Physics. — *Intensity measurements with a reflection echelon.* By J. H. GISOLF and P. ZEEMAN.

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To observe hyperfine structures one is nearly always forced to use interference apparatus. Neither the distances nor the intensities of the satellites of a hyperfine structure are reproduced directly by an interference apparatus. The true data are obtained after calculations in which enter the caracteristic features of the apparatus.

The intensity measurement is more complicated than the frequency measurement because all observations are made photographically and the observed densities must be converted in intensities. The different methods of the photographical photometry are discussed by ORNSTEIN, MOLL and BURGER: "Objektive Spektralphotometrie." [1]).

For the hyperfine structures the problem is in some respect simplified, because the density curves are the same for the different components in the very small wavelength region of a hyperfine structure. Very few intensity measurements on hyperfine structures have been made. In most cases only rough estimations are given.

SCHÜLER and JONES described a method which avoids the density curves, but which can only be applied when the intensity of the light source is rigorously constant. The time of exposure is varied in such a way that different components become equal in density [2]).

In the following pages will be described a method to measure the relative intensities of hyperfine structure components with the help of a reflection echelon. The essence of the method, which eliminates the photographic photometry and which does not want a lightsource of constant intensity, is given by MERTON for a transmission echelon [3]).

The intensity of a diffraction maximum of an echelongrating, depends very strongly on the angle of diffraction. The intensity distribution in the diffraction pattern of a grating is given by the formula:

$$I = I_0 \frac{\sin^2 X}{X^2} \cdot \frac{\sin^2 NY}{\sin^2 Y} \quad \text{. . . . . (1)}$$

where X is the phase difference of the extreme rays of each diffracting aperture, and Y the phase difference of the corresponding rays of the

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1) Samlung Vieweg, Heft 108/109.

2) SCHÜLER and JONES, Zs. für Phys. **74**, 631 (1932).

3) T. R. MERTON, Proc. Roy. Soc. A **97**, 307 (1920).

successive apertures. N is the number of apertures. The factor $\sin^2 X/X^2$ gives the intensity distribution of one single diffracting aperture. The width of the apertures of an echelon grating is equal to the step s, thus:

$$X = \frac{\pi s \sin\theta}{\lambda} \quad \ldots\ldots\ldots \quad (2)$$

where X is the angle of diffraction.

The expression $\sin X/X$ has a maximum-value when $\theta = 0$, and zero-value when $\theta = \pm\frac{\lambda}{s}$. The angle difference of two successive maxima of the factor $\sin NY/\sin Y$ is $\triangle\theta = \frac{\lambda}{s}$. We see therefore two interference orders in the region between $\theta_1 = -\frac{\lambda}{s}$ and $\theta_2 = +\frac{\lambda}{s}$, except when $\sin NY/\sin Y$ has a maximum at $\theta = 0$, for then the next orders occur when $\theta = \pm\frac{\lambda}{s}$, where the factor $\sin X/X = 0$. In this case we say that the line is observed in the single-order position.

In case of a reflection echelon, the directions of the diffraction maxima are found from the equation:

$$2t - s\theta - t\theta^2/2 = m\lambda. \quad \ldots\ldots\ldots \quad (3)$$

where s is the step, t the thickness of the echelon plates and m the order of interference[1]).

In this formula the wave length λ depends on the density of the air. The echelon is therefore placed in an airtight box with a large window of quartz. The directions of the maxima are displaced by changing the pressure inside the box.

The refractive index for a pressure P and a temperature T is given by:

$$n_T^P - 1 = \frac{(n_0^{76} - 1)}{(1 + aT)} \frac{P}{76} \quad \ldots\ldots\ldots \quad (4)$$

where $a = 0{,}00367$.

The change of refractive index $\triangle n$ with a change of pressure $\triangle P$ is

$$\triangle n = \frac{(n-1)\triangle P}{76} \quad \ldots\ldots\ldots \quad (5)$$

The corresponding displacement of the maxima is found from (3):

$$\frac{d\theta}{dn} = \frac{m\lambda_0}{n^2} \cdot \frac{1}{(s + t\theta)}.$$

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1) An extensive discussion of the theory and the use of the reflexion echelon is given by:

W. E. WILLIAMS: Proceedings of the Optical Convention 1926 part II page 982 and Proc. of the Phys. Soc. **45**, 699 (1933).

