Physics. — *Separation of substances by flotation.* I. By F. K. TH. VAN ITERSON.

(Communicated at the meeting of January 30, 1937.)

1. *Introduction.*

The specific weight of coal is about 1.4. Put a small piece of coal on the rim of a saucer with water. It floats when you let the water rise. It is interesting to repeat the experiment with paperclips, pinheads and scraps of metal. Materials which float readily are those of metallic, resinous or adamantine lustre; those which readily sink are the vitreous or earthy gangue minerals. But by application of the proper flotation reagents it is possible to separate most solid substances by flotation.

If the needle or the piece of coal is chemically cleansed it sinks. A partial touch of the fingers, the use of sparingly oiled tools in the mines materially effects the floatability of the above mentioned materials, but has no influence on the easy wettability of the gangue minerals and such like substances.

As flotation is an effect of superficial tension, i.e. 0,07 gramme per cm. or less, the method only operates on small particles. Although separate pieces of coal of 4 mm. and larger float, we consider at the State Mines 2 mm. and less a practical coarseness. For the heavy base-metal sulphides 0,5 or 0,3 mm. is taken as a technical limit.

2. *Geometry of flotation* [1]).

It would be useful for a good understanding of the phenomena to make a thorough study of the geometry of bodies heavier than water, floating by the action of superficial tension, but we must confine our exposure to a few instances.

Figure 1 shows that the angle of contact of a free airwater surface with a fatty steel surface is $\vartheta = 90°$. This enables us to construct the surface-curve for the floating needle.

We call the surface tension γ, and use the notations indicated in figure 3.

The equilibrium of the surface-tension with the waterpressure acting on the depth x against a curved surface gives

$$x\, s\, \varrho\, d\varphi = \gamma\, d\varphi$$
$$(s = \text{sp.w.liquid, for water } s = 1)$$

$$\frac{\gamma}{\varrho} = x\, s \quad \frac{1}{\varrho} = \frac{\dfrac{d^2y}{dx^2}}{\pm\left\{1 + \left(\dfrac{dy}{dx}\right)^2\right\}^{3/2}} = \frac{s}{\gamma}\, x.$$

[1]) The reader may pass over this paragraph and start at paragraph 3 but then his insight into the phenomena remains imperfect.

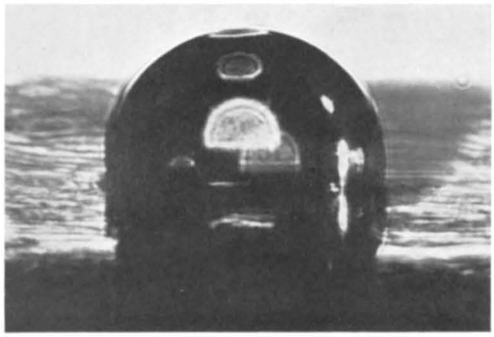

Fig. 1. Enlarged photograph of a bubble of air of 0,5 mm. diameter adhering to a greased steel surface.

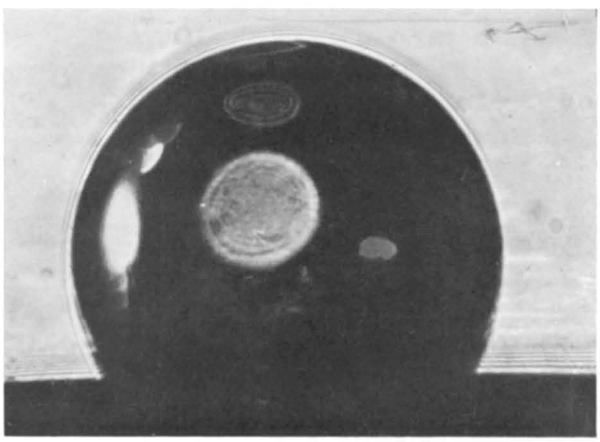

Fig. 6. Enlarged photograph of an air bubble adhering to an oiled coal-surface. Angle of contact $\vartheta = 60°$. Diameter of bubble $^1/_3$ mm.

