gefunden werden. Wegen der Identität

$$\sigma - \frac{P}{Q} = \left(\frac{P_s}{Q_s} - \frac{P}{Q} \right) + R_s$$

ist alsdann entweder $\frac{P_s}{Q_s} = \frac{P}{Q}$ und also

$$\sigma - \frac{P}{Q} = R_s,$$

oder $\frac{P_s}{Q_s} \neq \frac{P}{Q}$, also $|P_s Q - P Q_s| \geqslant 1$ und somit wegen (9):

$$\left| \sigma - \frac{P}{Q} \right| \geqslant \frac{1}{Q Q_s} - |R_s| \geqslant \frac{1}{2 Q Q_s} \geqslant \frac{1}{2} |R_s|,$$

so dass in jedem Fall

$$\left| \sigma - \frac{P}{Q} \right| \geqslant \frac{1}{2} |R_s| \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

folgt.

Wenn nun aber Q und also auch s genügend gross ist, so gilt wegen (9) und (7)

$$\frac{1}{Q} \leqslant q^{-\frac{2}{3} \alpha s (q^{1/m} - 1) q^{\frac{s-2}{m}}},$$

und wegen (10) und (4)

$$\left| \sigma - \frac{P}{Q} \right| \geqslant q^{-\frac{4}{3} \alpha s q^{\frac{s}{m}}},$$

also erst recht

$$(B): \qquad \left| \sigma - \frac{P}{Q} \right| \geqslant Q^{-\frac{2 q^{2/m}}{q^{1/m} - 1}}.$$

Damit ist bewiesen:

Satz 2: *Alle Näherungsbrüche von σ mit genügend grossem Nenner genügen der Ungleichung (B). Die Zahl σ ist also Nicht-Liouvillesch.*

Herrn Dr. med. A. HEILBRONN gewidmet.

Krefeld, März 1937.

Mathematics. — *Two remarks on* VAN DER CORPUT's *generalisation of* KNOPP's *inequality.* By V. LEVIN. (Communicated by J. G. VAN DER CORPUT).

(Communicated at the meeting of April 24, 1937).

1. Let $a_n \geqq o$ (not all $a_n = o$), $0 < u < 1$, $q > 0$, $0 < p \leqq 1$. Then

$$F(u) = \sum_{n=0}^{\infty} u^n \left(a_0^{\binom{n}{0} q^n} a_1^{\binom{n}{1} q^{n-1}} \ldots a_n^{\binom{n}{n}} \right)^{\frac{p}{(q+1)^n}} \leqq \frac{q\lambda + 1}{(1-\lambda)^{1-p}} \left\{ \sum_{n=0}^{\infty} a_n \right\}^p . \quad (1)$$

where $0 < \lambda = \lambda(u, p, q) < 1$ is uniquely determined by

$$u = \frac{(q+1) \lambda^{1 - \frac{p}{q+1}}}{q\lambda + 1} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1, 1)$$

The sign of equality in (1) holds for $a_n = c\lambda^n$ $(c > 0)$. VAN DER CORPUT's inequality [1]) is (1) with $p = 1$, in which case $u = 1$ is admitted (KNOPP's inequality). But for $p = 1$, $u = 1$ equality cannot occur in (1), the constant $q + 1$ remaining, however, the best possible.

The proof of (1) runs as follows:

$$F(u) = \sum_{n=0}^{\infty} u^n \lambda^{\frac{np}{q+1}} \left(\left(a_0^p \lambda^{-p.0}\right)^{\binom{n}{0} q^n} \left(a_1^p \lambda^{-p.1}\right)^{\binom{n}{1} q^{n-1}} \ldots \left(a_n^p \lambda^{-pn}\right)^{\binom{n}{n}} \right)^{\frac{1}{(q+1)^n}}$$

$$\leqq \sum_{n=0}^{\infty} u^n \lambda^{\frac{np}{q+1}} \frac{1}{(q+1)^n} \sum_{m=0}^{n} \binom{n}{m} q^{n-m} a_m^p \lambda^{-mp}$$

$$= \sum_{m=0}^{\infty} a_m^p \left(\frac{u \lambda^{-\frac{pq}{q+1}}}{q+1} \right)^m \sum_{n=m}^{\infty} \binom{n}{m} \left(\frac{q u \lambda^{\frac{p}{q+1}}}{q+1} \right)^{n-m}$$

$$= \sum_{m=0}^{\infty} a_m^p \left(\frac{u \lambda^{-\frac{pq}{q+1}}}{q+1} \right)^m \left(1 - \frac{q u \lambda^{\frac{p}{q+1}}}{q+1} \right)^{-m-1}$$

$$= (q\lambda + 1) \sum_{m=0}^{\infty} a_m^p \lambda^{m(1-p)}$$

---

1) "Generalisation of an inequality by KNOPP", these Proceedings **39** (1936), 1053–1055.

by (1, 1). Hence

$$F(u) \leqq (q\lambda+1)\left\{\sum_{n=0}^{\infty} a_n\right\}^p \left\{\sum_{n=0}^{\infty} \lambda^n\right\}^{1-p} = \frac{q\lambda+1}{(1-\lambda)^{1-p}}\left\{\sum_{n=0}^{\infty} a_n\right\}^p.$$

