be left unchanged. Both properties can be changed independently by altering different parts of the molecule.

A number of substances is found only having growth stimulating properties, but not showing polar transport in the plant. These substances do not act in the avena bending test, nor do they show the regulative power of the auxins for example in bud inhibition.

A substance showing the highest cell elongation capacity is found in allo-cinnamic acid, which can be compared with that of β indole-acetic acid.

The steric isomer cinnamic acid has no activity.

A preliminary study was made of the effect of substitution on the cell elongation property.

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Astronomy. — *The theorem of minimum loss of energy due to viscosity in steady motion and the origin of the planetary system from a rotating gaseous disc.* By H. P. BERLAGE Jr. (Royal Magnetical and Meteorological Observatory Batavia).

(Communicated at the meeting of September 28, 1935).

Recent papers by GUSTAF STRÖMBERG [1]) and G. DEDEBANT, PH. SCHERESCHEWSKY and PH. WEHRLÉ [2]) suggested to me that with the aid of a hydrodynamical theorem due to HELMHOLTZ and KORTEWEG, the solution could be found of a problem, which has occupied me ever since I became convinced that the planetary system has evolved from a gaseous disc surrounding the sun [3]).

As was previously shown [4]), the equilibrium of a rotating gaseous

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1) G. STRÖMBERG, The origin of the galactic rotation and of the connection between physical properties of the stars and their motion, Astroph. J. **79**, 460 (**1934**); Formation of galaxies, stars and planets, Astroph. J. **80**, 327 (1934).

2) G. DEDEBANT, PH. SCHERESCHEWSKY and PH. WEHRLÉ, Sur une classe de mouvements naturels de fluides visqueux caractérisée par un minimum de la puissance dissipée, C.R. **199**, 1287 (1934).

3) These Proceedings **33**, 614 (1930); **33**, 719 (1930).

4) These Proceedings **35**, 554 (1932).

This theorem may be applied to our solar envelope in steady motion, although it is not liquid but gaseous, because the gas does not expand or contract during its pure rotational motion, so that its compressibility does not enter into question.

If u, v, w are the velocity components and η the coefficient of viscosity, the rate at which kinetic energy is converted into heat is expressed by the dissipation function

$$\left.\begin{aligned}\frac{dE}{dt}=\iiint\eta\Big\{2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\\+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\Big\}dx\,dy\,dz\end{aligned}\right\} \quad . \quad (3)$$

As the motion is in planes parallel to the equator, we have

$$w=0,\frac{\partial u}{\partial z}=\frac{\partial v}{\partial z}=\frac{\partial w}{\partial z}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the dissipation function reduces to

$$\frac{dE}{dt}=\iiint\eta\left\{2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right\}dx\,dy\,dz \quad . \quad (5)$$

A transformation to cylindrical coordinates r, φ, h makes

$$u=-\omega\, r \sin\varphi \quad , \quad v=\omega\, r\cos\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Moreover

$$\left.\begin{aligned}\frac{\partial u}{\partial x}=\frac{\partial u}{\partial r}\cos\varphi-\frac{\partial u}{\partial\varphi}\frac{\sin\varphi}{r} \quad &, \quad \frac{\partial v}{\partial x}=\frac{\partial v}{\partial r}\cos\varphi-\frac{\partial v}{\partial\varphi}\frac{\sin\varphi}{r}\\ \frac{\partial u}{\partial y}=\frac{\partial u}{\partial r}\sin\varphi+\frac{\partial u}{\partial\varphi}\frac{\cos\varphi}{r} \quad &, \quad \frac{\partial v}{\partial y}=\frac{\partial v}{\partial r}\sin\varphi+\frac{\partial v}{\partial\varphi}\frac{\cos\varphi}{r}\end{aligned}\right\} \quad . \quad . \quad (7)$$

After execution of the partial differentiations and introduction of the expressions (7) in (5), we get

$$\frac{dE}{dt}=\iiint\eta\left(\frac{d\omega}{dr}\right)^2 r^3\,dr\,d\varphi\,dh \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

According to the theorem just mentioned this function decreases during the evolution of the disc and tends to a finite minimum. It does not finally disappear, as would be the case with a freely rotating body. Uniform angular velocity is never attained, because our nebula has no finite boundary and merges into interstellar matter.

Hence, in the final steady state the variation of (8) is zero, or

$$\delta \iiint \eta \left(\frac{d\omega}{dr}\right)^2 r^3\, dr\, d\varphi\, dh = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

As the motion is cylinderwise, we may take for the closed surface over which the integration is to be extended any ring bounded by two planes parallel to the equator and two cylinders coaxial with the disc. Hence

$$\delta \int \eta \left(\frac{d\omega}{dr}\right)^2 r^3\, dr = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

whereas the integral is indefinite. If follows that

$$\delta \eta \left(\frac{d\omega}{dr}\right)^2 r^3 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

However, the steady state of the disc may as well be characterized by the equation

$$\frac{d}{dr} \left\{ \eta r^3 \frac{d\omega}{dr} \right\} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

which was developed in a previous paper [1]). Hence

$$\eta r^3 \frac{d\omega}{dr} = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and

$$\delta \left(\frac{d\omega}{dr}\right) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Let us now suppose, as was always done, that the pressure gradient of the gas is small in comparison with the attraction of the sun. It then follows from (1) that (14) should hold, when ω is varied from

$$\omega = \left(\frac{fM}{r^3}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

to

$$\omega = \left(\frac{fM}{r^3} + \frac{RT}{r} \frac{d\, lg\, \varrho_e}{dr}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

[1]) These Proceedings **37**, 221 (1934).