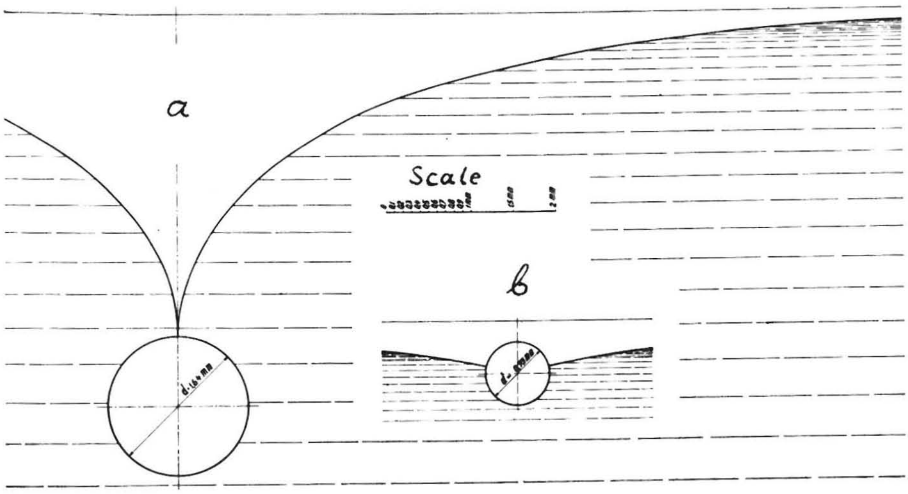

Fig. 2. Needles floating in water.
a. (Ultimate case, supposing that both wings remain separated. Angle of contact $\vartheta = 90°$, superficial tension $\gamma = 0{,}0722$ gramme per cm. Specific weight steel 7.8.
b. Normal case, diameter needle 0,75 mm.)

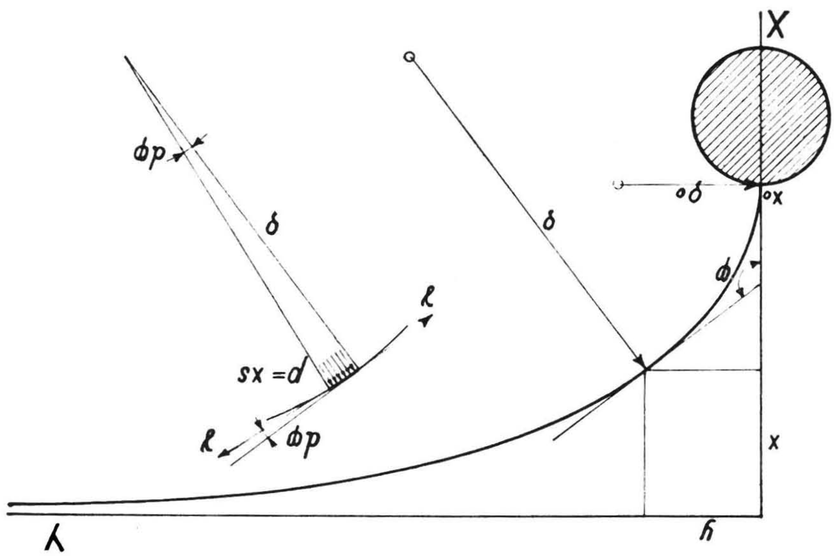

Fig. 3. Notations for the mathematical problem of the floating needle.

Substitute

$$\frac{dy}{dx} = tg\,\varphi = \frac{sin\,\varphi}{cos\,\varphi} \quad \frac{d^2y}{dx^2} = \frac{1}{cos^2\,\varphi}\,\frac{d\varphi}{dx}.$$

Then our differential equation becomes

$$-\cos\varphi\, d\varphi = \frac{s}{\gamma}\, x\, dx.$$

Integrated and taken into consideration that for $x=0$ $\varphi=90°$

$$1-\sin\varphi=\frac{s}{\gamma}\frac{x^2}{2} \qquad \sin\varphi=1-\frac{s}{\gamma}\frac{x^2}{2}.$$

For $\varphi=0$

$$x_0=\sqrt{\frac{2\gamma}{s}} \text{ and as } \varrho=\frac{\gamma}{sx} \quad \varrho_0=\sqrt{\frac{\gamma}{2s}}.$$

We can go on and put $\dfrac{dy}{dx}=tg\,\varphi=\dfrac{\sin\varphi}{\sqrt{1-\sin^2\varphi}}$

$$\frac{dy}{dx}=\frac{1-\frac{s}{\gamma}\frac{x^2}{2}}{\sqrt{\frac{s}{\gamma}x^2-\frac{s^2}{\gamma^2}\frac{x^4}{4}}}.$$

Which gives an elliptic integral for y

But more instructive and easier than solving this integral with a table is to construct the curve by graphical integration according to SOLIN. The surface curve figure 2 is obtained in this way. It is simply the curve for which the radius of curvature varies reciprocally with the depth x.