2. From his inequality VAN DER CORPUT[2]) deduces the following:

$$\sum_{n=0}^{\infty} \frac{1}{n+1}\left(a_0^{\binom{n}{0}} a_1^{\binom{n}{1}} \cdots a_n^{\binom{n}{n}}\right)^{\frac{1}{2^n}} < (\pi-2)\sum_{n=0}^{\infty} a_n,$$

where $(\pi-2)$ is not the best possible constant [3]). We prove

$$\sum_{n=0}^{\infty} \frac{1}{n+1}\left(a_0^{\binom{n}{0} q^n} a_1^{\binom{n}{1} q^{n-1}} \cdots a_n^{\binom{n}{n}}\right)^{\frac{1}{(q+1)^n}} < \sum_{n=0}^{\infty} \frac{n+2+q}{(n+1)(n+2)} a_n, \quad . \quad (2)$$

where the factor 1 of the right hand side is the best possible one. Also

$$\sum_{n=0}^{\infty} \frac{1}{n+1}\left(a_0^{\binom{n}{0} q^n} a_1^{\binom{n}{1} q^{n-1}} \cdots a_n^{\binom{n}{n}}\right)^{\frac{1}{(q+1)^n}} < \frac{q+1}{q}\log(q+1)\sum_{n=0}^{\infty}\frac{a_n}{n+1} \quad (2.\ 1)$$

but here $\frac{q+1}{q}\log(q+1)$ is not the best possible constant [4]).

Both (2) and (2. 1) reduce to identities for $q \to 0$.

Proof of (2):

$$\left.\begin{aligned}
&\sum_{n=0}^{\infty} \frac{1}{n+1}\left(a_0^{\binom{n}{0} q^n} a_1^{\binom{n}{1} q^{n-1}} \cdots a_n^{\binom{n}{n}}\right)^{\frac{1}{(q+1)^n}} \\
&\leqq \sum_{n=0}^{\infty} \frac{1}{n+1}\frac{1}{(q+1)^n}\sum_{m=0}^{n}\binom{n}{m} q^{n-m} a_m = \sum_{m=0}^{\infty} a_m q^{-m} \sum_{n=m}^{\infty}\frac{1}{n+1}\binom{n}{m}\left(\frac{q}{q+1}\right)^n \\
&= (q+1)\sum_{m=0}^{\infty} a_m \int_0^1 \frac{t^m}{1+qt}\,dt < \sum_{m=0}^{\infty} a_m \int_0^1 t^m (q+1-qt)\,dt \\
&= \sum_{m=0}^{\infty} \frac{m+2+q}{(m+1)(m+2)} a_m.
\end{aligned}\right\} \quad (2.\ 2)$$

2) Loc. cit.

3) The best possible constant in this inequality lies thus between $\underset{0<x<1}{Max}\ \frac{1-x^2}{x}\log\frac{1}{1-x} = 1.082\ldots$ and $\pi - 2 = 1.141\ldots$. The lower limit is obtained by taking $a_n = x^{2n} (0<x<1)$.

4) The best possible constant in (2.1) for $q=1$ lies between $\underset{0<x<1}{Max}\ x\frac{\log\frac{1}{1-x}}{\log\frac{1}{1-x^2}} = 1.260..$ and $2\log 2 = 1.386\ldots$, the lower limit being obtained by taking $a_n = x^{2n}$.

To prove that (2) is a sharp inequality it is sufficient to take $a_n = x^n$ $(0 < x < 1)$ and let $x \to 1$.

The proof of (2. 1) proceeds from (2. 2) as follows:

$$\sum_{n=0}^{\infty} \frac{1}{n+1}\left(a_0^{\binom{n}{0} q^n} a_1^{\binom{n}{1} q^{n-1}} \cdots a_n^{\binom{n}{n}}\right)^{\frac{1}{(q+1)^n}} \leqq (q+1)\sum_{m=0}^{\infty} a_m \int_0^1 \frac{t^m}{1+qt}\,dt$$

$$= (q+1)\sum_{m=0}^{\infty}\frac{a_m}{m+1}\left(\frac{1}{q+1} + q\int_0^1 \frac{t^{m+1}}{(1+qt)^2}\,dt\right)$$

$$< (q+1)\left(\frac{1}{q+1} + q\int_0^1 \frac{t}{(1+qt)^2}\,dt\right)\sum_{m=0}^{\infty}\frac{a_m}{m+1} = \frac{q+1}{q}\log(q+1)\sum_{m=0}^{\infty}\frac{a_m}{m+1}.$$

Botany. — *On the influence of aggregation on the transport of asparagine and caffeine in the tentacles of Drosera capensis. L.* By W. H. ARISZ and J. OUDMAN. (Communicated by Prof. J. C. SCHOUTE).

(Communicated at the meeting of April 24, 1937).

§ 1. *Introduction.*

Most of the investigators who have occupied themselves with the carnivorous nutrition of Drosera are of the opinion that the tentacles play a part in the absorption by the leaf of the animal substances split by enzymes. (For literature see OUDMAN). MANGENOT also assumes a transport through the tentacles, and considers that aggregation has something to do with this. HOMÈS sees more connection between aggregation and secretion. More especially the phenomena of aggregation and secretion point to the substances being taken up by the tentacles. SCHMID and RUSCHMANN demonstrated that phosphorus and potassium were present in the tentacles after nutrition, whilst prior to it they were not so, or only in small quantities. A. KOK investigated the absorption of caffeine by the tentacles and the rapidity with which this substance penetrated from which she concluded that this process is a diffusion process. OUDMAN investigated the absorption of asparagine and caffeine quantitatively by means of the Micro-Kjeldahl method. His experiments showed that if agar with asparagine or caffeine were placed carefully on the marginal tentacles, the leaf a few hours later has become richer in nitrogen. In 24 hours with equimolecular solutions about 1½ times as much caffeine was taken up as asparagine. OUDMAN (l. c. p. 405) discusses the possibility of this being connected with the pathway of the movement of both substances, that for caffeine being more especially the vacuole and for