Substituting (16) and (15) in (14), we get the following equation

$$\frac{\left(\frac{-3fM}{r^4}-\frac{RT}{r^2}\frac{d\,lg\,\varrho_e}{dr}+\frac{RT}{r}\frac{d^2\,lg\,\varrho_e}{dr^2}\right)}{\left(\frac{fM}{r^3}+\frac{RT}{r}\frac{d\,lg\,\varrho_e}{dr}\right)^{\frac{1}{2}}}=\frac{\left(-\frac{3fM}{r^4}\right)}{\left(\frac{fM}{r^3}\right)^{\frac{1}{2}}} \quad . \quad . \quad (17)$$

As far as squares and products of small quantities may be neglected (17) reduces to

$$\frac{d\,lg\,\varrho_e}{dr}+2r\frac{d^2\,lg\,\varrho_e}{dr^2}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The solution of this equation is

$$\varrho_e=\varrho_0\,e^{-ar^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

when ϱ_0 and a are integration constants to be determined from observational data.

Hence, the density of the gas in the equatorial plane of the disc in the state immediately preceding the formation of planets will be distributed according to (19). Comparing (19) with (2), we see that our provisional assumption concerning this distribution erred in the power of r only.

Now, (2) was based on the empirical statement that the masses of the planets are such that $lg\,\varrho_e$ is nearly a linear function of r, or

$$lg\,\varrho_e=lg\,\varrho_0-ar \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

From (19) follows the parabolical rule

$$lg\,\varrho_e=lg\,\varrho_0-ar^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

In Fig. 1, which is a reproduction of Fig. 2 in a previous paper [1]), the logarithm of the density which follows from the planetary masses has been represented in function of r. The straight line (dashes) was drawn as a first approximation to the empirical curve showing the fluctuations which were previously discussed. I have now adjusted a parabola (dots) as closely as possible to the same empirical curve.

It immediately appears that the question whether the straight line or the parabola is the best first approximation to the empirical curve is difficult to answer. The straight line functions better near the centre, the parabola better near the periphery, but when theory proves that

[1]) These Proceedings **37**, 221 (1934).

the parabola should be chosen, we can only be pleased that all ambiguity has been removed.

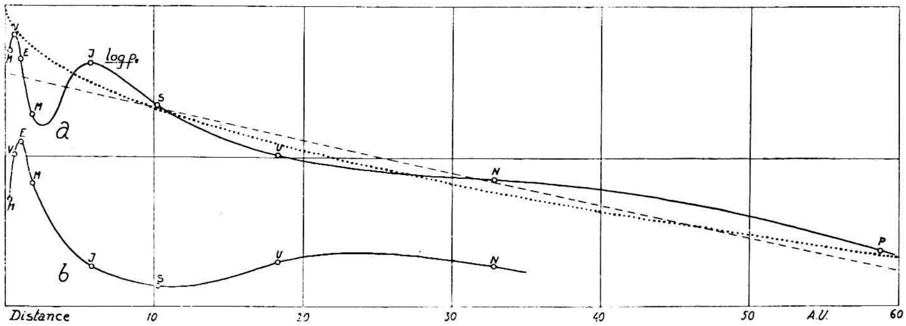

Fig. 1.

a. Empirical (fulldrawn) and theoretical (dotted) curves of the logarithm of the density of the gas in the equatorial plane of the disc; previous provisional approximation by straight line (dashes).
b. Planetary density curve.

There is no need of repeating here previous calculations with the present function ϱ_e. As the method has not changed I only give some results.

From the known total mass of the planets and the known total moment of momentum of the system, the following constants are deduced

$$a = 6.86 \times 10^{-7}\ \mathrm{cm}^{-\frac{1}{2}}$$

$$\varrho_0 = 4.02 \times 10^{-8}\ \mathrm{g/cm^3}.$$

If we do not take account of the fluctuations of the real density curve about its mean theoretical course, we now get the following masses of the planets

	Mass (Earth = 1)	
	According to (19)	Observed
Mercury	0.71	0.03
Venus.	3.24	0.82
Earth	12.7	1.00
Mars	30.5	0.11
Planetoids . . .	87.2	—
Jupiter	133	317
Saturn	118	95
Uranus	51.4	14.8
Neptune	8.6	17.3
Pluto	0.4	0.7

Bearing in mind that the only empirical quantities were the invariable total mass and total moment of momentum of the system, whereas the rest is theory, we may be content with the general agreement.

It may be that the question why the masses of the central planets are all smaller than the computed values, with the most conspicuous discrepancy in Mars, presents a problem which will have to be treated separately. On the other hand there is an essential improvement in the calculated masses of the peripheral planets, especially Neptune and Pluto. This induces me to venture an extrapolation which has now become urgent. Indeed, if we assume as before that the outer radius of the disc corresponds with $\varrho = 10^{-24}$, which is the density of the interstellar matter within the galaxy, we now find that it amounts to 207 Astronomical Units. Hence there would be room for a planet beyond Pluto and as we are now for the first time in the possession of the theoretical mean density curve, it is of importance to calculate its mass. However, it works out to be only 0.003 times the mass of the Earth, or too small to be of any significance, so that our previous conclusion that with Pluto we have reached the outer limit of the planetary system, remains unchanged.

I wish to add one remark, which is perhaps not superfluous. If (19) is inserted in (1), we get

$$\frac{fM}{r^2} - RT\,a r^{\frac{1}{2}} = \omega^2 r \quad . \; . \; . \; . \; . \; . \; . \; . \quad (22)$$

This equation suggests that at a certain finite distance from the sun ω would become zero, but this is only apparent, because the equation only holds as long as the second term on the left side remains considerably smaller than the first. It should not be used beyond its limit of validity.

Batavia, August 14, 1935.