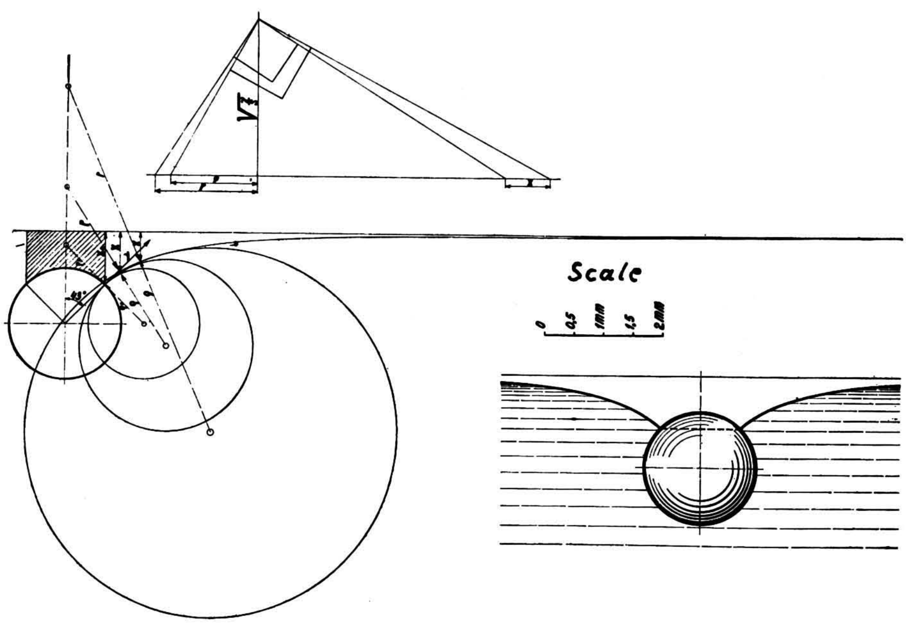

Fig. 4. Graphical construction depression in water-surface for a floating small fatty ball.
Specific weight steel 7.8.
Angle of contact $\vartheta=90°$.
Superficial tension $\gamma=0.0722$ gramme per cm.
Diameter of largest floating ball $=0{,}18$ cm.

Differential equation for water surface $\dfrac{1}{\varrho}-\dfrac{1}{r}=\dfrac{s}{\gamma}x$.

$s=$ specific weight liquid $=1$.

The aggregate displacement of a floating body and the depression in the water-surface are equal to the weight of the body.

We now proceed to the problem of floating bodies with circular horizontal section; i.e. small balls.

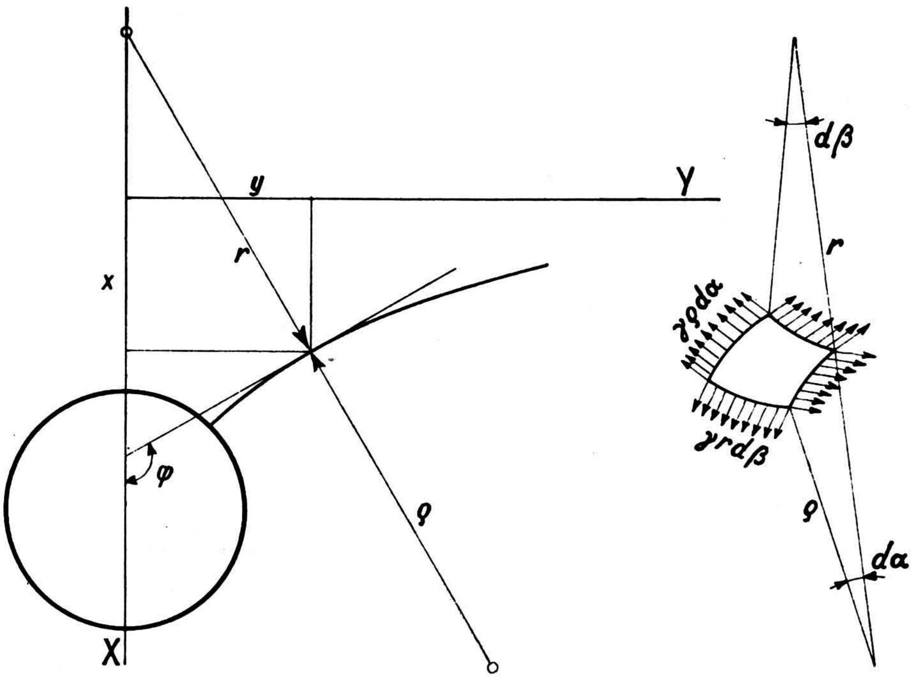

Fig. 5. Notations for the mathematical problem of the floating small steel ball.

