
Physics. — *On the scattering of neutrons in matter.* (II). By Prof. L. S. ORNSTEIN. (Communication from the Physical Institute of the University of Utrecht).

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In this paper we will discuss the frequency law for the energy for the case that the nucleus can be captured by the proton. The distribution law in velocity has been discussed by FERMI in the book offered to ZEEMAN on the 25th of May 1935 (p. 128) for the case of a source, as a problem of diffusion. We will treat the problem of a parallel beam entering in matter of which we follow the individual particles. We assume that at each collision a probability p for capture exists; the probability for reflection being $1—p$, the reflection is assumed to have the character of the collision of rigid spheres.

If we assume that the probability of collision pro unit of time is a, the number of neutrons which at the time has suffered n collisions can be deduced in a way analogous to that of the POISSON—BATEMAN-formula. If N_0 is the number of neutrons at the time $t = 0$, the formula

$$N = N_0\, e^{-\alpha t}$$

represents the number N of neutrons which have suffered no collision.

The number of neutrons which have suffered one collision only can be deduced in the following way.

During the interval $d\xi$ between ξ and $\xi + d\xi$ the number of collisions is given by

$$N_0\, a e^{-\alpha \xi}\, d\xi$$

A fraction $1—p$ of the neutrons which suffered a collision proceeds and therefore the number which suffered only one collision belonging to the chosen group amounts to

$$(1-p)\, N_0\, a\, e^{-\alpha\xi}\, d\xi\, e^{-\alpha(t-\xi)}.$$

Integrating from 0 to t, we find the number of neutrons which during the time t suffered one collision and are not captured. The result is

$$(1-p)\, N_0\, e^{-\alpha t}\, a t.$$

In the same way we find for the number of neutrons which suffer n collisions and are not captured

$$\frac{N_0\, e^{-\alpha t} \{a t (1-p)\}^n}{n!} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (I)$$

Now the probability that the energy which is ε_0 at the time zero is between ε and $\varepsilon + d\varepsilon$ after n collisions (without capture) is given by the function $W_n(\varepsilon)$ introduced in paper (I). Using (I) we get for the probability of an energy ε at the time t:

$$\left.\begin{aligned} P(\varepsilon_0, \varepsilon, t) &= \sum_0^\infty \frac{e^{-\alpha t}}{n!} \{\alpha t (1-p)\}^n (W_n(\varepsilon) \\ \text{or} \qquad & \\ &= e^{-\alpha p t} \sum_0^\infty e^{-\alpha t (1-p)} \{\alpha t (1-p)\}^n W_n(\varepsilon) \end{aligned}\right\} \quad . \quad . \quad . \quad (II)$$

We see, therefore, at once that the solution can be given by a formula analogous to (3) of paper (I). Introducing $at = \nu$, ν being the mean number of collisions during the time t, and $\nu_a = (1-p)\,\nu$ being the mean number of collisions without capture, we get:

$$P(\varepsilon, \nu) = e^{-p\nu} \frac{\nu_a\, e^{-\nu_a}}{\varepsilon_0} \frac{2}{i\,x} J_1(i\,x) \quad . \quad . \quad . \quad . \quad . \quad . \quad (III)$$

where x is given by the relation

$$\nu_a \ln \frac{\varepsilon_0}{\varepsilon} = \frac{x^2}{4}.$$

Instead of (II) we can also write:

$$P\left(\varepsilon, \frac{\nu_a}{1-p}\right) = \frac{\nu_a\, e^{-\frac{\nu_a}{1-p}}}{\varepsilon_0} \frac{2}{i\,x} J_1(i\,x)$$

for $\varepsilon = \varepsilon_0$ we have:

$$P(\varepsilon_0\, \nu) = e^{-\frac{\nu_a}{1-p}}.$$

From relation (II) we can derive a differention-integral equation in the same way as in paper (I). Taking the differential quotient with respect to t and using formula (1) of that paper, we get

$$\frac{\delta P}{\delta t} = -\,\alpha P + \alpha (1-p) \int_\varepsilon^{\varepsilon_0} \frac{d\xi}{\xi} P(\xi, t).$$

Or introducing ν

$$\frac{\delta P}{\delta \nu} + P = (1-p) \int_\varepsilon^{\varepsilon_0} \frac{d\xi}{\xi} P(\xi, t). \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV)$$

For $\varepsilon = \varepsilon_0$ the equation takes the form $\dfrac{\delta P}{\delta \nu} + P = 0$ which has the solution given above.

It is possible to take into account the dependence of p on velocity, which has been neglected in this paper; we hope to do this elsewhere.

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