

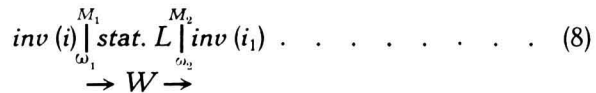




there are two invariant liquids  $i$  and  $i_1$ , a variable liquid  $L$  and two membranes with the surfaces  $\omega_1$  and  $\omega_2$ . As we shall begin by assuming besides that both membranes are permeable for water only, it is of no importance for our further considerations whether these three liquids contain the same substances yes or no. We can keep the liquids  $i$  and  $i_1$ , practically invariant by refreshing them at short intervals during the osmosis or by conducting a current of these liquids along the membranes.

As the two membranes are permeable for water only, so that the variable liquid can only take in or give off water, the concentrations of the substances  $X, Y, Z$  etc. present in it, will, therefore, change continuously during the osmosis in such a way that their ratio remains the same. Consequently, when in system (7) we start from a definite liquid  $L$ , we only have to consider its  $W$ -amount as an independent variable.

If we now leave system (7) alone, the variable liquid will at last get a definite composition, which will not change any more during the further osmosis; then a stationary state has set in, which we represent by :



Then the same quantity of water will flow through the one membrane as through the other, so that not only the composition but also the quantity of the stationary liquid will change no more. If for the sake of concentration we assume that liquid  $i_1$ , has a greater  $O.W.A.$  than liquid  $i$ , the water will diffuse according to the arrows in (8).

In order to elucidate this we imagine that per second  $\gamma_1$  and  $\gamma_2$  quantities of water diffuse  $\rightarrow$  through  $1 \text{ cM}^2$  of the membranes  $M_1$  and  $M_2$  respectively; then liquid  $L$  will change no more, when

$$\omega_1 \gamma_1 = \omega_2 \gamma_2 \dots \dots \dots (9)$$

The quantity  $\gamma_1$  depends upon the composition of liquid  $i$  and the  $W$ -amount  $= w$  of liquid  $L$ ; the quantity  $\gamma_2$  depends upon the composition of liquid  $i_1$  and the  $W$ -amount  $= w$  of liquid  $L$ ; consequently we may put :

$$\gamma_1 = \varphi_1(i, w) \quad \text{and} \quad \gamma_2 = \varphi_2(w, i_1) \dots \dots \dots (10)$$

in which  $\varphi_1$  besides contains the magnitudes, determining the nature of the membrane  $M_1$  and  $\varphi_2$  those of  $M_2$ . It now appears from (9) and (10) that

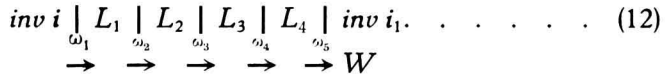
$$\omega_1 \varphi_1(i, w) = \omega_2 \varphi_2(i_1, w) \dots \dots \dots (11)$$

must be satisfied, so that the  $W$ - amount  $= w$  and also, therefore the composition of the stationary liquid  $L$  has been completely determined. From this it follows that the composition of the stationary liquid depends upon :

- 10. the compositions of the invariant liquids  $i$  and  $i_1$ .
- 20. the nature of the two membranes and the ratio  $\omega_1 : \omega_2$  of their surfaces.
- 30. the ratio of the concentrations of the substances  $X, Y, Z$  etc. in the original variable liquid, which is the same in the stational liquid of course.

Later on we shall see that what has been mentioned sub 3 does not play a part, when the membranes are permeable for all substances.

Corresponding considerations obtain also for systems with 2 and more variable liquids. If e.g. we leave to itself the system :

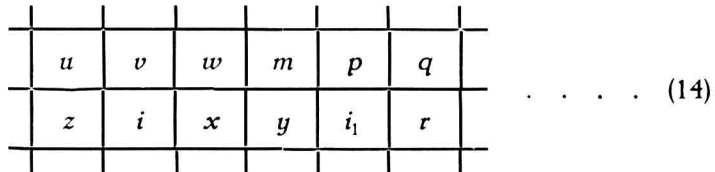


then a stational state will set in here also, in which the same quantity of water will flow through each of the 5 membranes, so that not only the composition, but also the quantity of each of these 4 liquids will change no more. The  $W$ -amount and consequently also the composition of each of these 4 liquids is defined by the 4 equations :

$$\omega_1 \gamma_1 = \omega_2 \gamma_2 = \omega_3 \gamma_3 = \omega_4 \gamma_4 = \omega_5 \gamma_5. \dots \dots (13)$$

in which  $\gamma_1, \gamma_2$  etc. represent similar functions as in (10).