The hydrostatic pressure $p = sx$ on a double curved elemental surface of the sides $\varrho d\alpha$ and $r d\beta$ is in equilibrium with the surface tension acting at the sides.

$$s x \varrho \, d\alpha \, r \, d\beta = \gamma r \, d\beta \, d\alpha - \gamma \varrho \, d\alpha \, d\beta$$

$\frac{1}{\varrho} - \frac{1}{r} = \frac{s}{\gamma} x$. This is the condition for the surface.

The radius of curvature of the curve of revolution in the plane of our paper being

$$\frac{1}{\varrho} = \frac{\frac{d^2y}{dx^2}}{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}.$$

The radius of curvature of a section at right angle with the plane of our paper is the descriptive line of r of the cone touching the circle with radius ϱ as this circle moves outward at a right angle with the plane of our paper.

Now

$$\frac{1}{r} = \frac{\cos\varphi}{y} = \frac{1}{y\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{1/2}}.$$

Substituting $\frac{1}{\varrho}$ and $\frac{1}{r}$ in the formula for the condition of the water surface gives us the differential equation of the descriptive curve for the depression in the water surface.

$$y\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\} - \frac{x\,y}{a^2}\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}.$$

This equation, however, is insoluble.

If we call $\frac{\gamma}{s} = \frac{\text{superficial tension}}{\text{density of the liquid}} = a^2$ our fundamental equation is written

$$\frac{a^2}{\varrho} - \frac{a^2}{r} = x.$$

Much easier and more exact than solving the differential equation by approximations is to build up the curve by parts of circles as indicated in figure 4. If we adopt the depth x_0, then we know the radius ϱ_0 at the contact with the ball and also that the water-level is an asymptote. We extend by using a draughtsmans curve; then test in a point with the fundamental equation whether the curvature is right and if not, we correct. By this graphical method all the problems of the hanging or lying drops are readily solved.

The maximum diameter of the floating ball, the centri-angle of the waterline (43°), the initial radius of curvature ϱ_0 and the depth of the water-line x_0 are found by trying successive diameters and depths x.

The upward pull of the superficial tension γ on the ball plus the volume of the dotted body in figure 4 are in equilibrium with the weight of the submerged ball.

At the same time with the curve of depression in the water surface we obtain the evolute or path of the centres of the curve.

Figure 6 is a photograph of an air-bubble adhering to the flat surface of an oiled piece of coal under water. The angle of capillarity or angle of contact ϑ is about 60°.

Of practical importance is the answer to the question (see figure 7): When the angle of contact ϑ is known, for which center angle φ of the waterline of a floating ball, the upward pull P of the surface-tension reaches its maximum?

$$P = 2\pi r \sin\varphi\,\gamma\cos(90^\circ + \varphi - \vartheta)$$
$$P = 2\pi r\,\gamma\sin\varphi\sin(\vartheta - \varphi).$$

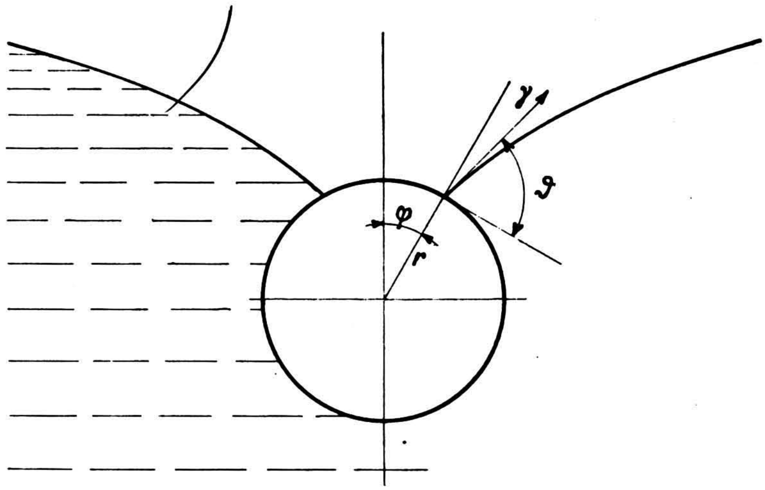

Fig. 7. Notations for calculating the maximum upward pull of surface tension acting on a floating ball.

P is maximum for $\frac{dP}{d\varphi} = 0$ gives $\varphi = \frac{1}{2}\vartheta$.

For coal $\varphi = \frac{60^\circ}{2} = 30^\circ$.