$(n-1)$ and θ being very small, this can be reduced to:

$$\frac{d\theta}{dn}=\frac{m\lambda}{s}=\frac{2t}{s},$$

thus:

$$\delta\theta=\frac{2t}{s}\frac{(n-1)\,\triangle P}{76} \quad \ldots \ldots \ldots \quad (6)$$

The intensities of the diffraction maxima depend on the diffraction angle according to:

$$I=\frac{I_0 \sin^2 \frac{\pi s}{\lambda}\theta}{\left(\frac{\pi s}{\lambda}\theta\right)^2} \quad \ldots \ldots \ldots \quad (7)$$

To measure the relative intensities of two satellites λ_1 and λ_2, the pressure in the echelon box is changed untill the maxima of λ_1 and λ_2 have the same intensity. Denote the corresponding diffraction angles by θ_1 and θ_2. Then the relative intensities $I_0^{(1)}$ and $I_0^{(2)}$ are given by:

$$\frac{I_0^{(1)}}{I_0^{(2)}}=\frac{\sin^2 \frac{\pi s}{\lambda}\theta_2}{\sin^2 \frac{\pi s}{\lambda}\theta_1}\cdot\frac{\theta_1^2}{\theta_2^2} \quad \ldots \ldots \ldots \quad (8)$$

The angles θ_1 and θ_2 can be measured with the barometer in the following manner. First a pressure is chosen so that one of the satellites shows two orders of equal density. In this position the angle θ is $\lambda/2s$. With this value of θ as a starting point, the diffraction angles for each other pressure may be calculated from (6). The pressure can be measured sufficiently accurate with an ordinary mercury barometer. The photometer is only used to indicate equal density of two components of one pattern. To find the pressure for which the components become exactly equal, it will be allowed to use a linear intrapolation within a region of some per cent.

In stead of measuring the pressures one might measure the displacements of the maxima. However the superposition of the own intensity distribution of the line, and the intensity curve of the echelon, makes the line asymetrical in the steep parts of the distribution curve of the echelon. For broad lines it is possible to estimate the width of the lines from this displacement of the center of gravity [1]).

The Amsterdam laboratory possesses a reflection echelon of 25 plates with a thickness $t=7.05$ mm and a step $s=1$ mm. The echelon is used in a Littrow Mounting with a 170 cm quartzfluorite doublet. Both the echelon and the lens are made by Adam Hilger Ltd.

The photograms are made with a Zeiss microphotometer.

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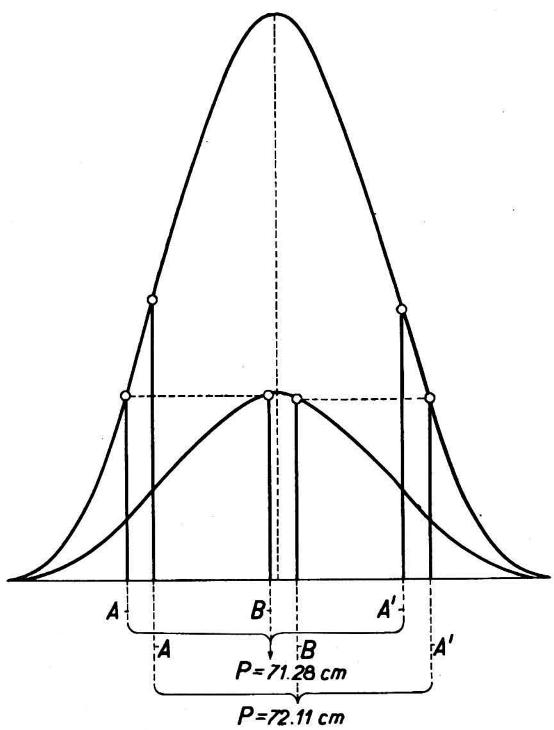
[1]) T. R. MERTON, l.c.

As an example we give the intensity measurements of the satellites of the Cadmium line $\lambda\,4678\ {}^3P_0$—${}^3S_1$. This structure consists of one strong component X, originating from the even isotopes, and two faint satellites A and B, coming from the odd isotopes with $I = 1/2$. The structure is photographed at different pressures in the echelon box. The pressures are sought for which $A = A'$, $A = B$ and $A' = B$. The positions of the satellites in the distribution curve of the echelon are indicated in the figure by A and B.



As B is nearly exactly in the one-order position, there is only one order of B visible. The corresponding photograms are shown in the plate.

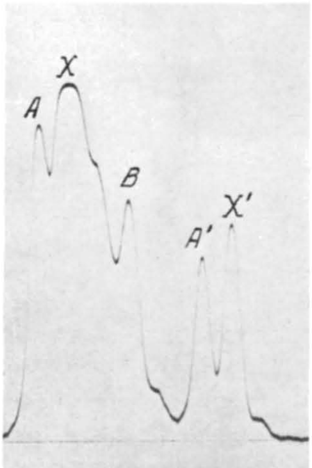
The strong component X stands in the centre of gravity of A and B. The positions of A and B with respect to X are: $A = -0.137\ \text{cm}^{-1}$, $B = +0.260\ \text{cm}^{-1}$.