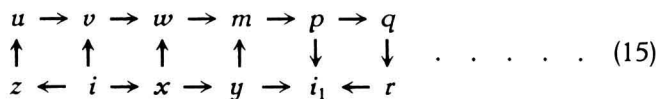
We may also extend these considerations to systems in which the liquids are in any way in contact with one another as e.g. in the system :



Here we find besides the invariant liquids  $i$  and  $i_1$ , the 10 variable liquids  $x, y, z$  etc. besides. We may imagine that these var. liquids are enclosed in spaces with walls, either elastic or not elastic and that the invariant liquids  $i$  and  $i_1$  move perpendicularly to the plane of drawing. For the sake of simplicity we shall call a similar system a tissue of cells.

It is easy to deduce now that a stationary state will set in also in these tissues. Then a continuous current of water will flow from  $i$  towards  $i_1$  and in such a way that through every cell as much water will flow in as flows out, so that the composition and the quantity of each of the liquids will change no more. From this also follows that [as e.g. is the case in system 12] an equal quantity of water will no longer flow through each membrane.

Omitting the walls of the cells we now shall represent (14) by :



in which the arrows indicate the directions in which the water moves. As however, these directions etc. depend upon the nature and the surfaces of the membranes etc., (15) only represents one of the many cases possible.

It would be possible to determine the partition of the current of the water in (15), if  $\gamma_1, \gamma_2$  etc. were known. As, however, these functions are unknown, we shall suppose that the quantity of water, diffusing per second through 1 cM<sup>2</sup> of a membrane, is proportional with the difference of the *O.W.A.*'s of the two adjacent liquids<sup>1)</sup>.

Although of course this supposition is quantitatively incorrect, we shall yet use it in order to get some idea of the phenomena.

For the sake of simplicity we shall now assume that the letters in (14) and (15) indicate not only the liquids, but also their *O.W.A.* We shall assume besides that the water diffuses according to the arrows in (15); if this should not be the case, the results will tell us.

We can represent the quantity of water, which liquid *x* takes in per second from liquid *i* by :

$$\omega K(x - i) \dots \dots \dots (16)$$

in which  $\omega$  is the surface of the membrane between *x* and *i* and *K* is a constant, determined by the nature of the membrane. As has been assumed in (15) liquid *x* gives off water to *y* and *w*; this quantity is per second :

$$\omega_1 K_1(y - x) + \omega_2 K_2(w - x) \dots \dots \dots (17)$$

As the quantity of water taken in and given off must be the same, (16) and (17) must be equal to one another. If for the sake of simplicity we now suppose that all membranes have the same surface and the same nature, so that  $\omega K = \omega_1 K_1 = \omega_2 K_2$ , then follows :

$$x - i = y - x + w - x \text{ or } 3x - y - w = i \dots \dots (18)$$

Acting in a corresponding way for each of the 10 cells, we find the 10 equations (19)

$$\left. \begin{array}{ll} 3x - y - w = i & (x) \quad -x - v + 3w - m = 0 \quad (w) \\ -x + 3y - m = i_1 & (y) \quad -y + 3m - p - w = 0 \quad (m) \\ 2z - u = i & (z) \quad -m + 3p - q = i_1 \quad (p) \\ -z - v + 2u = 0 & (u) \quad -p + 2q - r = 0 \quad (q) \\ -u + 3v - w = i & (v) \quad 2r - q = i_1 \quad (r) \end{array} \right\} (19)$$

The letters placed between parentheses indicate for which cell an

---

1) For binary liquids we might also take the *W*-amount instead of the *O.W.A.*'s; for liquids with 3 or more substances this is not possible, however. Two liquids *L* and *L'*, containing the same substances *W, X, Y* etc. and having the same *W*-amount, will generally yet have a different *O.W.A.* so that also in system *L|L'* water will diffuse.

equation obtains. Solving these equations, we find the 10 relations (20)

$$\left. \begin{aligned} 93 x &= 61 i + 32 i_1 & 93 v &= 78 i + 15 i_1 & 93 q &= 10 i + 83 i_1 \\ 93 y &= 32 i + 61 i_1 & 93 w &= 58 i + 35 i_1 & 93 r &= 5 i + 88 i_1 \\ 93 z &= 88 i + 5 i_1 & 93 m &= 35 i + 58 i_1 & & \\ 93 u &= 83 i + 10 i_1 & 93 p &= 15 i + 78 i_1 & & \end{aligned} \right\} \quad (20)$$

by which the *O.W.A.* of each of the 10 cells is determined. We see, as was indeed to be expected, that each of these *O.W.A.*'s is greater than *i* but smaller than *i*<sub>1</sub>. From this it follows:

$$x - i = 32 (i_1 - i) : 93 ; y - x = 29 (i_1 - i) : 93 ; \text{etc.}$$

If everywhere the factor  $(i_1 - i) : 93$ , occurring in all these results is omitted, we find the 16 relations (21)

$$\left. \begin{aligned} x - i &= 32 & u - z &= 5 & w - x &= 3 & i_1 - p &= 15 \\ y - x &= 29 & v - u &= 5 & m - w &= 23 & q - p &= 5 \\ i_1 - y &= 32 & v - i &= 15 & m - y &= -3 & r - q &= 5 \\ z - i &= 5 & w - v &= 20 & p - m &= 20 & i_1 - r &= 5 \end{aligned} \right\} \quad (21)$$

The quantity of water diffusing from *i* towards *x* or from *x* towards *y* per second through 1 cM<sup>2</sup> of the membranes is:

$$\omega K (x - i) \text{ or } \omega K (y - x)$$

From this it follows that we may find the quantity of water diffusing per sec. through each of the membranes by multiplying the numbers of (21) with  $\omega K (i_1 - i) : 93$ .

