333 I in stead of 273 I, so that now the result of the calculation for the highest layers is in better agreement with observation. The values given in the table were those taken from REGENER's curve in the paper cited.

the region where the saturation with secondary rays may be considered complete, the calculated relative values are in sufficient agreement with REGENER's measurements. We therefore conclude, that the assumptions we made are in the main not in contradiction with stratospheric measurements of ionisation in closed vessels.

§ 5. Having obtained, in this way, a suitable means to account for the relative ionisation curve for measurements in a closed vessel in the upper troposphere and the stratosphere, as far as data are known, we may now follow the same method for investigating ionisation and conductivity in the highest layers of the free atmosphere, no further assumptions or data being needed.

In the very highest layers, secondary radiation produced by the primaries incident from above may be expected to be practically absent. It is at this point, that the insight just obtained concerning the dip of the horizon, helps us out. The peculiar situation is, that in the highest layers *nearly all of the secondary radiation comes from below.* The number of these secondaries coming from below increases with altitude. In the second column of Table 2, the influence of the dip of the horizon is expressed. We have seen above that the solid angle of the horizontal dip, expressed in the hemisphere

TABLE II

Conductivity of the upper atmosphere.

| altitude $H$ km. | *sin.* $\beta$ $\beta$=dip of horizon above 10 km. | $p$ in mm. Hg | $\frac{\text{ionic number } H}{\text{ionic nr. at 10 km.}}$ | $\frac{\text{conductivity } H}{\text{cond. at 10 km.}}$ | conductivity E.S.U. $\times 10^7$ |
|---|---|---|---|---|---|
| 10 | | 200 | 1 | 1 | $0.0_{9}691$ |
| 20 | 0.0564 | 50.2 | 2.81 | 1.118 . 10 | $0.0_{8}772$ |
| 30 | 0.0797 | 10.6 | 4.65 | 8.79 . 10 | $0.0_{7}607$ |
| 40 | 0.0976 | 2.29 | 8.52 | $7.44 \, . \, 10^{2}$ | $0.0_{6}514$ |
| 50 | 0.1127 | 0.496 | 4.54 . 10 | $1.83 \, . \, 10^{4}$ | 0.0000126 |
| 60 | 0.1260 | 0.108 | $2.33 \, . \, 10^{2}$ | $4.32 \, . \, 10^{5}$ | 0.000298 |
| 70 | 0.1380 | 0.024 | $1.15 \, . \, 10^{3}$ | $9.58 \, . \, 10^{6}$ | 0.00662 |
| 80 | 0.1491 | 0.005 | $5.96 \, . \, 10^{3}$ | $2.38 \, . \, 10^{8}$ | 0.164 |
| 90 | 0.1594 | 0.0013 | $2.45 \, . \, 10^{4}$ | $3.77 \, . \, 10^{9}$ | 2.61 |
| 100 | 0.1691 | 0.00040 | $8.46 \, . \, 10^{4}$ | $4.23 \, . \, 10^{10}$ | 29.2 |
| 120 | 0.1869 | 0.000118 | $3.17 \, . \, 10^{5}$ | $5.37 \, . \, 10^{11}$ | 371 |
| 140 | 0.2032 | 0.000073 | $5.57 \, . \, 10^{5}$ | $1.53 \, . \, 10^{12}$ | 1060 |
| 160 | 0.2182 | 0.00005 | $8.73 \, . \, 10^{5}$ | $3.49 \, . \, 10^{12}$ | 2410 |
| 200 | 0.2457 | 0.000024 | $2.047 \, . \, 10^{6}$ | $1.71 \, . \, 10^{13}$ | 11810 |

as a unit, is given by $sin\ \beta$. Its value is given in the second column, and the intensity of the secondaries is proportional to this. To this intensity, of course, the action of primaries coming from above and from below, must be added. So the intensity of primaries increases with altitude for two reasons.

In calculating the number of ions in the higher layers of the atmosphere, we may most suitably express it in the ionisation found by REGENER at 10 km as unit. Let $q$ be the number of ions per cc per second produced by radiation in the free atmosphere. In the state of equilibrium, we have $q = \alpha n^2$, where $\alpha$ the coefficient of recombination for the local pressure. Hence $n = \sqrt{\frac{q}{\alpha}}$. Now, since both $q$ and $\alpha$ are proportional to the pressure, $n$ will be independent of the pressure, for a given number of ionising rays.

The specific conductivity of the air is given by $\lambda = n\, e\, u$, where $e$ the elementary charge and $u$ the mobility. Since, for small pressures, $u$ is indirectly proportional to the pressure, $\lambda$ also increases indirectly proportionally to the pressure. We now are able to calculate the conductivity at various heights in the atmosphere. Its calculated value is given by the last column of Table 2. The values for the pressure used are those which STÖRMER has calculated according to assumptions by JEANS, and are given in column 3. They were taken from VEGARD's treatise in the Handbuch der Experimentalphysik XXV, p. 435.

For our calculation we start from the conductivity measured at an altitude of 10 km as a basis. For altitudes of 20 and 30 km we yet have to take into consideration the ionisation by secondaries from higher layers, but at 40 km and higher, this radiation is no more of any importance. For these higher altitudes, all of the secondary radiation comes from the side of the Earth. The interesting and rather unexpected point about the result is that the conductivity, somewhere between 80 and 90 km rises rapidly, and reaches, at 90 km a value corresponding to the conductivity of the humid Earth. Before this, BENNDORF [1]) had carried out similar calculations and reached about the same conclusions. However, at that time very few data concerning the intensity of ultraradiation at high altitudes were available, whereas at present the foundation is more certain.

The foregoing shows that the secondary radiation coming from below is capable of giving a complete explanation of the high conductivity at very large altitudes, and that our assumptions are suitable for calculating its magnitude.

*Amsterdam*, January 18, 1933.

*Natuurkundig Laboratorium.*

[1]) H. BENNDORF. Phys. Zs. 27, p. 686, (1926).