Two series of measurements gave the following four values for the relative intensity.

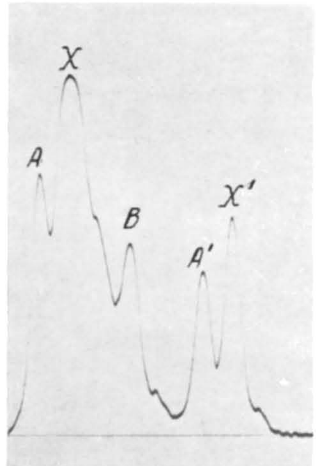
$$I_A/I_B = 2{,}82$$
$$2{,}95$$
$$2{,}92$$
$$\underline{3{,}05}$$
$$\text{mean } I_A/I_B = 2{,}94$$

The component X is much stronger than A and B. In the same manner

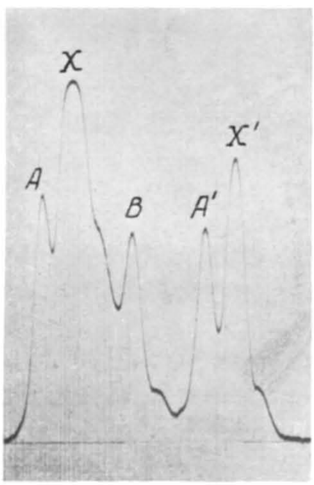
J. H. GISOLF AND P. ZEEMAN: INTENSITY MEASUREMENTS WITH A REFLECTION ECHELON.



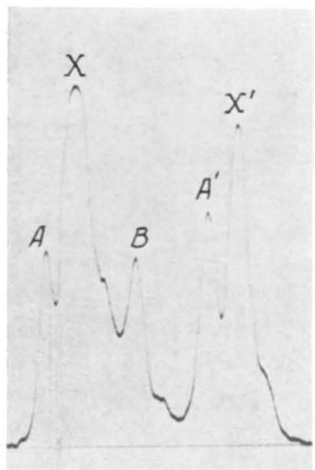
P = 73.08 cm



P = 72.57 cm



P = 72.06 cm



P = 71.38 cm

we can measure the relative intensities of A and B with respect to X. The following values were found:

$$I_A/I_X = 0{,}162 \qquad I_B/I_X = 0{,}052.$$

These give for the relative intensity of A and B the value 3.07, in perfect agreement with the former value. It is rather difficult to find a check on these measurements. In the older literature the intensity ratio of these satellites is always estimated as 3, while the intensity rules for the hyper-multiplets give the value 2. SCHÜLER found however that the relative intensity depends very strongly on the type of lightsource used [1]). This discrepency is not a matter of absorption in the light source, because this would smooth the intensity differences down. The older investigations were made on the Cadmium vapour arc. SCHÜLER found that in a cooled hollow cathode the relative intensity diminishes to 2 in agreement with the value expected by theory. We used a low-voltage arc with a tension of 17 volts and a alternating current of 2 amp., while the diameter of the tube was 1.5 cm. The (Osram) tube had a vacuum-jacket, to prevent self-reversal of the lines.

The method just described is build upon the intensity distribution curve of the echelon. How far is this distribution actually given by the equation (7)? BURGER and VAN CITTERT determined the shape of this curve for a transmission echelon, by a photographical photometric method [2]). They found a fair agreement for the upper part of the curve. At the foot of the curve on the contrary they found deviations of some hundreds per cent.

We have determined the relative intensities of X with respect to the faint components, in the foot of the curve. If the real intensity curve of the reflexion echelon deviates from the theoretical, the fault must be much smaller than the deviations found by BURGER and VAN CITTERT. This will be tested by measuring isotope concentrations under varying conditions of lightsource, temperature and pressure.

A further difficulty one always meets when measuring the intensities of satellites, is the superposition of the different satellites. To account for this effect it is necessary to know the shape of a single line. With the described method the intensity distribution in a line can be constructed without difficulty. However, a faint ghost of the grating at the long wave number side must be taken into account.

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[1]) H. SCHÜLER and J. E. KEYSTON, Zs. für Phys. **71**, 413 (1931).

[2]) H. C. BURGER and P. H. VAN CITTERT, Versl. Kon. Akad. Amsterdam **29**, 394 (1920).