From this it appears that all arrows in (15) indicate the correct direction, except those between *m* and *y*; as *m*—*y* is negative, the direction must be turned about. Then we get for the scheme of the movement of the water:

$$\begin{array}{cccccccc} & 5 & 20 & 23 & 20 & 5 & & \\ u & \rightarrow & v & \rightarrow & w & \rightarrow & m & \rightarrow & p & \rightarrow & q \\ 5 \uparrow & & 15 \uparrow & & 3 \uparrow & & \downarrow 3 & & \downarrow 15 & & \downarrow 5 & \dots \\ z & \leftarrow & i & \rightarrow & x & \rightarrow & y & \rightarrow & i_1 & \leftarrow & r \\ & 5 & & 32 & & 29 & & 32 & & 5 & \end{array} \quad (22)$$

The numbers placed with the arrows indicate the ratio of the diffusing quantities of water; [if we suppose these numbers multiplied by  $(i_1 - i) \omega K : 93$ , we have the quantities diffusing per second]. From this it appears that through every membrane does not flow the same quantity of water, that every cell takes in as much water as it gives off and that all the water given off to the tissue by the one invariant liquid, is taken in by the other invariant liquid.

Above we have assumed that all membranes have the same surface ( $\omega$ ) and the same nature ( $K$ ). Of course every change in a  $\omega$  or  $K$  of one or more of the membranes will also cause a change in the *O.W.A.* of every cell and of the partition of the current of the water. Let us imagine e.g. that between  $i$  and  $v$  of (14) the membrane becomes less permeable, then  $K$  will become smaller for this membrane; if for the sake of simplicity we imagine the membrane impermeable, then its  $K$  becomes  $= 0$ . As cell  $v$  will then no longer be in osmotic contact with  $i$  but only with  $u$  and  $w$ , relation ( $v$ ) of (19) must be replaced now by :

$$v - u = w - v \quad \text{or} \quad -u + 2v - w = 0 \quad . . . . \quad (23)$$

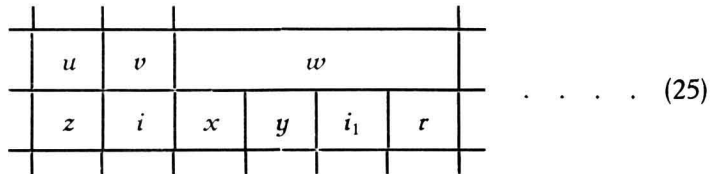
all other relations remain, however.

If  $x, y$  etc. are now calculated once more, we find that the *O.W.A.* of each cell and the partition of the current of the water in the tissue have changed. We find this partition in the scheme :

$$\begin{array}{cccccc}
 & 25 & 25 & 43 & 40 & 10 \\
 u & \rightarrow & v & \rightarrow & w & \rightarrow & m & \rightarrow & p & \rightarrow & q \\
 25 \uparrow & & | & 18 \uparrow & & \downarrow 3 & \downarrow 30 & \downarrow 10 & . & . & . & . \\
 z & \leftarrow & i & \rightarrow & x & \rightarrow & y & \rightarrow & i_1 & \leftarrow & r \\
 & & 25 & & 82 & & 64 & & 67 & & 10
 \end{array} \quad (24)$$

In order to learn from this the quantities of water diffusing per second, we have to multiply these numbers by  $\omega K (i_1 - i) : 213$ .

We now imagine in (14) the walls between the cells  $w, m, p$  and  $q$  taken away; we then get a tissue which we represent by



If we now assume the same rules for this again as for (14), we find for the partition of the current of the water the scheme

$$\begin{array}{cccccc}
 & 32 & 128 & & & \\
 u & \rightarrow & v & \rightarrow & w & \rightarrow & w & \rightarrow & w \\
 32 \uparrow & 96 \uparrow & 69 \uparrow & 17 \downarrow & 120 \downarrow & \downarrow 60 & . & . & . \\
 z & \leftarrow & i & \rightarrow & x & \rightarrow & y & \rightarrow & i_1 & \leftarrow & r \\
 & & 32 & & 155 & & 86 & & 103 & & 60
 \end{array} \quad (26)$$

In order to learn from this the quantities of water diffusing per second, these numbers must be multiplied by  $(i_1 - i) \omega K : 344$ .