$\varphi = \frac{1}{2}\vartheta$ substituted in the formula for P gives

$$P_{max.} = \pi r \gamma (1 - \cos\vartheta)$$

(for coal $P_{max.} = \pi r \gamma (1 - \frac{1}{2}\sqrt{3})$).

Which angle of contact ϑ gives the greatest value to $P_{max.}$?

This is $\vartheta = 180^\circ$ $_{max.}P_{max.} = 2\pi r \gamma$.

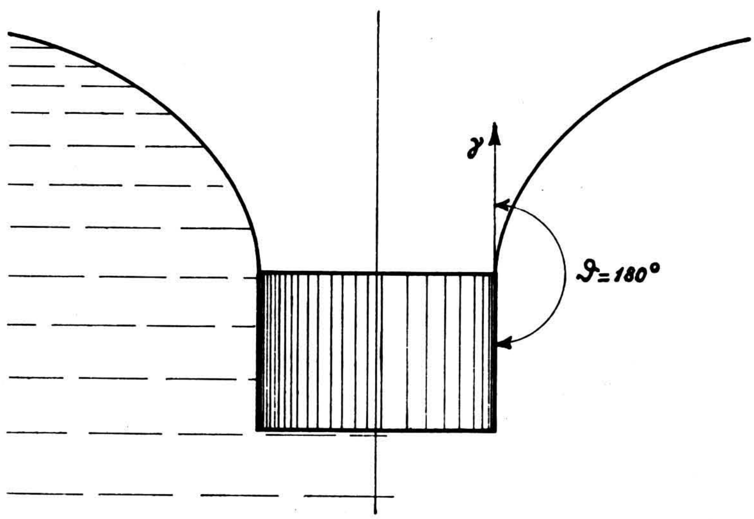

Fig. 8. Floating cylinder, or cube, or flat piece for angle of contact $\vartheta = 180^\circ$.

Figure 8 represents the case of a floating piece of material when the angle of contact $\vartheta = 180°$. This angle never occurs in practice.

The practical conclusion of our calculation, demonstrated by the figures is: The greater the angle of contact ϑ, the better the floatability of the material

The reverse also holds true: The smaller the angle of contact ϑ the greater the tendency to sink.

No material with an angle of contact $\vartheta = 0°$ i.e. completely wettable, can float by the action of surface tension.

Small particles will float even with small angles of contact.

In order to separate substances by flotation, the angle of contact of one of them must be zero. This means that this substance must be completely wettable like clean glass. Then it will sink. If for the other material the angle of contact is small and the specific weight is high then it must be finely ground in order that it should float.

(To be continued.)

Mathematics. — *Zur Differentialgeometrie der Gruppe der Berührungstransformationen. I. Doppelthomogene Behandlung von Berührungstransformationen.* Von J. A. SCHOUTEN.

(Communicated at the meeting of January 30, 1937).

I. *Einleitung.*

Bekanntlich hat LIE [1]) gezeigt, dass man eine allgemeine Berührungstransformation in den $2n-1$ Variablen $\xi^1, \ldots, \xi^n, \zeta_2, \ldots, \zeta_n$ schreiben kann als „homogene" Berührungstransformation in den $2n$ Variablen $\xi^1, \ldots, \xi^n, \eta_1, \ldots, \eta_n$, wo $\zeta_2 = -\eta_2/\eta_1$ u.s.w. Mit homogen ist hier gemeint, dass die transformierten ξ und η homogene Funktionen nullten bzw. ersten Grades in den alten η sind. Nun zeigen die Formeln der also entstandenen homogenen Berührungstransformation eine merkwürdige Neigung zur Dualität zwischen ξ und η, die sich aber nicht vollständig entfalten kann, schon aus dem einfachen Grunde weil ja die transformierten ξ und η keineswegs homogene Funktionen in den alten ξ sind. Es erhebt sich da die Frage ob sich die Koordinaten nicht noch anders wählen lassen und zwar so, dass eine in jeder Beziehung vollständige Dualität zu Tage tritt. Diese Frage lässt sich bejahend beantworten. Wählen wir statt der gewöhnlichen Punktkoordinaten ξ^h die aus der Geometrie der H_n [2]) bekannten VAN DANTZIG'schen homogenen Punkt-

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1) S. LIE, Theorie der Transformationsgruppen, Bd. II, S. 139 u.f.

2) D. v. DANTZIG, Theorie des projektiven Zusammenhangs n-dimensionaler Räume, Math. Ann. **106**, 400—454 (1932); J. A. SCHOUTEN und J. HAANTJES, Zur allgemeinen projektiven Differentialgeometrie, Comp. Math. **3**, 1—51 (1936), daselbst auch weitere Litteratur.